

**ADVANCED GCE**

**MATHEMATICS (MEI)**

Further Methods for Advanced Mathematics (FP2)

**4756**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**

None

**Friday 9 January 2009**  
**Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

## Section A (54 marks)

## Answer all the questions

- 1 (a) (i) By considering the derivatives of  $\cos x$ , show that the Maclaurin expansion of  $\cos x$  begins

$$1 - \frac{1}{2}x^2 + \frac{1}{24}x^4. \quad [4]$$

- (ii) The Maclaurin expansion of  $\sec x$  begins

$$1 + ax^2 + bx^4,$$

where  $a$  and  $b$  are constants. Explain why, for sufficiently small  $x$ ,

$$\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4\right)(1 + ax^2 + bx^4) \approx 1.$$

Hence find the values of  $a$  and  $b$ . [5]

- (b) (i) Given that  $y = \arctan\left(\frac{x}{a}\right)$ , show that  $\frac{dy}{dx} = \frac{a}{a^2 + x^2}$ . [4]

- (ii) Find the exact values of the following integrals.

(A)  $\int_{-2}^2 \frac{1}{4 + x^2} dx$  [3]

(B)  $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{4}{1 + 4x^2} dx$  [3]

- 2 (i) Write down the modulus and argument of the complex number  $e^{j\pi/3}$ . [2]

- (ii) The triangle OAB in an Argand diagram is equilateral. O is the origin; A corresponds to the complex number  $a = \sqrt{2}(1 + j)$ ; B corresponds to the complex number  $b$ .

Show A and the two possible positions for B in a sketch. Express  $a$  in the form  $re^{j\theta}$ . Find the two possibilities for  $b$  in the form  $re^{j\theta}$ . [5]

- (iii) Given that  $z_1 = \sqrt{2}e^{j\pi/3}$ , show that  $z_1^6 = 8$ . Write down, in the form  $re^{j\theta}$ , the other five complex numbers  $z$  such that  $z^6 = 8$ . Sketch all six complex numbers in a new Argand diagram. [6]

Let  $w = z_1 e^{-j\pi/12}$ .

- (iv) Find  $w$  in the form  $x + jy$ , and mark this complex number on your Argand diagram. [3]

- (v) Find  $w^6$ , expressing your answer in as simple a form as possible. [2]

- 3 (a) A curve has polar equation  $r = a \tan \theta$  for  $0 \leq \theta \leq \frac{1}{3}\pi$ , where  $a$  is a positive constant.
- (i) Sketch the curve. [3]
- (ii) Find the area of the region between the curve and the line  $\theta = \frac{1}{4}\pi$ . Indicate this region on your sketch. [5]
- (b) (i) Find the eigenvalues and corresponding eigenvectors for the matrix  $\mathbf{M}$  where

$$\mathbf{M} = \begin{pmatrix} 0.2 & 0.8 \\ 0.3 & 0.7 \end{pmatrix}. \quad [6]$$

- (ii) Give a matrix  $\mathbf{Q}$  and a diagonal matrix  $\mathbf{D}$  such that  $\mathbf{M} = \mathbf{QDQ}^{-1}$ . [3]

### Section B (18 marks)

#### Answer one question

#### Option 1: Hyperbolic functions

- 4 (a) (i) Prove, from definitions involving exponentials, that

$$\cosh^2 x - \sinh^2 x = 1. \quad [2]$$

- (ii) Given that  $\sinh x = \tan y$ , where  $-\frac{1}{2}\pi < y < \frac{1}{2}\pi$ , show that

(A)  $\tanh x = \sin y$ ,

(B)  $x = \ln(\tan y + \sec y)$ . [6]

- (b) (i) Given that  $y = \operatorname{artanh} x$ , find  $\frac{dy}{dx}$  in terms of  $x$ .

Hence show that  $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{1-x^2} dx = 2 \operatorname{artanh} \frac{1}{2}$ . [4]

- (ii) Express  $\frac{1}{1-x^2}$  in partial fractions and hence find an expression for  $\int \frac{1}{1-x^2} dx$  in terms of logarithms. [4]

- (iii) Use the results in parts (i) and (ii) to show that  $\operatorname{artanh} \frac{1}{2} = \frac{1}{2} \ln 3$ . [2]

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 The limaçon of Pascal has polar equation  $r = 1 + 2a \cos \theta$ , where  $a$  is a constant.

- (i) Use your calculator to sketch the curve when  $a = 1$ . (You need not distinguish between parts of the curve where  $r$  is positive and negative.) [3]
- (ii) By using your calculator to investigate the shape of the curve for different values of  $a$ , positive and negative,
- (A) state the set of values of  $a$  for which the curve has a loop within a loop,
- (B) state, with a reason, the shape of the curve when  $a = 0$ ,
- (C) state what happens to the shape of the curve as  $a \rightarrow \pm\infty$ ,
- (D) name the feature of the curve that is evident when  $a = 0.5$ , and find another value of  $a$  for which the curve has this feature. [7]
- (iii) Given that  $a > 0$  and that  $a$  is such that the curve has a loop within a loop, write down an equation for the values of  $\theta$  at which  $r = 0$ . Hence show that the angle at which the curve crosses itself is  $2 \arccos\left(\frac{1}{2a}\right)$ .

Obtain the cartesian equations of the tangents at the point where the curve crosses itself. Explain briefly how these equations relate to the answer to part (ii)(A). [8]