OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4755

Further Concepts For Advanced Mathematics (FP1)

Tuesday **7 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 3 printed pages and 1 blank page.

Section A (36 marks)

1 **(i)** Find the inverse of the matrix $\mathbf{A} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$. [2]

(ii) Use this inverse to solve the simultaneous equations

$$\begin{aligned} 4x + 3y &= 5, \\ x + 2y &= -4, \end{aligned}$$

showing your working clearly. [3]

2 Find the roots of the quadratic equation $x^2 - 8x + 17 = 0$ in the form $a + b\mathrm{j}$.

Express these roots in modulus-argument form. [5]

3 Find the equation of the line of invariant points under the transformation given by the matrix $\mathbf{M} = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$. [3]

4 The quadratic equation $x^2 - 2x + 4 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [1]

(ii) Hence find the value of $\alpha^2 + \beta^2$. [2]

(iii) Find a quadratic equation which has roots 2α and 2β. [2]

5 **(i)** Sketch the locus $|z - (3 + 4\mathrm{j})| = 2$ on an Argand diagram. [2]

(ii) On the same diagram, sketch the locus $\arg(z - 4) = \frac{1}{2}\pi$. [2]

(iii) Indicate clearly on your sketch the points which satisfy **both**

$$|z - (3 + 4\mathrm{j})| = 2 \quad \textbf{and} \quad \arg(z - 4) = \tfrac{1}{2}\pi.$$ [1]

6 Prove by induction that $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n + 1)^2$. [7]

7 Find $\sum_{r=1}^{n} 3r(r - 1)$, expressing your answer in a fully factorised form. [6]

Section B (36 marks)

8 A curve has equation $y = \dfrac{x^2 - 4}{(3x-2)^2}$.

(i) Find the equations of the asymptotes. [2]

(ii) Describe the behaviour of the curve for large positive and large negative values of x, justifying your description. [3]

(iii) Sketch the curve. [5]

(iv) Solve the inequality $\dfrac{x^2 - 4}{(3x-2)^2} \geqslant -1$. [4]

9 The quartic equation $x^4 + Ax^3 + Bx^2 + Cx + D = 0$, where A, B, C and D are real numbers, has roots $2 + \mathrm{j}$ and $-2\mathrm{j}$.

(i) Write down the other roots of the equation. [2]

(ii) Find the values of A, B, C and D. [8]

10 (i) You are given that

$$\frac{2}{r(r+1)(r+2)} = \frac{1}{r} - \frac{2}{r+1} + \frac{1}{r+2}.$$

Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}.$$ [9]

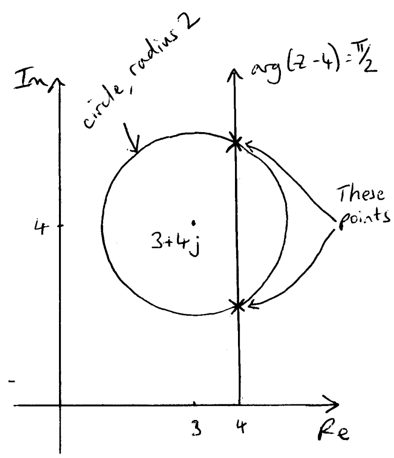
(ii) Hence find the sum of the infinite series

$$\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \dots .$$ [3]

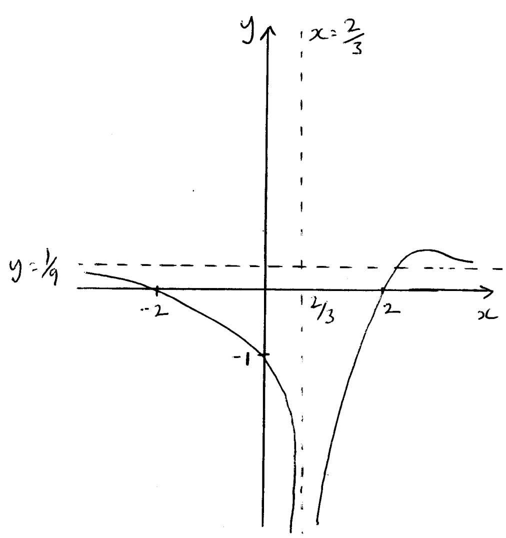
Mark Scheme 4755
June 2005

Section A			
1(i)	$\mathbf{A}^{-1}=\frac{1}{5}\begin{pmatrix}2 & -3\\ -1 & 4\end{pmatrix}$	M1 A1	Dividing by determinant
1(ii)	$\frac{1}{5}\begin{pmatrix}2 & -3\\ -1 & 4\end{pmatrix}\begin{pmatrix}5\\ -4\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}=\frac{1}{5}\begin{pmatrix}22\\ -21\end{pmatrix}$ $\Rightarrow x=\frac{22}{5},\ y=\frac{-21}{5}$	M1 A1(ft) , A1(ft) **[5]**	Pre-multiplying by their inverse Follow through use of their inverse No marks for solving without using inverse matrix
2	$4-\mathrm{j},\ 4+\mathrm{j}$ $\sqrt{17}\left(\cos 0.245+\mathrm{j}\sin 0.245\right)$ $\sqrt{17}\left(\cos 0.245-\mathrm{j}\sin 0.245\right)$	M1 A1 **[2]** M1 F1, F1 **[3]**	Use of quadratic formula Both roots correct Attempt to find modulus and argument One mark for each root Accept $\left(r,\ \theta\right)$ form Allow any correct arguments in radians or degrees, including negatives: 6.04, 14.0°, 346°. Accuracy at least 2s.f. S.C. F1 for consistent use of their incorrect modulus or argument (not both, F0)
3	$\begin{pmatrix}3 & -1\\ 2 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}\Rightarrow x=3x-y,\ y=2x$ $\Rightarrow y=2x$	M1 A1 A1 **[3]**	M1 for $\begin{pmatrix}3 & -1\\ 2 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}$ (allow if implied) $\begin{pmatrix}3 & -1\\ 2 & 0\end{pmatrix}\begin{pmatrix}k\\ mk\end{pmatrix}=\begin{pmatrix}K\\ mK\end{pmatrix}$ can lead to full marks if correctly used. Lose second A1 if answer includes two lines
4(i) **4(ii)** **4(iii)**	$\alpha+\beta=2,\ \alpha\beta=4$ $\alpha^2+\beta^2=\left(\alpha+\beta\right)^2-2\alpha\beta=4-8=-4$ Sum of roots = $2\alpha+2\beta=2\left(\alpha+\beta\right)=4$	B1 M1A1 (ft) M1	Both Accept method involving calculation of roots Or substitution method, or method

	Product of roots = $2\alpha \times 2\beta = 4\alpha\beta = 16$ $x^2 - 4x + 16 = 0$	A1(ft) **[5]**	involving calculation of roots The = 0, or equivalent, is necessary for final A1

5(i)	Sketch of Argand diagram with: Point $3+4\mathrm{j}$. Circle, radius 2.	B1 B1 **[2]**	Circle must not touch either axis. B1 max if no labelling or scales. Award even if centre incorrect.
5(ii)	Half-line: Starting from (4, 0) Vertically upwards	B1 B1 **[2]**	
5(iii)	Points where line crosses circle clearly indicated.  	B1 **[1]**	Identifying 2 points where their line cuts the circle

Qu	Answer	Mark	Comment
Section A (continued)			
6	For $k=1$, $1^3=1$ and $\frac{1}{4}1^2(1+1)^2=1$, so true for $k=1$ Assume true for $n=k$ Next term is $(k+1)^3$ Add to both sides RHS = $\frac{1}{4}k^2(k+1)^2+(k+1)^3$ $=\frac{1}{4}(k+1)^2\left[k^2+4(k+1)\right]$ $=\frac{1}{4}(k+1)^2(k+2)^2$ $=\frac{1}{4}(k+1)^2((k+1)+1)^2$ But this is the given result with $(k+1)$ replacing *k*. Therefore if it is true for *k* it is true for $(k+1)$. Since it is true for $k=1$ it is true for *k* = 1, 2, 3, … .	B1 B1 B1 M1 M1 A1 E1 **[7]**	Assuming true for *k*, $(k+1)^{\text{th}}$ term - for alternative statement, give this mark if whole argument logically correct Add to both sides Factor of $(k+1)^2$ Allow alternative correct methods For fully convincing algebra leading to true for $k \Rightarrow$ true for *k* + 1 Accept 'Therefore true by induction' only if previous A1 awarded S.C. Give E1 if convincing explanation of induction following acknowledgement of earlier error
7	$3\sum r^2-3\sum r$ $=3\times\frac{1}{6}n(n+1)(2n+1)-3\times\frac{1}{2}n(n+1)$ $=\frac{1}{2}n(n+1)[(2n+1)-3]$ $=\frac{1}{2}n(n+1)(2n-2)$ $=n(n+1)(n-1)$	M1,A1 M1,A1 M1 A1 c.a.o. **[6]**	Separate sums Use of formulae Attempt to factorise, only if earlier M marks awarded Must be fully factorised
			Section A Total: 36

8(i)	$x=\frac{2}{3}$ and $y=\frac{1}{9}$	B1, B1 **[2]**	-1 if any others given. Accept min of 2s.f. accuracy
8(ii)	Large positive x, $y \to \frac{1}{9}^{+}$ (e.g. consider $x=100$) Large negative x, $y \to \frac{1}{9}^{-}$ (e.g. consider $x=-100$)	M1 A1 E1 **[3]**	Approaches horizontal asymptote, not inconsistent with their (i) Correct approaches Reasonable attempt to justify approaches
8(iii)	Curve $x=\frac{2}{3}$ shown with correct approaches $y=\frac{1}{9}$ shown with correct approaches (from below on left, above on right). (2, 0), (-2, 0) and (0, -1) shown 	B1(ft) B1(ft) B1(ft) B1 B1 **[5]**	1 for each branch, consistent with horizontal asymptote in (i) or (ii) Both x intercepts y intercept (give these marks if coordinates shown in workings, even if not shown on graph)
8(iv)	$-1=\frac{x^2-4}{(3x-2)^2} \Rightarrow -9x^2+12x-4=x^2-4$ $\Rightarrow 10x^2-12x=0$ $\Rightarrow 2x(5x-6)=0$ $\Rightarrow x=0$ or $x=\frac{6}{5}$ From sketch, $y \ge -1$ for $x \le 0$ and $x \ge \frac{6}{5}$	M1 A1 B1 F1 **[4]**	Reasonable attempt at solving inequality Both values – give for seeing 0 and $\frac{6}{5}$, even if inequalities are wrong For $x \le 0$ Lose only one mark if any strict inequalities given

9(i)	2 - j 2j	B1 B1 **[2]**	
9(iii)	$(x-2-\mathrm{j})(x-2+\mathrm{j})(x+2\mathrm{j})(x-2\mathrm{j})$ $=(x^2-4x+5)(x^2+4)$ $=x^4-4x^3+9x^2-16x+20$ So A = -4, B = 9, C = -16 and D = 20	M1, M1 A1,A1 A4 **[8]**	M1 for each attempted factor pair A1 for each quadratic - follow through sign errors Minus 1 each error – follow through sign errors only
OR	$-\mathrm{A}=\sum\alpha=4\Rightarrow\mathrm{A}=-4$ $\mathrm{B}=\sum\alpha\beta=9\Rightarrow\mathrm{B}=9$ $-\mathrm{C}=\sum\alpha\beta\gamma=16\Rightarrow\mathrm{C}=-16$ $\mathrm{D}=\sum\alpha\beta\gamma\delta=20\Rightarrow\mathrm{D}=20$	M1, A1 M1, A1 M1, A1 M1, A1 **[8]**	M1s for reasonable attempt to find sums S.C. If one sign incorrect, give total of A3 for A, B, C, D values If more than one sign incorrect, give total of A2 for A, B, C, D values
OR	Attempt to substitute two correct roots into $x^4+Ax^3+Bx^2+Cx+D=0$ Produce 2 correct equations in two unknowns A = -4, B = 9, C = -16, D = 20	M1 M1 A2 A4	One for each root One for each equation One mark for each correct. S.C. If one sign incorrect, give total of A3 for A, B, C, D values If more than one sign incorrect, give total of A2 for A, B, C, D values

10(i)	$\sum_{r=1}^{n}\frac{2}{r(r+1)(r+2)}=\sum_{r=1}^{n}\left[\frac{1}{r}-\frac{2}{(r+1)}+\frac{1}{(r+2)}\right]$	M1	Give if implied by later working
	$=\left(\frac{1}{1}-\frac{2}{2}+\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{2}{4}+\frac{1}{5}\right)+$ $\ldots\ldots+\left(\frac{1}{n-1}-\frac{2}{n}+\frac{1}{n+1}\right)+\left(\frac{1}{n}-\frac{2}{n+1}+\frac{1}{n+2}\right)$	M1 A2	Writing out terms in full, at least three terms All terms correct. A1 for at least two correct
	$=\frac{1}{1}-\frac{2}{2}+\frac{1}{2}+\frac{1}{n+1}-\frac{2}{n+1}+\frac{1}{n+2}$ $=\frac{1}{2}-\frac{1}{n+1}+\frac{1}{n+2}$	M1 A3	Attempt at cancelling terms Correct terms retained (minus 1 each error)
	$=\frac{1}{2}-\frac{1}{(n+1)(n+2)}$	M1	Attempt at single fraction leading to given answer.
		[9]	
10(ii)	$\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\ldots$ $=\frac{1}{2}\sum_{r=1}^{n}\frac{2}{r(r+1)(r+2)}=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{(n+1)(n+2)}\right)$	M1 M1	M1 relating to previous sum, M1 for recognising that $\frac{1}{(n+1)(n+2)}\to 0$ as $n\to\infty$ (could be implied)
	$\Rightarrow \frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\ldots=\frac{1}{4}$	A1	
		[3]	

4755 - Further Concepts For Advanced Mathematics

General Comments

This was a successful paper, which performed according to design. Many clearly talented students scored highly but this was not true of the weaker candidates and there was a realistic tail of low marks.

The general standard of work was high and the majority of candidates showed a good level of competence, including in handling algebra, though weaker candidates lost marks through poor algebraic manipulation.

There was evidence, in the form of a fair number of fragmentary answers to the last question, that some candidates found the paper rather long. The paper was very appropriate for its candidature.

Comments on Individual Questions

1) Many candidates got this question fully right. Among those who did not get full marks, the most common cause was not knowing how to use matrices to solve simultaneous equations; some used Gaussian elimination instead and they were awarded no marks

2) While many candidates got this question fully right, there were also many who lost marks through careless mistakes. A particularly common error was to say that $\frac{8\pm\sqrt{-4}}{2}=4\pm 2\mathrm{j}$. There were also some weaker candidates who did not seem to know the meaning of modulus-argument form and so did not attempt the second part of the question. Several candidates calculated modulus and argument but failed to get full marks because they did not give their answers in modulus-argument form, even though this was specifically required by the question. A few candidates gave a negative modulus for one of the roots.

3) Most candidates either got this question fully right or did not know the appropriate method. A common error was to equate $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ instead of to $\begin{pmatrix} x \\ y \end{pmatrix}$.

4) On the whole this question was well answered.

Nearly all candidates got part (i) right.

Part (ii) produced more errors including a distressing number of answers in which it was claimed that $\alpha^2 + \beta^2 = (\alpha + \beta)^2$.

Part (iii) was rather better answered than part (ii) but several candidates lost a mark by giving a quadratic expression rather than a quadratic equation.

5) Many candidates got this question fully right.

In part (i) almost all candidates drew a circle of radius 2, but several had the wrong centre.

Part (ii) was less well answered with several candidates drawing a circle rather than a line. Among those who knew it should be a line, several drew a full line instead of a half line.

A common error in part (iii) was to give the answer as the sector formed by the half-line and the circle, rather than as the two points of intersection.

6) This question was generally well answered. Many candidates got it fully right. However a significant proportion lost marks by missing out logical steps in the argument, or by not stating them adequately. There was also a significant proportion who failed to manipulate the algebra to establish the result for $k + 1$ terms.

7) Many candidates answered this question well up to the last line, where many candidates lost a mark by failing to give it in a fully factorised form. However, a significant minority failed to separate the summation correctly and several tried to multiply standard sums rather than add them.

8) Virtually all candidates were able to obtain some marks on this question but only the very best achieved full marks.

(i) Almost all candidates got the vertical asymptote right but many failed to get the horizontal one. Assuming $y = 0$ was a horizontal asymptote was a very common error.

(ii) This was not well answered; many candidates did not realise that they were expected to show from which sides the curve approaches the horizontal asymptote.

(iii) Many candidates drew a curve that was at least nearly right but the approaches to the horizontal asymptote were often incorrect. Some candidates did not appreciate that they are expected to indicate the asymptotes and to give the points where the curve cuts the co-ordinate axes.

(iv) Candidates were required to solve an inequality and while most knew what to do, many spoilt their answers by making careless mistakes in their algebra.

9) There were many fully correct answers to this question.

Almost everyone got part (i) right.

There were three possible strategies for part (ii). Those who used the pairs of conjugate roots to form two quadratic factors, and then multiplied them together, were mostly successful. A more common approach was to find $\sum\alpha$, $\sum\alpha\beta$, $\sum\alpha\beta\gamma$ and $\alpha\beta\gamma\delta$ and use them to find coefficients of the quartic; those using this method often lost marks from careless mistakes. The third method involved substituting two roots into the equation and so forming real and imaginary equations for the coefficients; those who chose this method were seldom successful.

10) Most candidates knew what to do in part (i) of this question and a very encouraging number got it fully right. The most common errors were not correctly identifying the terms left over after the summation, and failing to demonstrate the last step combining the algebraic fractions into the form in the given answer.

A few candidates 'simplified' the terms and so could not see the cancelling pattern.

Few candidates were successful in part (ii). Most did not consider what happened to the fraction as $n \to \infty$ and many of the very best still failed to spot that the required sum was half of the sum in part (i), so they gave a final answer of $\frac{1}{2}$ rather than $\frac{1}{4}$.