FP2 JUNE 2006 Q4 HYPERBOLIC FUNCTIONS

4 i)

$$1 + 2\sinh^2 x = 1 + 2\left(\tfrac{1}{2}(e^x - e^{-x})\right)^2$$
$$= 1 + 2\left(\tfrac{1}{4}(e^{2x} - 2 + e^{-2x}\right)$$
$$= 1 + \tfrac{1}{2}(e^{2x} + e^{-2x}) - 1$$
$$= \tfrac{1}{2}(e^{2x} + e^{-2x})$$
$$= \cosh 2x$$

ii)

$$2\cosh 2x + \sinh x = 5$$
$$2(1 + 2\sinh^2 x) + \sinh x = 5$$
$$2 + 4\sinh^2 x + \sinh x = 5$$
$$4\sinh^2 x + \sinh x - 3 = 0$$
$$(4\sinh x - 3)(\sinh x + 1) = 0$$

$\Rightarrow$ $4\sinh x - 3 = 0$ or $\sinh x + 1 = 0$

$4\sinh x = 3$ $\qquad$ $\sinh x = -1$

$\sinh x = \frac{3}{4}$

$x = \operatorname{arsinh}\left(\frac{3}{4}\right)$ $\qquad$ $x = \operatorname{arsinh}(-1)$

$x = \ln\left(\frac{3}{4} + \sqrt{\left(\frac{3}{4}\right)^2 + 1}\right)$ $\qquad$ $x = \ln\left(-1 + \sqrt{(-1)^2 + 1}\right)$

$x = \ln\left(\frac{3}{4} + \frac{5}{4}\right)$ $\qquad$ $x = \ln(-1 + \sqrt{2})$

$x = \ln 2$

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4 iii) $$\int_0^{\ln 3} \sinh^2 x \, dx = \int_0^{\ln 3} \frac{1}{2}(\cosh 2x - 1)\, dx$$

$$= \left[\frac{1}{4}\sinh 2x - \frac{x}{2}\right]_0^{\ln 3}$$

$$= \left(\frac{1}{4}\sinh(2\ln 3) - \frac{1}{2}\ln 3\right) - (0 - 0)$$

$$= \frac{1}{4}\sinh(\ln 9) - \frac{1}{2}\ln 3$$

$$= \frac{1}{4} \times \frac{1}{2}\left(e^{\ln 9} - e^{-\ln 9}\right) - \frac{1}{2}\ln 3$$

$$= \frac{1}{8}\left(9 - \frac{1}{9}\right) - \frac{1}{2}\ln 3$$

$$= \frac{1}{8}\left(\frac{81-1}{9}\right) - \frac{1}{2}\ln 3$$

$$= \frac{10}{9} - \frac{1}{2}\ln 3$$

4 iv) $$\int_3^5 \sqrt{x^2 - 9}\, dx$$

Let $x = 3\cosh u$

$\frac{dx}{du} = 3\sinh u$

$dx = 3\sinh u \, du$

Also $\frac{x}{3} = \cosh u$

so $\operatorname{arcosh}\left(\frac{x}{3}\right) = u$

when $x = 5$, $u = \operatorname{arcosh}(5/3) = \ln\left(5/3 + \sqrt{(5/3)^2 - 1}\right)$

$= \ln\left(\frac{5}{3} + \frac{4}{3}\right) = \ln 3$

4iv cont) when $x = 3$, $u = \text{arcosh}\,1 = \ln\left(1 + \sqrt{1^2 - 1}\right)$

$= \ln(1) = 0$

Integral becomes

$$\int_0^{\ln 3} \sqrt{9\cosh^2 u - 9} \times 3\sinh u \, du$$

$$= \int_0^{\ln 3} 3\sqrt{\cosh^2 u - 1} \times 3\sinh u \, du$$

$$= \int_0^{\ln 3} 3\sinh u \times 3\sinh u \, du$$

$$= \int_0^{\ln 3} 9\sinh^2 u \, du$$

using result from part (iii)

$$= 9\left[\frac{10}{9} - \frac{1}{2}\ln 3\right]$$

$$= 10 - \frac{9}{2}\ln 3$$