FP2 JUNE 2011 Q2 COMPLEX NUMBERS

2 a) By de Moivre's theorem

$$\cos 5\theta + j\sin 5\theta = (\cos\theta + j\sin\theta)^5$$

Let $c = \cos\theta$
Let $s = \sin\theta$

$$= c^5 + 5c^4js + 10c^3j^2s^2 + 10c^2j^3s^3 + 5cj^4s^4 + j^5s^5$$

$$= c^5 + 5c^4js - 10c^3s^2 - 10c^2js^3 + 5cs^4 + js^5$$

$$= c^5 - 10c^3s^2 + 5cs^4 + j(5c^4s - 10c^2s^3 + s^5)$$

Equating real and imaginary parts

$$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$$

$$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$$

$$\tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta} = \frac{5c^4s - 10c^2s^3 + s^5}{c^5 - 10c^3s^2 + 5cs^4}$$

Diving top and bottom by c^5

$$\tan 5\theta = \frac{\frac{5c^4s}{c^5} - \frac{10c^2s^3}{c^5} + \frac{s^5}{c^5}}{\frac{c^5}{c^5} - \frac{10c^3s^2}{c^5} + \frac{5cs^4}{c^5}}$$

$$\tan 5\theta = \frac{5t - 10t^3 + t^5}{1 - 10t^2 + 5t^4}$$ where $t = \frac{s}{c}$

$$\tan 5\theta = \frac{t(t^4 - 10t^2 + 5)}{5t^4 - 10t^2 + 1}$$ as required

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2 b) i

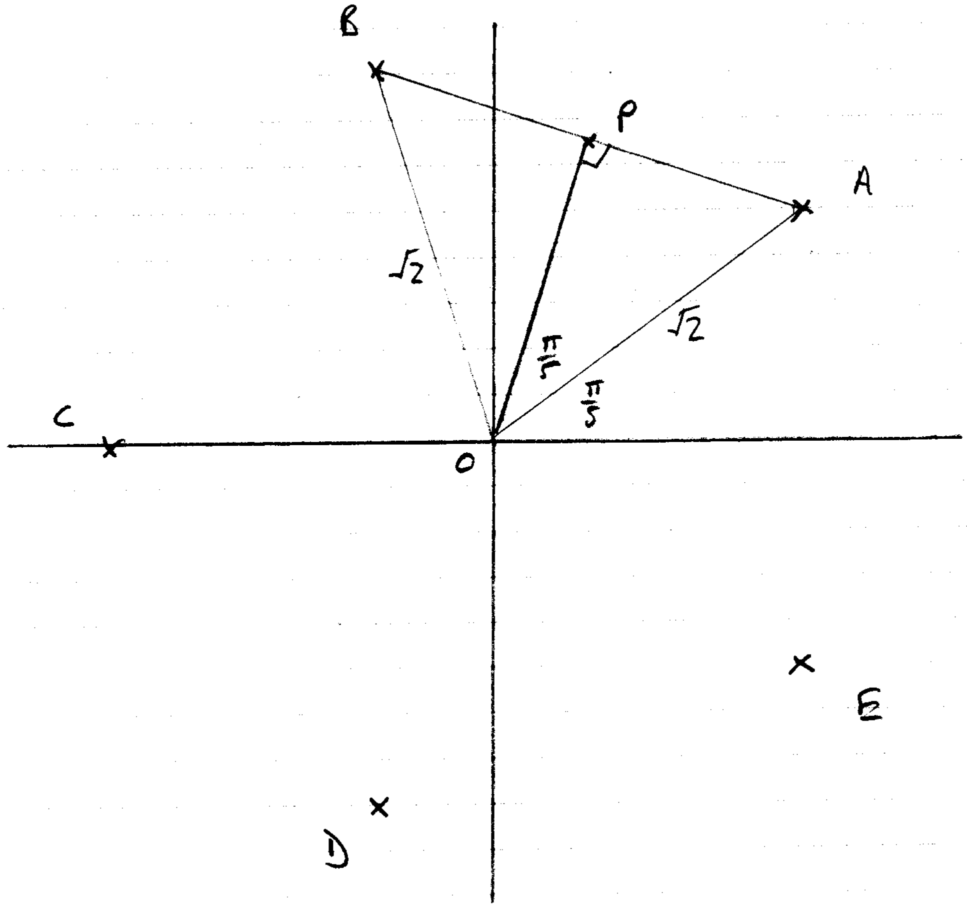
$$-4\sqrt{2} = \sqrt{2}^5 e^{j\pi}$$

fifth roots are

$$\sqrt{2}\, e^{j\left(\frac{\pi}{5} + \frac{2n\pi}{5}\right)} \quad \text{for} \quad n = 0, 1, 2, 3, 4$$

$$\sqrt{2}\, e^{j\frac{\pi}{5}}, \quad \sqrt{2}\, e^{j\frac{3\pi}{5}}, \quad \sqrt{2}\, e^{j\pi}, \quad \sqrt{2}\, e^{j\frac{7\pi}{5}}, \quad \sqrt{2}\, e^{j\frac{9\pi}{5}}$$

ii)



iii)

$$\arg(w) = \frac{\pi}{5} + \frac{\frac{2\pi}{5}}{2} = \frac{\pi}{5} + \frac{\pi}{5} = \frac{2\pi}{5}$$

$$\cos\frac{\pi}{5} = \frac{|w|}{\sqrt{2}} \qquad \text{from } \Delta\ OPA$$

$$\therefore\ |w| = \sqrt{2}\cos\frac{\pi}{5}$$

2b iv)

$$w = \left(\sqrt{2}\cos\frac{\pi}{5}\right) e^{j\frac{2\pi}{5}}$$

w^n is real for first time when $n = 5$ (need argument to be whole multiple of π)

$$a = w^5 = \left(\sqrt{2}\cos\frac{\pi}{5}\right)^5 e^{j2\pi} = \left(\sqrt{2}\cos\frac{\pi}{5}\right)^5$$

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