FP2 JUNE 2014 Q2 COMPLEX NUMBERS

2a) $C = a\cos\theta + a^2\cos 2\theta + a^3\cos 3\theta + \ldots$ $|a|<1$

$S = a\sin\theta + a^2\sin 2\theta + a^3\sin 3\theta + \ldots$

$$C + jS = a(\cos\theta + j\sin\theta) + a^2(\cos 2\theta + j\sin\theta) + \ldots$$

$$C + jS = ae^{j\theta} + a^2e^{j2\theta} + a^3e^{j3\theta} + \ldots$$

This is an infinite GP, first term $ae^{j\theta}$
common ratio $ae^{j\theta}$

$$S_\infty = \frac{a}{1-r} \qquad C + jS = \frac{ae^{j\theta}}{1-ae^{j\theta}}$$

$$C + jS = \frac{ae^{j\theta}}{1-ae^{j\theta}} \times \frac{1-ae^{-j\theta}}{1-ae^{-j\theta}}$$

$$C + jS = \frac{ae^{j\theta}(1-ae^{-j\theta})}{1-a(e^{j\theta}+e^{-j\theta})+a^2}$$

$$C + jS = \frac{ae^{j\theta}-a^2}{1-2a\cos\theta+a^2}$$

$$C + jS = \frac{a\cos\theta + ja\sin\theta - a^2}{1-2a\cos\theta+a^2}$$

Equating real and imaginary parts

$$S = \frac{a\sin\theta}{1-2a\cos\theta+a^2} \qquad C = \frac{a\cos\theta-a^2}{1-2a\cos\theta+a^2}$$

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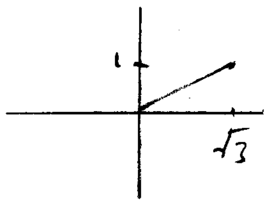
2b) $\sqrt{3} + j$

i)



$|\sqrt{3}+j| = 2$

$\arg(\sqrt{3}+j) = \frac{\pi}{6}$

Vertices of hexagon

$$2e^{j\frac{\pi}{6}},\ 2e^{j\frac{\pi}{6}} \times e^{j\frac{\pi}{3}},\ 2e^{j\frac{\pi}{6}} \times e^{j\frac{2\pi}{3}},\ 2e^{j\frac{\pi}{6}} \times e^{j\pi},$$
$$2e^{j\frac{\pi}{6}} \times e^{\frac{4\pi}{3}},\ 2e^{j\frac{\pi}{6}} \times e^{j\frac{5\pi}{3}}$$

$$= 2e^{j\frac{\pi}{6}},\ 2e^{j\frac{\pi}{2}},\ 2e^{j\frac{5\pi}{6}},\ 2e^{j\frac{7\pi}{6}},\ 2e^{j\frac{3\pi}{2}},\ 2e^{j\frac{11\pi}{6}}$$

Vertices:

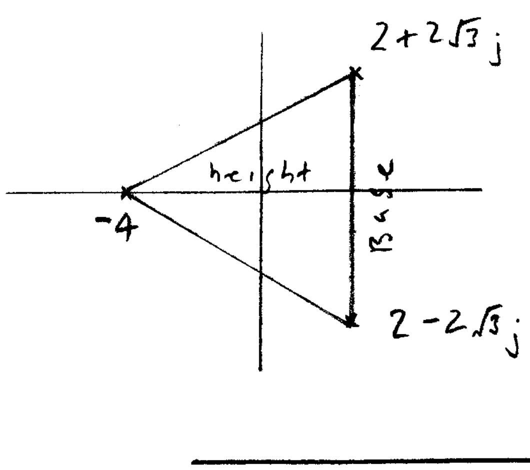
$$\sqrt{3}+j,\ 2j,\ -\sqrt{3}+j,\ -\sqrt{3}-j,\ -2j,\ \sqrt{3}+j$$

ii) New vertices

$$4e^{j\frac{\pi}{3}},\ 4e^{j\pi},\ 4e^{j\frac{5\pi}{3}},\ 4e^{j\frac{7\pi}{3}},\ 4e^{j3\pi},\ 4e^{j\frac{11\pi}{3}}$$

Vertices

$$2+2\sqrt{3}j,\ -4,\ 2-2\sqrt{3}j,\ 2+2\sqrt{3}j,\ -4,\ 2-2\sqrt{3}j$$



Area $= \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} \times 4\sqrt{3} \times 6$

$= 12\sqrt{3}$ units^2