

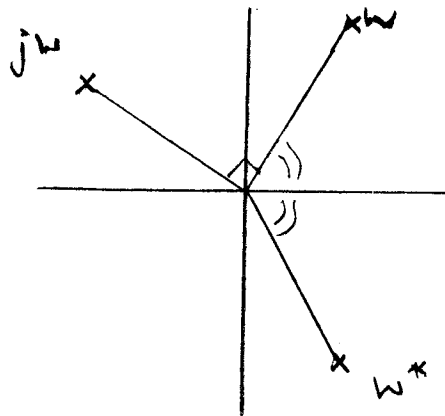
$$2i) \quad 0 < \theta < \frac{\pi}{6}, \quad w = \frac{1}{2} e^{3j\theta}$$

$$|w| = \frac{1}{2} \quad \arg(w) = 3\theta$$

$$|w^*| = \frac{1}{2} \quad \arg(w^*) = -3\theta$$

$$jw = e^{j\frac{\pi}{2}} \times \frac{1}{2} e^{3j\theta} = \frac{1}{2} e^{j(\frac{\pi}{2} + 3\theta)}$$

$$|jw| = \frac{1}{2} \quad \arg(jw) = \frac{\pi}{2} + 3\theta$$



$$2ii) \quad (1+w)(1+w^*) = \left(1 + \frac{1}{2} e^{3j\theta}\right) \left(1 + \frac{1}{2} e^{-3j\theta}\right)$$

$$= 1 + \frac{1}{2} e^{3j\theta} + \frac{1}{2} e^{-3j\theta} + \frac{1}{4}$$

$$= \frac{5}{4} + \frac{1}{2} \left( e^{3j\theta} + e^{-3j\theta} \right)$$

$$= \frac{5}{4} + \frac{1}{2} (2 \cos 3\theta)$$

$$= \frac{5}{4} + \cos 3\theta$$

$$2\text{iii)} \quad C = \cos 2\theta - \frac{1}{2} \cos 5\theta + \frac{1}{4} \cos 8\theta - \frac{1}{8} \cos 11\theta + \dots$$

$$S = \sin 2\theta - \frac{1}{2} \sin 5\theta + \frac{1}{4} \sin 8\theta - \frac{1}{8} \sin 11\theta + \dots$$

$$C + jS = (\cos 2\theta + j \sin 2\theta) - \frac{1}{2} (\cos 5\theta + j \sin 5\theta) + \frac{1}{4} (\cos 8\theta + j \sin 8\theta) - \dots$$

$$C + jS = e^{2j\theta} - \frac{1}{2} e^{5j\theta} + \frac{1}{4} e^{8j\theta} - \frac{1}{8} e^{11j\theta} + \dots$$

This is a GP with first term  $a = e^{2j\theta}$   
 common ratio  $r = -\frac{1}{2} e^{3j\theta}$

since  $|r| = \frac{1}{2} < 1 \quad S_{\infty} = \frac{a}{1-r}$

$$\Rightarrow C + jS = \frac{e^{2j\theta}}{1 + \frac{1}{2} e^{3j\theta}}$$

$$\Rightarrow C + jS = \frac{e^{2j\theta} (1 + \frac{1}{2} e^{-3j\theta})}{(1 + \frac{1}{2} e^{3j\theta})(1 + \frac{1}{2} e^{-3j\theta})}$$

$$\Rightarrow C + jS = \frac{e^{2j\theta} + \frac{1}{2} e^{-j\theta}}{\frac{5}{4} + \cos 3\theta}$$

$$= \frac{\cos 2\theta + j \sin 2\theta + \frac{1}{2} \cos \theta - \frac{1}{2} j \sin \theta}{\frac{5}{4} + \cos 3\theta}$$

$$= \frac{4 \cos 2\theta + 4j \sin 2\theta + 2 \cos \theta - 2j \sin \theta}{5 + 4 \cos 3\theta}$$

2 iii)  
cont)

Equating real and imaginary parts

$$C = \frac{4 \cos 2\theta + 2 \cos \theta}{5 + 4 \cos 3\theta}$$

$$S = \frac{4 \sin 2\theta - 2 \sin \theta}{5 + 4 \cos 3\theta}$$

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