FP2 JANUARY 2013 Q2 COMPLEX NUMBERS

2a) i)

$$1 + e^{j2\theta} = e^{j\theta}\left(e^{-j\theta} + e^{j\theta}\right)$$
$$= (\cos\theta + j\sin\theta) \times 2\cos\theta$$
$$= 2\cos\theta(\cos\theta + j\sin\theta)$$

ii)

$$C = 1 + \binom{n}{1}\cos 2\theta + \binom{n}{2}\cos 4\theta + \ldots + \cos 2n\theta$$

$$S = \binom{n}{1}\sin 2\theta + \binom{n}{2}\sin 4\theta + \ldots + \sin 2n\theta$$

$$C + jS = 1 + \binom{n}{1}\left(\cos 2\theta + j\sin 2\theta\right) + \binom{n}{2}\left(\cos 4\theta + j\sin 4\theta\right) + \ldots + \binom{n}{n}\left(\cos 2n\theta + j\sin 2n\theta\right)$$

$$C + jS = 1 + \binom{n}{1}e^{j2\theta} + \binom{n}{2}e^{j4\theta} + \ldots + \binom{n}{n}e^{j2n\theta}$$

$$C + jS = \left(1 + e^{j2\theta}\right)^n$$

$$C + jS = \left(2\cos\theta(\cos\theta + j\sin\theta)\right)^n$$

$$C + jS = 2^n\cos^n\theta\left(\cos n\theta + j\sin n\theta\right)$$

Equating real and imaginary parts gives

$$C = 2^n\cos^n\theta\cos(n\theta)$$
$$S = 2^n\cos^n\theta\sin(n\theta)$$

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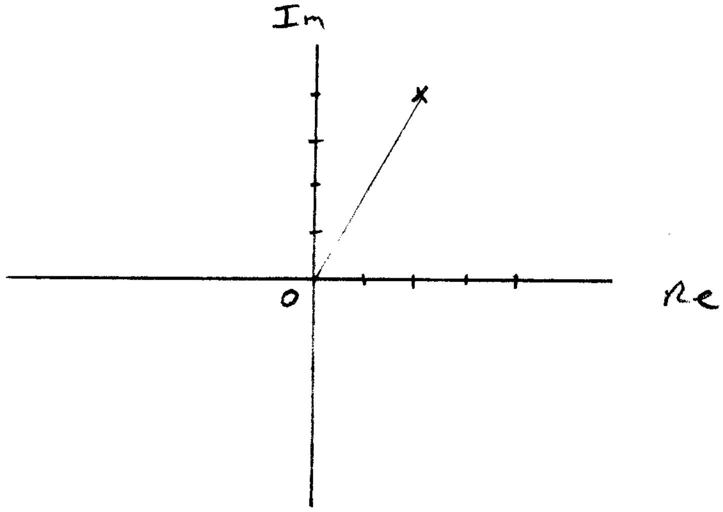
2b) i) $e^{j\frac{2\pi}{3}} = \cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3}$

$= -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

ii)



Other vertices obtained by rotating $2+4j$ by $\frac{2\pi}{3}$ anti-clockwise and clockwise about 0

2nd vertex $= (2+4j)e^{j\frac{2\pi}{3}}$

$= (2+4j)\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$

$= -1 - 2j + \sqrt{3}j - 2\sqrt{3}$

2nd vertex $= (-1-2\sqrt{3}) + (\sqrt{3}-2)j$

3rd vertex $= (2+4j)e^{-j\frac{2\pi}{3}}$

$= (2+4j)\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$

$= -1 - 2j - \sqrt{3}j + 2\sqrt{3}$

$= (2\sqrt{3}-1) + (-2-\sqrt{3})j$

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iii) Length of side = distance between 1st and 2nd vertices

$$= \sqrt{\left((-1-2\sqrt{3})-2\right)^2 + \left((\sqrt{3}-2)-4\right)^2}$$

$$= \sqrt{(-3-2\sqrt{3})^2 + (\sqrt{3}-6)^2}$$

$$= \sqrt{9+12\sqrt{3}+12+3-12\sqrt{3}+36}$$

$$= \sqrt{60} = \sqrt{4\times 15} = 2\sqrt{15}$$