FP2 JUNE 2007 Q4 HYPERBOLIC FUNCTIONS

4 a) $$\int_0^1 \frac{1}{\sqrt{9x^2+16}}\,dx = \int_0^1 \frac{1}{\sqrt{9\left(x^2+\frac{16}{9}\right)}}\,dx$$

$$= \frac{1}{3}\int_0^1 \frac{1}{\sqrt{x^2+\left(\frac{4}{3}\right)^2}}\,dx$$

$$= \frac{1}{3}\left[\ln\left(x+\sqrt{x^2+\left(\frac{4}{3}\right)^2}\right)\right]_0^1$$

$$= \frac{1}{3}\left[\ln\left(1+\sqrt{1+\tfrac{16}{9}}\right) - \ln\left(0+\tfrac{4}{3}\right)\right]$$

$$= \frac{1}{3}\left[\ln\left(1+\tfrac{5}{3}\right) - \ln\left(\tfrac{4}{3}\right)\right]$$

$$= \frac{1}{3}\left[\ln\left(\tfrac{8}{3}\right) - \ln\left(\tfrac{4}{3}\right)\right]$$

$$= \frac{1}{3}\ln\left(\frac{8/3}{4/3}\right)$$

$$= \frac{1}{3}\ln 2$$

4b) i) $$2\sinh x\cosh x = 2\left(\tfrac{1}{2}(e^x-e^{-x})\times\tfrac{1}{2}(e^x+e^{-x})\right)$$

$$= \frac{1}{2}\left(e^{2x}-1+1-e^{-2x}\right)$$

$$= \frac{1}{2}\left(e^{2x}-e^{-2x}\right)$$

$$= \sinh 2x$$

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4b) $y = 20\cosh x - 3\cosh 2x$

ii) $\frac{dy}{dx} = 20\sinh x - 6\sinh 2x$

At st.pt $\frac{dy}{dx} = 0$

$\Rightarrow \quad 20\sinh x - 6\sinh 2x = 0$

$20\sinh x - 12\sinh x\cosh x = 0$

$4\sinh x(5 - 3\cosh x) = 0$

$\Rightarrow \quad \sinh x = 0 \quad$ or $\quad 5 - 3\cosh x = 0$

$x = 0 \qquad 5 = 3\cosh x$

$\frac{5}{3} = \cosh x$

$x = \pm \operatorname{arcosh}\frac{5}{3}$

$x = \pm \ln\left(\frac{5}{3} + \sqrt{\left(\frac{5}{3}\right)^2 - 1}\right)$

$x = \pm \ln\left(\frac{5}{3} + \frac{4}{3}\right)$

$x = \pm \ln 3$

When $x = \ln 3$,

$y = 20 \times \frac{5}{3} - 3\cosh(2\ln 3)$

$y = \frac{100}{3} - 3\cosh(\ln 9)$

$y = \frac{100}{3} - \frac{3}{2}\left(9 + \frac{1}{9}\right)$

$y = \frac{100}{3} - \frac{3}{2} \times \frac{82}{9}$

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4bii cont)

$$y = \frac{100}{3} - \frac{41}{3} = \frac{59}{3}$$

st pt at $\left(\ln 3, \frac{59}{3}\right)$

When $x = -\ln 3$

$$y = 20 \times \frac{5}{3} - 3\cosh(-2\ln 3)$$

$$y = \frac{100}{3} - 3\cosh\left(\ln\left(\tfrac{1}{9}\right)\right)$$

$$y = \frac{100}{3} - \frac{3}{2}\left(\frac{1}{9} + 9\right)$$

$$y = \frac{59}{3} \quad \text{as before}$$

st pt at $\left(-\ln 3, \frac{59}{3}\right)$

When $x = 0$

$$y = 20\cosh 0 - 3\cosh 0$$

$$y = 20 - 3 = 17$$

st.pt. at $(0, 17)$

4b) iii)

$$\int_{-\ln 3}^{\ln 3} \left(20\cosh x - 3\cosh 2x\right) dx$$

$$= \left[\, 20\sinh x - \frac{3}{2}\sinh 2x \,\right]_{-\ln 3}^{\ln 3}$$

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4b iii cont)

$$= \left[20 \times \tfrac{1}{2}\left(e^{x} - e^{-x}\right) - \tfrac{3}{2} \times \tfrac{1}{2}\left(e^{2x} - e^{-2x}\right)\right]_{-\ln 3}^{\ln 3}$$

$$= \left[10\left(e^{\ln 3} - e^{-\ln 3}\right) - \tfrac{3}{4}\left(e^{2\ln 3} - e^{-2\ln 3}\right)\right]$$

$$- \left[10\left(e^{-\ln 3} - e^{\ln 3}\right) - \tfrac{3}{4}\left(e^{-2\ln 3} - e^{2\ln 3}\right)\right]$$

$$= \left[10\left(3 - \tfrac{1}{3}\right) - \tfrac{3}{4}\left(9 - \tfrac{1}{9}\right)\right]$$

$$- \left[10\left(\tfrac{1}{3} - 3\right) - \tfrac{3}{4}\left(\tfrac{1}{9} - 9\right)\right]$$

$$= \left[\tfrac{80}{3} - \tfrac{20}{3}\right] - \left[-\tfrac{80}{3} + \tfrac{20}{3}\right]$$

$$= 20 - -20$$

$$= 40$$