OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4771

Decision Mathematics 1

Thursday **15 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Questions **1** and **6**.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 7 printed pages, 1 blank page and an insert.

HN/4

Section A (24 marks)

1 Answer this question on the insert provided.

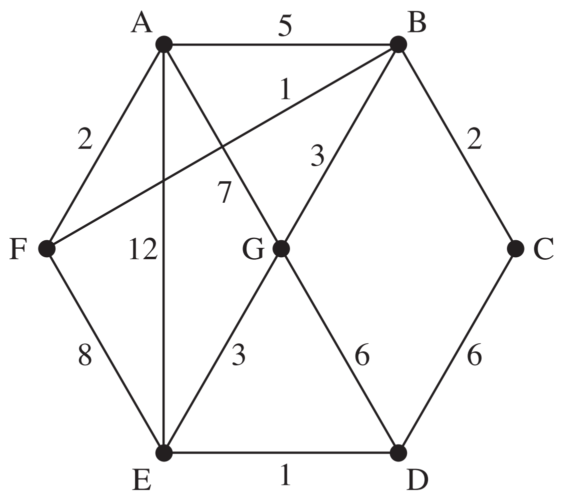


Fig. 1

(i) Apply Dijkstra's algorithm to the copy of Fig. 1 in the insert to find the least weight route from A to D.

Give your route and its weight. [6]

(ii) Arc DE is now deleted. Write down the weight of the new least weight route from A to D, and explain how your working in part **(i)** shows that it is the least weight. [2]

2 Fig. 2.1 represents the two floors of a house. There are 5 rooms shown, plus a hall and a landing, which are to be regarded as separate rooms. Each "×" represents an internal doorway connecting two rooms. The "⊗" represents the staircase, connecting the hall and the landing.

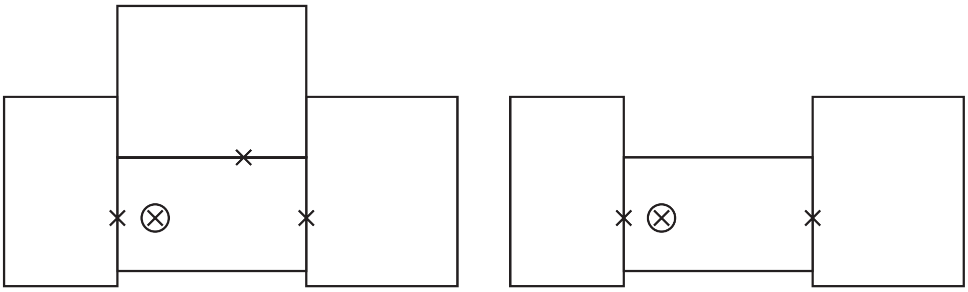


Fig. 2.1

(i) Draw a graph representing this information, with vertices representing rooms, and arcs representing internal connections (doorways and the stairs).

What is the name of the type of graph of which this is an example? [3]

(ii) A larger house has 12 rooms on two floors, plus a hall and a landing. Each ground floor room has a single door, which leads to the hall. Each first floor room has a single door, which leads to the landing. There is a single staircase connecting the hall and the landing.

How many arcs are there in the graph of this house? [1]

(iii) Another house has 12 rooms on three floors, plus a hall, a first floor landing and a second floor landing. Again, each room has a single door on to the hall or a landing. There is one staircase from the hall to the first floor landing, and another staircase joining the two landings.

How many arcs are there in the graph of this house? [1]

(iv) Fig. 2.2 shows the graph of another two-floor house. It has 8 rooms plus a hall and a landing. There is a single staircase.

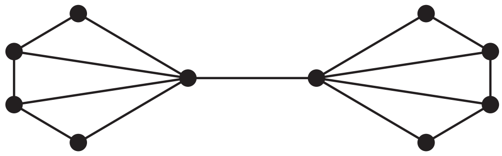


Fig. 2.2

Draw a possible floor plan, showing internal connections. [3]

3 An incomplete algorithm is specified in Fig. 3.

$f(x) = x^2 - 2$
Initial values: $L = 0, R = 2$.
Step 1 Compute $M = \dfrac{L+R}{2}$.
Step 2 Compute $f(M)$.
Step 3 If $f(M) < 0$ change the value of L to that of M.
Otherwise change the value of R to that of M.
Step 4 Go to Step 1.

Fig. 3

(i) Apply two iterations of the algorithm. [6]

(ii) After 10 iterations $L = 1.414063, R = 1.416016, M = 1.416016$ and $f(M) = 0.005100$.

Say what the algorithm achieves. [1]

(iii) Say what is needed to complete the algorithm. [1]

Section B (48 marks)

4 Table 4.1 shows some of the activities involved in preparing for a meeting.

	Activity	Duration (hours)	Immediate predecessors
A	Agree date	1	–
B	Construct agenda	0.5	–
C	Book venue	0.25	A
D	Order refreshments	0.25	C
E	Inform participants	0.5	B, C

Table 4.1

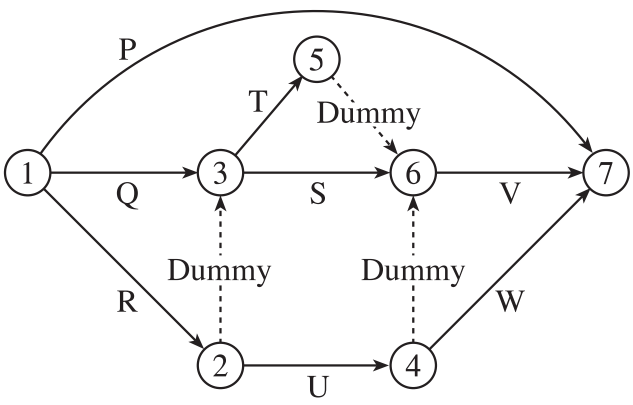
(i) Draw an activity-on-arc network to represent the precedences. [4]

(ii) Find the early event time and the late event time for each vertex of your network, and list the critical activities. [3]

(iii) Assuming that each activity requires one person and that each activity starts at its earliest start time, draw a resource histogram. [2]

(iv) In fact although activity A has duration 1 hour, it actually involves only 0.5 hours work, since 0.5 hours involves waiting for replies. Given this information, and the fact that there is only one person available to do the work, what is the shortest time needed to prepare for the meeting? [2]

Fig. 4.2 shows an activity network for the tasks which have to be completed after the meeting.



P: Clean room

Q: Prepare draft minutes

R: Allocate action tasks

S: Circulate draft minutes

T: Approve task allocations

U: Obtain budgets for tasks

V: Post minutes

W: Pay refreshments bill

Fig. 4.2

(v) Draw a precedence table for these activities. [5]

[Turn over

5 John is reviewing his lifestyle, and in particular his leisure commitments. He enjoys badminton and squash, but is not sure whether he should persist with one or both. Both cost money and both take time.

Playing badminton costs £3 per hour and playing squash costs £4 per hour. John has £11 per week to spend on these activities.

John takes 0.5 hours to recover from every hour of badminton and 0.75 hours to recover from every hour of squash. He has 5 hours in total available per week to play and recover.

(i) Define appropriate variables and formulate two inequalities to model John's constraints. [3]

(ii) Draw a graph to represent your inequalities.

Give the coordinates of the vertices of your feasible region. [6]

(iii) John is not sure how to define an objective function for his problem, but he says that he likes squash "twice as much" as badminton. Letting every hour of badminton be worth one "satisfaction point" define an objective function for John's problem, taking into account his "twice as much" statement. [1]

(iv) Solve the resulting LP problem. [2]

(v) Given that badminton and squash courts are charged by the hour, explain why the solution to the LP is not a feasible solution to John's practical problem. Give the best feasible solution. [2]

(vi) If instead John had said that he liked badminton more than squash, what would have been his best feasible solution? [2]

6 Answer parts (ii)(*A*) and (iii)(*B*) of this question on the insert provided.

A particular component of a machine sometimes fails. The probability of failure depends on the age of the component, as shown in Table 6.

Year of life	first	second	third	fourth	fifth	sixth
Probability of failure during year, given no earlier failure	0.10	0.05	0.02	0.20	0.20	0.30

Table 6

You are to simulate six years of machine operation to estimate the probability of the component failing during that time. This will involve you using six 2-digit random numbers, one for each year.

(i) Give a rule for using a 2-digit random number to simulate failure of the component in its first year of life.

Similarly give rules for simulating failure during each of years 2 to 6. [3]

(ii) (*A*) Use your rules, together with the random numbers given in the insert, to complete the simulation table in the insert. This simulates 10 repetitions of six years operation of the machine. Start in the first column working down cell-by-cell. In each cell enter a tick if there is no simulated failure and a cross if there is a simulated failure.

Stop and move on to the next column if a failure occurs. [5]

(*B*) Use your results to estimate the probability of a failure occurring. [1]

It is suggested that any component that has not failed during the first three years of its life should automatically be replaced.

(iii) (*A*) Describe how to simulate the operation of this policy. [2]

(*B*) Use the table in the insert to simulate 10 repetitions of the application of this policy. Re-use the same random numbers that are given in the insert. [3]

(*C*) Use your results to estimate the probability of a failure occurring. [1]

(iv) How might the reliability of your estimates in parts **(ii)** and **(iii)** be improved? [1]

Candidate Name	Centre Number	Candidate Number

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MEI STRUCTURED MATHEMATICS 4771

Decision Mathematics 1

INSERT

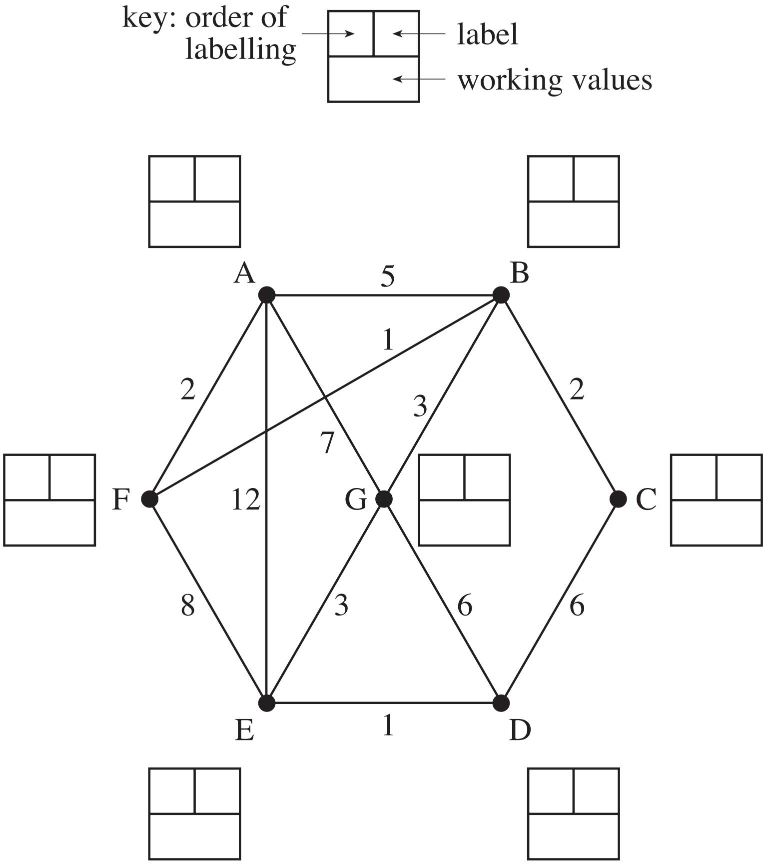
Thursday **15 JUNE 2006** Afternoon 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- This insert should be used in Questions **1** and **6**.
- Write your name, centre number and candidate number in the spaces provided at the top of this page and attach it to your answer booklet.

This insert consists of 4 printed pages.

1 **(i)**



Least weight route: ______________________________

Weight ______________________________

(ii) ______________________________

____

____

6 (ii) (*A*) and (iii)(*B*)

Random numbers

Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
60	16	11	59	00	77	46	02	77	59
95	41	89	82	25	19	43	62	08	13
19	51	76	22	65	18	06	86	69	84
25	81	32	13	15	71	57	20	08	23
06	65	23	37	25	69	97	82	52	60
82	32	03	45	22	50	87	72	08	84

(ii) (*A*)

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
Year 1										
Year 2										
Year 3										
Year 4										
Year 5										
Year 6										

[Question 6 (iii)(*B*) is printed overleaf.]

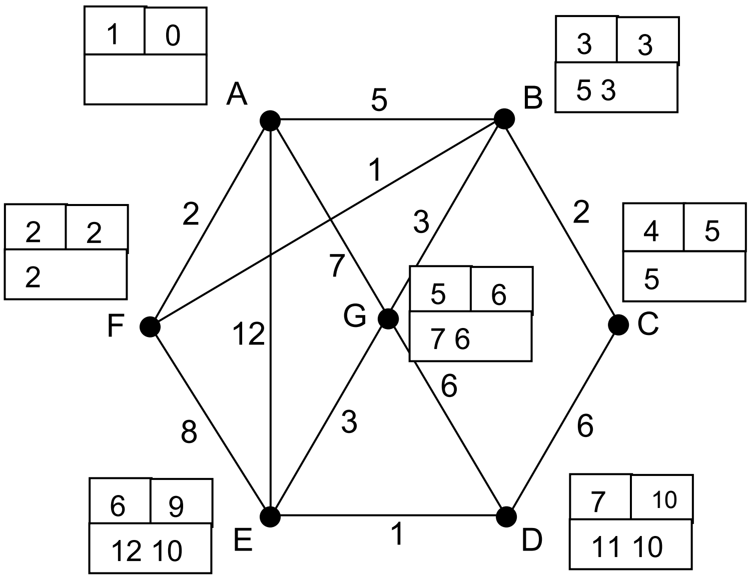
Random numbers

Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
60	16	11	59	00	77	46	02	77	59
95	41	89	82	25	19	43	62	08	13
19	51	76	22	65	18	06	86	69	84
25	81	32	13	15	71	57	20	08	23
06	65	23	37	25	69	97	82	52	60
82	32	03	45	22	50	87	72	08	84

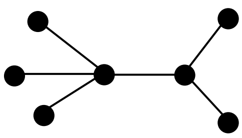
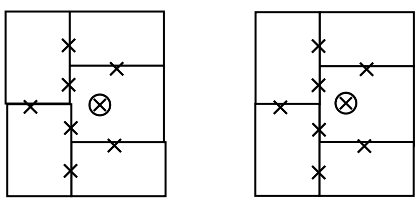
(iii) (*B*)

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
Year 1										
Year 2										
Year 3										
Year 4										
Year 5										
Year 6										

Mark Scheme 4771
June 2006

(i) 	M1 sca Dijkstra A1 labels A1 order of labelling A1 working values
Least weight route: A F B G E D Weight = 10	B1 B1
(ii) 11 From working value. Can't be bettered since new least weight must be bigger than 10.	B1 B1

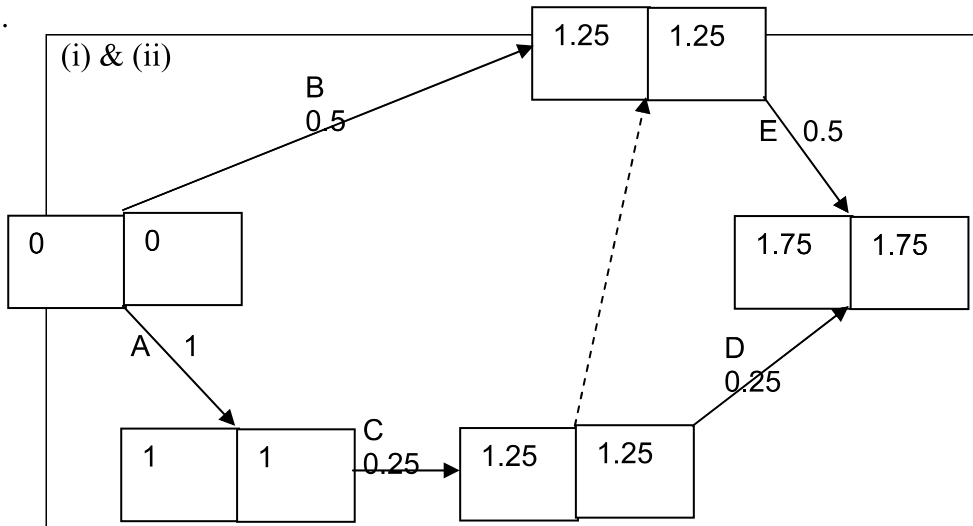
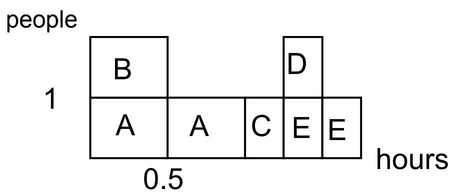
2.

(i) e.g. 	M1 A1
a tree	B1
(ii) 13	B1
(iii) 14	B1
(iv) e.g. 	M1 A1 A1

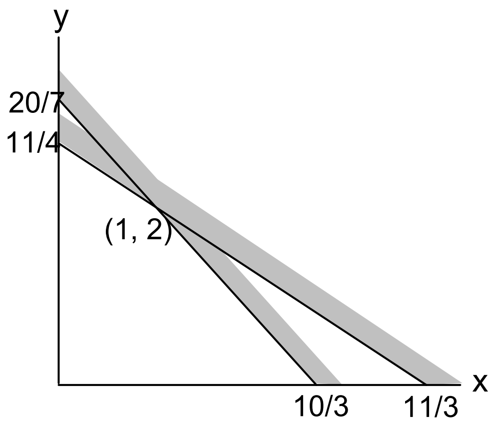
3.

(i) M = 1 f(M) = –1 L = 1 M = 1.5 f(M) = 0.25 R = 1.5	B1 B1 B1 B1 B1 B1
(ii) Solves equations (Allow "Finds root 2".)	B1
(iii) A termination condition	B1

4.

(i) & (ii)  Critical activities: A, C, E	M1 sca activity-on-arc A1 A, B, C A1 D A1 E B1 forward pass (1.25 at end of B/dummy) B1 backward pass (1.25 at start of dummy/D) B1
(iii) 	M1 A1
(iv) 2 hours (resource smoothing on A/B, but extra time needed for D/E).	M1 A1
(v) P – Q – R – S Q, R T Q, R U R V S, T, U W U	 B1 B1 B1 B1 B1

5.

(i) Let x be the number of hours spent at badminton Let y be the number of hours spent at squash	B1
$3x + 4y \le 11$ $1.5x + 1.75y \le 5$	B1 B1
(ii) 	B1 axes labelled and scaled B1 line B1 line B1 shading B1 intercepts B1 (1, 2)
(iii) $x + 2y$	B1
(iv) $22/4 > 5 > 10/3$, so 5.5 at (0, 11/4)	M1 A1
(v) Squash courts sold in whole hours 1 hour badminton and 2 hours squash per week	B1 B1
(vi) 3 hours of badminton and no squash	B1 B1

6.

(i) year 1: 00 – 09 failure, otherwise no failure — M1 A1
year 2: 00 – 04
year 3: 00 – 01
year 4: 00 – 19 — A1
year 5: 00 – 19
year 6: 00 – 29

(ii)(A)

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
year 1	√	√	√	√	x	√	√	x	√	√
year 2	√	√	√	√		√	√		√	√
year 3	√	√	√	√		√	√		√	√
year 4	√	√	√	x		√	√		x	√
		√	√			√	√			√

M1 ticks and crosses
A1 run 1
A1 runs 2–4
A1 runs 5–7
B1 runs 8–10

(B) 0.6 — B1

(iii)
(A) if no failure then continue after year 3 – but using rules for yrs 1 to 3 — B1 B1

(B)

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
year 1	√	√	√	√	x	√	√	x	√	√
year 2	√	√	√	√		√	√		√	√
year 3	√	√	√	√		√	√		√	√
year 4	√	√	√	√		√	√		x	√
	√	√	√	√		√	√			√

M1
A1 runs 1–5
A1 runs 6–10

(C) 0.3 — B1

(iv) more repetitions — B1

4771 - Decision Mathematics 1

General Comments

Performances on parts of this paper were disappointing. In particular many candidates were unable successfully to execute the algorithm in question 3 and many were unable to deal with the simulation in question 6. Conversely the graph question (question 2) and the CPA question (question 4) were both done well.

Comments on Individual Questions

Q 1 **Networks**

(i) A straightforward application of Dijkstra. As always there were some candidates, rather too many on this presentation, who either did not know the algorithm or could not convince the examiners that they knew it.

(ii) The instruction "… explain how your working …" caused difficulties. All that was required was a reference to working values, or to shortest paths to neighbouring vertices.

Q 2 **Graphs**

Candidates scored well on this question. It was pleasing to see this modelling dealt with so successfully.

Q 3 **Algorithms**

This was very badly done.

(i) At step 1 most candidates correctly computed $M = {(0+2)}/{2} = 1$. At step 2 most correctly had $f(M) = 1^2 - 2 = -1$. At step 3 the vast majority incorrectly put L equal to –1 (i.e. f(M)) instead of the correct value of 1 (i.e. M). It is difficult to see why this should have been the case, and it was costly.

(ii) Many candidates seemed to think that an adequate objective for the algorithm was "to make L equal to R". Some others, who had much more idea, thought that it was finding the square root of R.

(iii) All that was required was the observation that the algorithm was missing a stopping condition – candidates did not need to provide such a condition.

Q 4 **CPA**

(i) & (ii) These parts were well done. Again, it was pleasing to see an aspect of modelling being tackled so well.

(iii) Not many candidates knew what a resource histogram is.

(iv) The question was constructed so that candidates would be able to use their answer to part (iii) to help them in part (iv). Some were able to argue through to the correct answer having not succeeded with part (iii).

(v) Many candidates gratefully accepted the 5 marks which were on offer here.

Q 5 **LP**

(i) How **do** we persuade candidates to define their variables properly?
It was anticipated that the time constraint would create problems for some candidates, and indeed many gave 0.5b+0.75s<5 instead of 1.5b+1.75s<5.

(ii) Some very tiny graphs were seen.
Many candidates either overlooked or ignored the instruction to give the coordinates of their feasible region.

(iii) This revealed modelling weaknesses. Many gave "b=2s" or equivalent.

(iv) Too many candidates did not show how they were solving the LP. The instruction in part (ii), that they give the coordinates of their feasible region, was intended to help with this.

(v)&(vi) Many candidates showed weaknesses in interpretation in these parts. Good candidates answered them quickly and efficiently.

Q 6 **Simulation**

(i) The vast majority of candidates clearly did not read the question. They saw a set of probabilities and set off in a knee-jerk routine, simulating a supposed probability distribution with 6 outcomes, despite the fact that the probabilities did not add up to 1, and despite the fact that nothing much made sense thereafter.

(ii) It might have been thought that the provided tables would have helped candidates to understand this question, but that seemed not to be the case. This is well illustrated by the number of candidates who filled every cell with either a tick or a cross, and then gave an "out of 60" probability.

(iii) Very few candidates were able, in their comments, to distinguish between "reality" and the simulation model. Most comments parroted the question in referring to replacing the component, rather than to continuing the simulation.

(iv) A substantial minority of candidates thought that the reliability could be improved by using 3-digit random numbers.