OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 22 May 2009
Morning

Duration: 1 hour 30 minutes

* 4 7 5 5 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

2R–8J18

Section A (36 marks)

1 **(i)** Find the inverse of the matrix $\mathbf{M} = \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix}$. **[2]**

(ii) Use this inverse to solve the simultaneous equations

$$4x - y = 49,$$
$$3x + 2y = 100,$$

showing your working clearly. **[3]**

2 Show that $z = 3$ is a root of the cubic equation $z^3 + z^2 - 7z - 15 = 0$ and find the other roots. **[5]**

3 **(i)** Sketch the graph of $y = \frac{2}{x+4}$. **[2]**

(ii) Solve the inequality

$$\frac{2}{x+4} \leqslant x + 3,$$

showing your working clearly. **[5]**

4 The roots of the cubic equation $2x^3 + x^2 + px + q = 0$ are $2w$, $-6w$ and $3w$. Find the values of the roots and the values of p and q. **[6]**

5 **(i)** Show that $\frac{1}{5r-2} - \frac{1}{5r+3} \equiv \frac{5}{(5r-2)(5r+3)}$ for all integers r. **[2]**

(ii) Hence use the method of differences to show that $\sum_{r=1}^{n} \frac{1}{(5r-2)(5r+3)} = \frac{n}{3(5n+3)}$. **[4]**

6 Prove by induction that $3 + 10 + 17 + \ldots + (7n - 4) = \frac{1}{2}n(7n - 1)$ for all positive integers n. **[7]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{(x+2)(3x-5)}{(2x+1)(x-1)}$.

(i) Write down the coordinates of the points where the curve crosses the axes. **[3]**

(ii) Write down the equations of the three asymptotes. **[3]**

(iii) Determine whether the curve approaches the horizontal asymptote from above or below for

(A) large positive values of x,

(B) large negative values of x. **[3]**

(iv) Sketch the curve. **[3]**

8 Fig. 8 shows an Argand diagram.

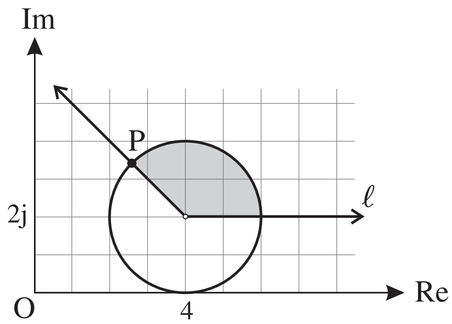


Fig. 8

(i) Write down the equation of the locus represented by the perimeter of the circle in the Argand diagram. **[3]**

(ii) Write down the equation of the locus represented by the half-line ℓ in the Argand diagram. **[3]**

(iii) Express the complex number represented by the point P in the form $a + b$j, giving the exact values of a and b. **[3]**

(iv) Use inequalities to describe the set of points that fall within the shaded region (excluding its boundaries) in the Argand diagram. **[3]**

[Question 9 is printed overleaf.]

9 You are given that $\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$, $\mathbf{N} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

(i) The matrix products $\mathbf{Q}(\mathbf{MN})$ and $(\mathbf{QM})\mathbf{N}$ are identical. What property of matrix multiplication does this illustrate?

Find **QMN**. **[4]**

M, **N** and **Q** represent the transformations M, N and Q respectively.

(ii) Describe the transformations M, N and Q. **[4]**

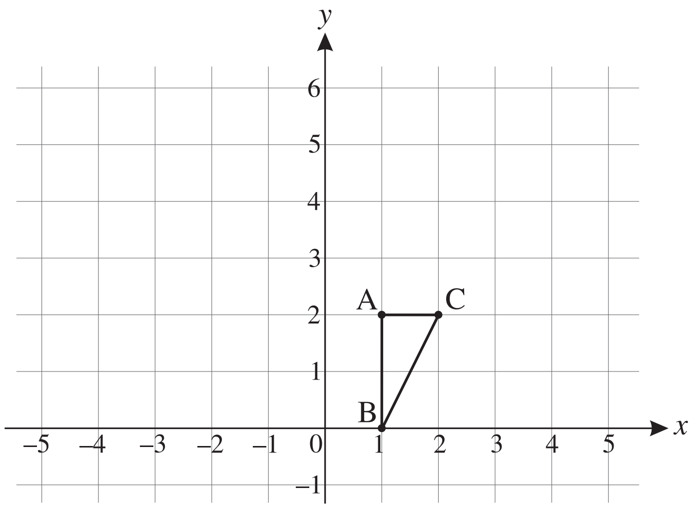
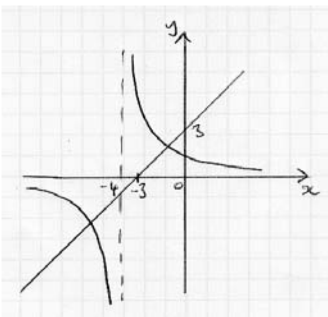


Fig. 9

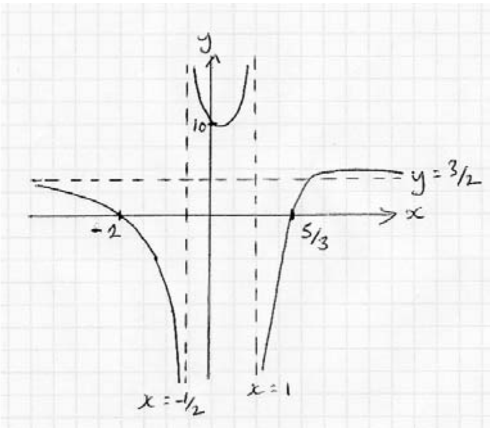
(iii) The points A, B and C in the triangle in Fig. 9 are mapped to the points A′, B′ and C′ respectively by the composite transformation N followed by M followed by Q. Draw a diagram showing the image of the triangle after this composite transformation, labelling the image of each point clearly. **[4]**

4755 (FP1) Further Concepts for Advanced Mathematics

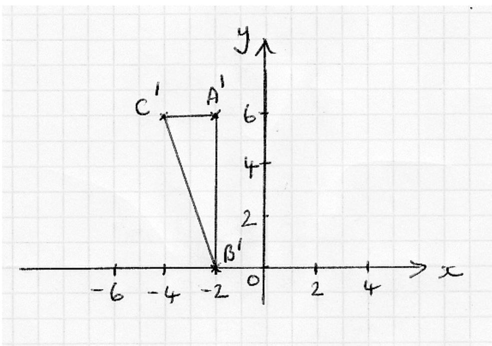
Section A			
1(i)	$\mathbf{M}^{-1} = \frac{1}{11}\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$	M1 A1 **[2]**	Dividing by determinant
1(ii)	$\frac{1}{11}\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 49 \\ 100 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 198 \\ 253 \end{pmatrix}$ $\Rightarrow x = 18, y = 23$	M1 A1(ft) A1(ft) **[3]**	Pre-multiplying by their inverse
2	$z^3 + z^2 - 7z - 15 = (z-3)(z^2 + 4z + 5)$ $z^2 + 4z + 5 = 0 \Rightarrow z = \frac{-4 \pm \sqrt{16-20}}{2}$ $\Rightarrow z = -2 + \mathrm{j}$ and $z = -2 - \mathrm{j}$	B1 M1 A1 M1 A1 **[5]**	Show $z = 3$ is a root; may be implied Attempt to find quadratic factor Correct quadratic factor Use of quadratic formula or other valid method Both solutions
3(i)		B1 B1 **[2]**	Asymptote at x = -4 Both branches correct
3(ii)	$\frac{2}{x+4} = x + 3 \Rightarrow x^2 + 7x + 10 = 0$ $\Rightarrow x = -2$ or $x = -5$ $x \geq -2$ or $-4 > x \geq -5$	M1 A1 A1 A2 **[5]**	Attempt to find where graphs cross or valid attempt at solution using inequalities Correct intersections (both), or -2 and -5 identified as critical values $x \geq -2$ $-4 > x \geq -5$ s.c. A1 for $-4 \geq x \geq -5$ or $-4 > x > -5$

4	$2w-6w+3w=\frac{-1}{2}$ $\Rightarrow w=\frac{1}{2}$ $\Rightarrow$ roots are $1, -3, \ \frac{3}{2}$ $\frac{-q}{2}=\alpha\beta\gamma=\frac{-9}{2}\Rightarrow q=9$ $\frac{p}{2}=\alpha\beta+\alpha\gamma+\beta\gamma=-6\Rightarrow p=-12$	M1 A1 A1 M1 A2(ft) **[6]**	Use of sum of roots – can be implied Correct roots seen Attempt to use relationships between roots s.c. M1 for other valid method One mark each for *p* = -12 and *q* = 9

5(i)	$\frac{1}{5r-2}-\frac{1}{5r+3}\equiv\frac{5r+3-5r+2}{(5r+3)(5r-2)}$ $\equiv\frac{5}{(5r+3)(5r-2)}$	M1 A1 **[2]**	Attempt to form common denominator Correct cancelling
5(ii)	$\sum_{r=1}^{30}\frac{1}{(5r-2)(5r+3)}=\frac{1}{5}\sum_{r=1}^{30}\left[\frac{1}{(5r-2)}-\frac{1}{(5r+3)}\right]$ $=\frac{1}{5}\left[\left(\frac{1}{3}-\frac{1}{8}\right)+\left(\frac{1}{8}-\frac{1}{13}\right)+\left(\frac{1}{13}-\frac{1}{18}\right)+\dots+\left(\frac{1}{5n-7}-\frac{1}{5n-2}\right)+\left(\frac{1}{5n-2}-\frac{1}{5n+3}\right)\right]$ $=\frac{1}{5}\left[\frac{1}{3}-\frac{1}{5n+3}\right]=\frac{n}{3(5n+3)}$	B1 B1 M1 A1 **[4]**	First two terms in full Last term in full Attempt to cancel terms
6	When *n* = 1, $\frac{1}{2}n(7n-1)=3$, so true for *n* = 1 Assume true for *n* = *k* $3+10+17+.....+(7k-4)=\frac{1}{2}k(7k-1)$ $\Rightarrow 3+10+17+.....+(7(k+1)-4)$ $=\frac{1}{2}k(7k-1)+(7(k+1)-4)$ $=\frac{1}{2}[k(7k-1)+(14(k+1)-8)]$ $=\frac{1}{2}[7k^2+13k+6]$ $=\frac{1}{2}(k+1)(7k+6)$ $=\frac{1}{2}(k+1)(7(k+1)-1)$ But this is the given result with *k* + 1 replacing *k*. Therefore if it is true for *k* it is true for *k* + 1. Since it is true for *n* = 1, it is true for *n* = 1, 2, 3 and so true for all positive integers.	B1 E1 M1 M1 A1 E1 E1 **[7]**	 Assume true for *n* = *k* Add $(k+1)$th term to both sides Valid attempt to factorise c.a.o. with correct simplification Dependent on previous E1 and immediately previous A1 Dependent on B1 and both previous E marks
			Section A Total: 36

Section B			
7(i)	$(0\ ,10),(-2\ ,0),\left(\frac{5}{3}\ ,\ 0\right)$	B1 B1 B1 [3]	
7(ii)	$x=\frac{-1}{2}\ ,\ x=1\ ,\ y=\frac{3}{2}$	B1 B1 B1 [3]	
7(iii)	Large positive *x*, $y\to\frac{3^+}{2}$ (e.g. consider $x=100$) Large negative *x*, $y\to\frac{3^-}{2}$ (e.g. consider $x=-100$)	M1 B1 B1 [3]	Clear evidence of method required for full marks
7(iv)	Curve 3 branches of correct shape Asymptotes correct and labelled Intercepts correct and labelled  	B1 B1 B1 [3]	

8 (i)	$\lvert z-(4+2\mathrm{j})\rvert = 2$	**B1** **B1** **B1** **[3]**	Radius = 2 $z-(4+2\mathrm{j})$ or $z-4-2\mathrm{j}$ All correct
8(ii)	$\arg\left(z-(4+2\mathrm{j})\right) = 0$	**B1** **B1** **B1** **[3]**	Equation involving the argument of a complex variable Argument = 0 All correct
8(iii)	$a = 4-2\cos\frac{\pi}{4} = 4-\sqrt{2}$ $b = 2+2\sin\frac{\pi}{4} = 2+\sqrt{2}$ $\mathrm{P} = 4-\sqrt{2}+\left(2+\sqrt{2}\right)\mathrm{j}$	**M1** **A2** **[3]**	Valid attempt to use trigonometry involving $\frac{\pi}{4}$, or coordinate geometry 1 mark for each of *a* and *b* s.c. A1 only for *a* = 2.59, *b* = 3.41
8(iv)	$\frac{3}{4}\pi > \arg\left(z-(4+2\mathrm{j})\right) > 0$ and $\lvert z-(4+2\mathrm{j})\rvert < 2$	**B1** **B1** **B1** **[3]**	$\arg\left(z-(4+2\mathrm{j})\right) > 0$ $\arg\left(z-(4+2\mathrm{j})\right) < \frac{3}{4}\pi$ $\lvert z-(4+2\mathrm{j})\rvert < 2$ Deduct one mark if only error is use of inclusive inequalities

Section B (continued)			
9(i)	Matrix multiplication is associative	B1 [1]	
	$\mathbf{MN}=\begin{pmatrix}3&0\\0&2\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}$	M1	Attempt to find **MN** or **QM**
	$\Rightarrow \mathbf{MN}=\begin{pmatrix}0&3\\2&0\end{pmatrix}$	A1	or $\mathbf{QM}=\begin{pmatrix}0&-2\\3&0\end{pmatrix}$
	$\mathbf{QMN}=\begin{pmatrix}-2&0\\0&3\end{pmatrix}$	A1(ft) [3]	
9(ii)	M is a stretch, factor 3 in the x direction, factor 2 in the y direction.	B1 B1	Stretch factor 3 in the x direction Stretch factor 2 in the y direction
	N is a reflection in the line $y = x$.	B1	
	Q is an anticlockwise rotation through 90° about the origin.	B1 [4]	
9(iii)	$\begin{pmatrix}-2&0\\0&3\end{pmatrix}\begin{pmatrix}1&1&2\\2&0&2\end{pmatrix}=\begin{pmatrix}-2&-2&-4\\6&0&6\end{pmatrix}$	M1 A1(ft)	Applying their **QMN** to points. Minus 1 each error to a minimum of 0.
		B2 [4]	Correct, labelled image points, minus 1 each error to a minimum of 0. Give B4 for correct diagram with no workings.
			Section B Total: 36
			Total: 72

4755 Further Concepts for Advanced Mathematics (FP1)

General comments

Overall the candidates performed well, showing a good grasp of the material.

The standard of written argument varied; many candidates demonstrated excellent use of mathematical notation but some seemed to lack familiarity with the standard conventions for setting out clear mathematical arguments.

As has been the case with previous papers, some candidates dropped marks through careless algebraic manipulation, and a smaller number by failing to label diagrams and graphs clearly.

Comments on Individual Questions

1) **Matrices**

Usually this was well answered and gave candidates a good start. Some forgot to use the determinant, some miscalculated the determinant, and some used the determinant correctly but did not adjust the elements of **M** when finding the inverse.

Some candidates ignored the instruction to use their inverse matrix to solve the equations and so lost 3 marks.

2) **Complex roots of a cubic**

The large majority of candidates were able to show that *z* = 3 was a root of the equation. Usually they demonstrated this by showing that substituting *z* = 3 did yield zero. Some showed that the expression factorised with (*z* - 3) as one factor.

Solving the quadratic equation resulting from division by *z* -3, either using the quadratic formula or by factorisation, was usually successful. Some candidates erred in the quadratic expression and those that then found real roots should have asked themselves why the examiner had posed the question in terms of *z*. A surprising number of candidates thought $\frac{\sqrt{4}}{2} = 2$.

3) **Graph sketch and inequality**

This question revealed some short-comings in dealing with inequalities. Few candidates scored highly on part (ii).

(i) This graph should have been simple to sketch, based on AS core work. However, a large minority of candidates failed to score any marks. Many candidates attempted to plot the graph, rather than to sketch it. This method was rarely successful.

(ii) Candidates who realised that drawing *y* = *x* + 3 on their graph would demonstrate where the solutions lay were the most successful by far. Solving a simple quadratic equation found the intersections and the regions could be seen from the graph, although dealing with the asymptote at *x* = -4 and the producing correct inequalities still needed some thought.

Most candidates multiplied the inequality given by (x + 4) without considering whether this was negative or positive and then incorrectly produced two inequalities for x with -5 and -2. This earned no marks, although some managed to recover the situation by realising that $x = -5$ and $x = -2$ were 'critical values' and then going on to solve the inequality correctly.

A few candidates correctly solved the inequality by considering the two instances of $x > -4$ and $x < -4$, or by multiplying both sides by $(x+4)^2$, or by subtracting one side of the inequality from the other, although these methods must have taken up considerable time compared with the method of using intersecting graphs.

Generally only the very best candidates scored full marks for this question, even though it was quite simple if a graphical approach was used.

4) **Roots of a cubic**

Usually this was successfully answered, although those who worked throughout with w to find p and q often forgot to state the values of the roots and lost one mark.

Frequent errors were: forgetting that $a = 2$ in the root relationships; expanding $(x - 2w)(x + 6w)(x - 3w)$ and forgetting that the coefficient of x^3 had to be adjusted to be 2; forgetting to use the sum of the roots, so that w was either not found, or was assumed to be one.

There were also surprising numbers of candidates who solved $-6 = \frac{p}{2}$ to give $p = -3$ and/or $-\frac{9}{2} = \frac{q}{2}$ to give $q = 2.25$.

5) **Method of differences**

Many candidates earned full marks for this question.

(i) This was usually correctly shown, although many candidates lost a mark by omitting brackets in the numerator of the algebraic fraction when forming a common denominator, resulting in incorrect workings.

(ii) This was usually started in the right way. Candidates who stated the correct relationship between the summation required and the expression in part (i) did not fall into the trap of forgetting the factor of $\frac{1}{5}$ needed to reach the solution. Some candidates obviously had to perform a rescue at the end. Some knew that 5 was a problem but put it in the wrong place, losing the accuracy mark. Many candidates wasted time and effort by multiplying out the denominator of the algebraic fraction, rather than preserving factors in their expressions.

A small number of candidates failed to begin this part correctly and tried either to start an inductive proof, or to use a series involving $\frac{1}{3\times8}+\frac{1}{8\times13}+ ...$, apparently believing that this could lead to cancellation of terms.

6) **Proof by induction**

Many candidates showed a good understanding of proof by induction. There were many excellent answers with full explanation and grammatical details included. Those who took shortcuts with the wording of the argument were generally less successful, often losing one or two marks at the end by failing to present a complete argument.

The case $n = 1$ should consider both the first term of the series and the expression for the sum of one term, *and* point out that these are the same.

'Assume true for $n = k$' is not easily abbreviated; 'Assume $n = k$' does not convey the same meaning. If at this stage in the proof the expression for the sum is shown with $n = k + 1$, it should be made clear that this is the target for the following analysis.

There were a number of candidates who do not appreciate the difference between $7k - 4$ and $\sum 7r - 4$ and some candidates were clearly confused by the distinction between r, k and n.

Various errors arose when adding the $(k + 1)$th term to the sum of the first k terms. Some candidates added the kth term, rather than the $(k + 1)$th term. Others added the $(k + 1)$th term to the expression for the sum of the first $k + 1$ terms.

Some candidates failed to manipulate the factor of ½ out of the expression, and some algebraic manipulation was incorrect or unconvincing and so did not receive full credit.

The final steps in the proof are more easily explained if the structure of the sum of the first $k + 1$ terms is shown unsimplified, with $k + 1$ replacing n. Students should make it clear that they understand that they have shown that '*if* the result is true for $n = k$ it is true for $n = k + 1$', which is not quite the same as 'the result is true for $n = k$ and $n = k + 1$'. Finally, it is essential to use the fact that the result is true for $n = 1$ to complete the chain of reasoning.

Some candidates seemed to believe that this proof is solely about algebraic manipulation and failed to appreciate that the logic of the argument is essential.

7) **Curve sketching**

This question was successfully answered by many candidates, but some answers would have been improved by using half a page to sketch the graph, rather than squashing it into a few lines. The use of graph paper is unnecessary. Some sketches were carelessly rough; intercepts with axes, asymptotes and approaches to asymptotes must be clearly shown.

(i) Coordinates were asked for explicitly but some candidates did not express their answers as coordinates.

(ii) This was usually fully correct. Weaker candidates sometimes assumed that $y = 0$ was the horizontal asymptote.

(iii) Many candidates did not show sufficient workings to earn the method mark, even if they did give the correct approaches to the horizontal asymptote.

(iv) Many candidates who could have earned full marks failed to do so because of sloppy sketches that did not show clearly the approaches to the asymptotes, the asymptotes themselves, or the points where the graph crosses the x and y axes.

8) **Loci on the Argand diagram**

This question caused the most problems for candidates.

Many candidates were unfamiliar with the notation needed to describe loci in the complex plane.

(i) Modulus signs were missing in many cases and $z + 4 + 2j$ or $z - 4 + 2j$ were often seen, rather than $z - (4 + 2j)$ or $z - 4 - 2j$.
(ii) This part was often omitted altogether. Weaker candidates were clearly uncertain how to deal with loci involving the argument of a complex number. Many of the strongest candidates also made errors; for example brackets are important here so $\arg z - 4 - 2j = 0$ is not a correct expression.
(iii) This was frequently omitted but when tackled was usually well done. Again it is important to use brackets; $4 - \sqrt{2} + 2 + \sqrt{2}j$ is incorrect. Some candidates tried this part using the coordinate geometry of an intersection of a line with the circle, usually successfully, but sometimes leading to intractable algebra.
(iv) This was generally answered correctly by the strongest candidates. Some candidates earned a mark for the inequality giving the interior of the circle, but could not cope with the inequalities involving the argument of a complex number. The weaker candidates generally scored no marks for this part of the question.

9) **Matrix transformations**

This question was generally answered well. A small number of candidates failed to complete this question, presumably due to time pressure.

(i) Few candidates understood that matrix multiplication is associative.

Most could multiply the matrices correctly, but a surprising number calculated **MNQ** rather than **QMN**. Many made simple arithmetic mistakes.
(ii) N was usually correctly described but Q was often thought to represent a reflection and M was often called an enlargement instead of a two-way stretch.
(iii) Some candidates thought that they had to calculate and use the matrix product **NMQ**, rather than proceeding with **QMN**, which they had already obtained in part (i). Many played safe and worked through each matrix in turn. Those that tried to apply the transformations directly to the triangle without using matrices usually made errors.

Candidates' diagrams sometimes lacked labels and/or scales, despite the explicit instruction to label the image of each point clearly.