

# **Mathematics (MEI)**

Advanced Subsidiary GCE 4755

Further Concepts for Advanced Mathematics (FP1)

## **Mark Scheme for June 2010**

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Facsimile: 01223 552610  
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Qu	Answer	Mark	Comment
<b>Section A</b>			
<b>1</b>	$4x^2 - 16x + C \equiv A(x^2 + 2Bx + B^2) + 2$ $\Leftrightarrow 4x^2 - 16x + C \equiv Ax^2 + 2ABx + AB^2 + 2$ $\Leftrightarrow A = 4, B = -2, C = 18$	B1 M1 A2, 1 <b>[4]</b>	$A = 4$ Attempt to expand RHS or other valid method (may be implied) 1 mark each for B and C, c.a.o.
<b>2(i)</b>	$2x - 5y = 9$ $3x + 7y = -1$	B1 B1 <b>[2]</b>	
<b>2(ii)</b>	$\mathbf{M}^{-1} = \frac{1}{29} \begin{pmatrix} 7 & 5 \\ -3 & 2 \end{pmatrix}$ $\frac{1}{29} \begin{pmatrix} 7 & 5 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 9 \\ -1 \end{pmatrix} = \frac{1}{29} \begin{pmatrix} 58 \\ -29 \end{pmatrix}$ $\Rightarrow x = 2, y = -1$	M1 A1 <b>[2]</b> M1 A1(ft) <b>[2]</b>	Divide by determinant c.a.o. Pre-multiply by their inverse For both
<b>3</b>	$z = 1 - 2j$ $1 + 2j + 1 - 2j + \alpha = \frac{1}{2}$ $\Rightarrow \alpha = -\frac{3}{2}$ $\frac{-k}{2} = -\frac{3}{2}(1 - 2j)(1 + 2j) = -\frac{15}{2}$ $k = 15$ <p><b>OR</b></p> $(z - (1 + 2j))(z - (1 - 2j)) = z^2 - 2z + 5$ $2z^3 - z^2 + 4z + k = (z^2 - 2z + 5)(2z + 3)$ $\alpha = -\frac{3}{2}$ $k = 15$	B1 M1 A1 M1 A1(ft) A1 <b>[6]</b> M1 A1 M1 A1(ft) A1 <b>[6]</b>	$A = 4$ Valid attempt to use sum of roots, or other valid method c.a.o. Valid attempt to use product of roots, or other valid method Correct equation – can be implied c.a.o. Multiplying correct factors Correct quadratic, c.a.o. Attempt to find linear factor c.a.o.

<p><b>4</b></p> $w = x + 1 \Rightarrow x = w - 1$ $x^3 - 2x^2 - 8x + 11 = 0, w = x - 1$ $\Rightarrow (w - 1)^3 - 2(w - 1)^2 - 8(w - 1) + 11 = 0$ $\Rightarrow w^3 - 5w^2 - w + 16 = 0$ <p><b>OR</b></p> $\alpha + \beta + \gamma = 2$ $\alpha\beta + \alpha\gamma + \beta\gamma = -8$ $\alpha\beta\gamma = -11$ <p>Let the new roots be <math>k, l</math> and <math>m</math> then</p> $k + l + m = \alpha + \beta + \gamma + 3 = 2 + 3 = 5$ $kl + km + lm = (\alpha\beta + \alpha\gamma + \beta\gamma) + 2(\alpha + \beta + \gamma) + 3$ $= -8 + 4 + 3 = -1$ $klm = \alpha\beta\gamma + (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha + \beta + \gamma) + 1$ $= -11 - 8 + 2 + 1 = -16$ $\Rightarrow w^3 - 5w^2 - w + 16 = 0$		<p>B1</p> <p>M1</p> <p>M1</p> <p>A3</p> <p>[6]</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A3</p> <p>[6]</p>	<p>Substitution. For <math>x = w + 1</math> give B0 but then follow for a maximum of 3 marks</p> <p>Attempt to substitute into cubic</p> <p>Attempt to expand</p> <p>-1 for each error (including omission of = 0)</p> <p>All 3 correct</p> <p>Valid attempt to use their sum of roots in original equation to find sum of roots in new equation</p> <p>Valid attempt to use their product of roots in original equation to find one of <math>\sum \alpha\beta</math> or <math>\alpha\beta\gamma</math></p> <p>-1 each error (including omission of = 0)</p>
<p><b>5</b></p> $\sum_{r=1}^n \frac{1}{(5r-1)(5r+4)} = \frac{1}{5} \sum_{r=1}^n \left( \frac{1}{5r-1} - \frac{1}{5r+4} \right)$ $= \frac{1}{5} \left( \left( \frac{1}{4} - \frac{1}{9} \right) + \left( \frac{1}{9} - \frac{1}{14} \right) + \dots + \left( \frac{1}{5n-1} - \frac{1}{5n+4} \right) \right)$ $= \frac{1}{5} \left( \frac{1}{4} - \frac{1}{5n+4} \right) = \frac{1}{5} \left( \frac{5n+4-4}{4(5n+4)} \right) = \frac{n}{4(5n+4)}$		<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[6]</p>	<p>Attempt to use identity – may be implied</p> <p>Terms in full (at least first and last)</p> <p>Attempt at cancelling</p> $\left( \frac{1}{4} - \frac{1}{5n+4} \right)$ <p>factor of <math>\frac{1}{5}</math></p> <p>Correct answer as a single algebraic fraction</p>

<p><b>6(i)</b></p>	$u_2 = \frac{2}{1+2} = \frac{2}{3}, u_3 = \frac{\frac{2}{3}}{1+\frac{2}{3}} = \frac{2}{5}$	<p>M1 A1 [2]</p>	<p>Use of inductive definition c.a.o.</p>
<p><b>6(ii)</b></p>	<p>When <math>n = 1</math>, <math>\frac{2}{2 \times 1 - 1} = 2</math>, so true for <math>n = 1</math></p> <p>Assume <math>u_k = \frac{2}{2k-1}</math></p> $\Rightarrow u_{k+1} = \frac{\frac{2}{2k-1}}{1 + \frac{2}{2k-1}}$ $= \frac{\frac{2}{2k-1}}{\frac{2k-1+2}{2k-1}} = \frac{2}{2k+1}$ $= \frac{2}{2(k+1)-1}$ <p>But this is the given result with <math>k + 1</math> replacing <math>k</math>. Therefore if it is true for <math>k</math> it is also true for <math>k + 1</math>. Since it is true for <math>k = 1</math>, it is true for all positive integers.</p>	<p>B1 E1  M1  A1  E1 E1 [6]</p>	<p>Showing use of <math>u_n = \frac{2}{2n-1}</math></p> <p>Assuming true for <math>k</math></p> <p><math>u_{k+1}</math></p> <p>Correct simplification</p> <p>Dependent on A1 and previous E1</p> <p>Dependent on B1 and previous E1</p>
<b>Section A Total: 36</b>			

Section B			
7(i)	$\left(0, -\frac{1}{2}\right)$ $(-3, 0), \left(\frac{1}{2}, 0\right)$	B1	
7(ii)	$x = 3, x = 2$ and $y = 2$	B1 B1 B1 [3]	For both
7(iii)	Large positive $x$ , $y \rightarrow 2^+$ (e.g. substitute $x = 100$ to give 2.15..., or convincing algebraic argument)	M1 A1	Must show evidence of method A0 if no valid method
		B1	Correct RH branch
7(iv)	$\frac{(2x-1)(x+3)}{(x-3)(x-2)} = 2$ $\Rightarrow (2x-1)(x+3) = 2(x-3)(x-2)$ $\Rightarrow x = 1$ <p>From graph <math>x &lt; 1</math> or <math>2 &lt; x &lt; 3</math></p>	M1 A1 B1 B1 [4]	Or other valid method to find intersection with horizontal asymptote
			For $x < 1$ For $2 < x < 3$

8(i)	$\arg \alpha = \frac{\pi}{6},  \alpha  = 2$ $\arg \beta = \frac{\pi}{2},  \beta  = 3$	B1 B1 B1 [3]	Modulus of $\alpha$ Argument of $\alpha$ (allow $30^\circ$ ) Both modulus and argument of $\beta$ (allow $90^\circ$ )
8(ii)	$\alpha\beta = (\sqrt{3} + j)3j = -3 + 3\sqrt{3}j$ $\frac{\beta}{\alpha} = \frac{3j}{\sqrt{3} + j} = \frac{3j(\sqrt{3} - j)}{(\sqrt{3} + j)(\sqrt{3} - j)}$ $= \frac{3 + 3\sqrt{3}j}{4} = \frac{3}{4} + \frac{3\sqrt{3}j}{4}$	M1 A1 M1 A1 A1 A1 [5]	Use of $j^2 = -1$ Correct Correct use of conjugate of denominator Denominator = 4 All correct
8(iii)		M1 A1(ft) [2]	Argand diagram with at least one correct point Correct relative positions with appropriate labelling

Qu	Answer	Mark	Comment
<b>Section B (continued)</b>			
9(i)	P is a rotation through 90 degrees about the origin in a clockwise direction.  Q is a stretch factor 2 parallel to the $x$ -axis	B1 B1	Rotation about origin 90 degrees clockwise, or equivalent
9(ii)	$\mathbf{QP} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$	B1 B1 [4] M1 A1 [2]	Stretch factor 2 Parallel to the $x$ -axis  Correct order c.a.o.
9(iii)	$\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 2 \\ -2 & -1 & -3 \end{pmatrix}$ $A' = (0, -2), B' = (4, -1), C' = (2, -3)$	M1  A1(ft) [2]	Pre-multiply by their $\mathbf{QP}$ - may be implied  For all three points
9(iv)	$\mathbf{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1 B1 [2]	One for each correct column
9(v)	$\mathbf{RQP} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$  $(\mathbf{RQP})^{-1} = \frac{-1}{2} \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$	M1 A1(ft)  M1 A1 [4]	Multiplication of their matrices in correct order  Attempt to calculate inverse of their $\mathbf{RQP}$ c.a.o.
			<b>Section B Total: 36</b>
			<b>Total: 72</b>



**OCR (Oxford Cambridge and RSA Examinations)**  
**1 Hills Road**  
**Cambridge**  
**CB1 2EU**

**OCR Customer Contact Centre**

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