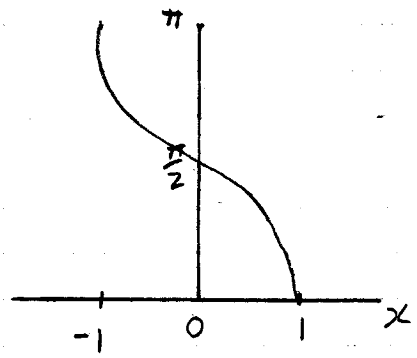
FP2 JUNE 2014 Q1 CALCULUS, POLAR, POWER SERIES

1a) i)



$y = f(x) = \arccos x$

ii)

$$y = \arccos x$$

$$\Rightarrow \quad \cos y = x$$

$$\Rightarrow \quad -\sin y \frac{dy}{dx} = 1$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{1}{\sin y}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{1}{\sqrt{1-\cos^2 y}} = -\frac{1}{\sqrt{1-x^2}}$$

iii)

$$f(x) = \arccos x$$

$$f'(x) = -\frac{1}{\sqrt{1-x^2}} = -(1-x^2)^{-\frac{1}{2}}$$

$$f''(x) = +\frac{1}{2}(1-x^2)^{-\frac{3}{2}} \times (-2x)$$

$$= -x(1-x^2)^{-3/2}$$

$$f'''(x) = -x\left(-\frac{3}{2}(1-x^2)^{-5/2} \times (-2x)\right) + (1-x^2)^{-3/2} \times (-1)$$

$$= -3x^2(1-x^2)^{-5/2} - (1-x^2)^{-3/2}$$

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1a iii) cont

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + + \dots$$

$$= \frac{\pi}{2} + x(-1) + \frac{x^2}{2}(0) + \frac{x^3}{6}(-1) + + \dots$$

$$= \frac{\pi}{2} - x - \frac{x^3}{6} + \dots$$

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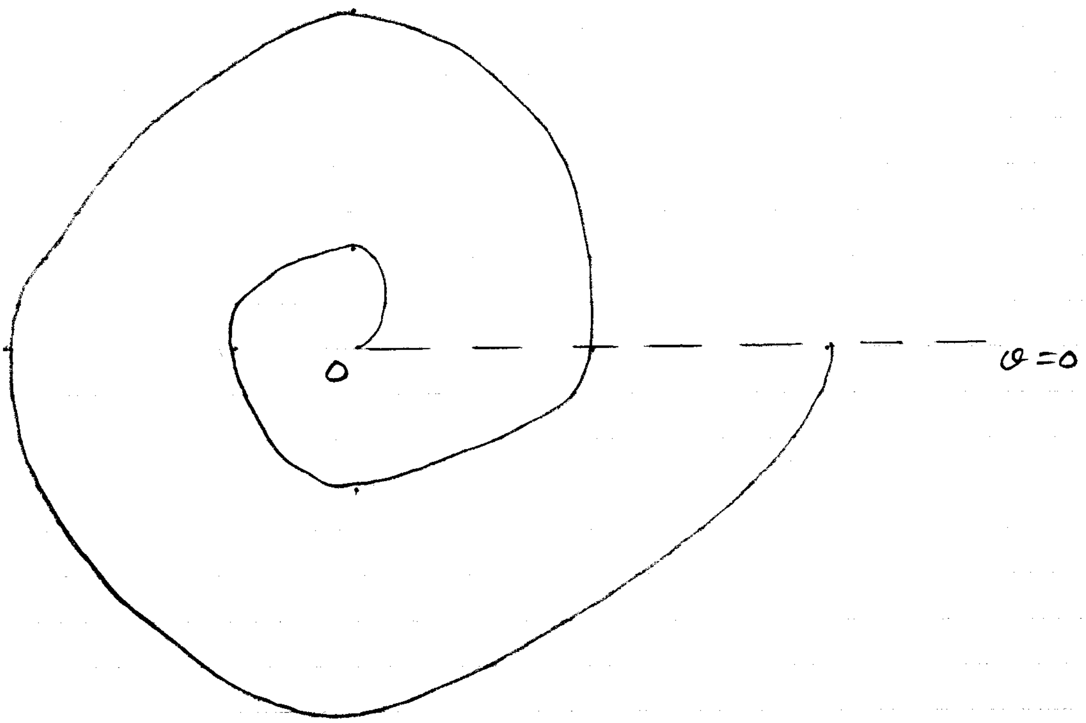
1 b) i)

$$r = \theta + \sin\theta, \quad \theta \geqslant 0$$

$$\frac{dr}{d\theta} = 1 + \cos\theta \geqslant 0 \text{ for all } \theta$$

$\therefore$ gradient $\geqslant 0$ for all θ and so r increases as θ increases

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π	$\frac{7\pi}{2}$	4π
r	0	2.57	3.14	3.71	6.28	8.85	9.42	10.00	12.56



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1 b) $\sin\theta \approx \theta$ for small θ

ii) $$\text{Area} = \int_0^\alpha \tfrac{1}{2} r^2 \, d\theta$$

$$= \int_0^\alpha \tfrac{1}{2}\left(\theta + \sin\theta\right)^2 d\theta$$

$$\approx \int_0^\alpha \tfrac{1}{2}\left(2\theta\right)^2 d\theta$$

$$\approx \int_0^\alpha 2\theta^2 \, d\theta$$

$$\approx \left[\frac{2\theta^3}{3}\right]_0^\alpha$$

$$\approx \frac{2\alpha^3}{3}$$