FP2 JUNE 2008 Q2 COMPLEX NUMBERS

2 i)

$$z = \sqrt{32}(1+j) \qquad w = 8\left(\cos\frac{7\pi}{12} + j\sin\frac{7\pi}{12}\right)$$

$$|z| = \sqrt{32} \times \sqrt{1^2+1^2} = \sqrt{32} \times \sqrt{2} = \sqrt{64} = 8$$

$$\arg(z) = \frac{\pi}{4}$$

$$|w| = 8$$

$$\arg(w) = \frac{7\pi}{12}$$

$$|zw| = |z| \times |w| = 8 \times 8 = 64$$

$$\arg(zw) = \arg(z) + \arg(w) = \frac{\pi}{4} + \frac{7\pi}{12} = \frac{5\pi}{6}$$

$$\left|\frac{z}{w}\right| = \frac{|z|}{|w|} = \frac{8}{8} = 1$$

$$\arg\left(\frac{z}{w}\right) = \arg(z) - \arg(w)$$

$$= \frac{\pi}{4} - \frac{7\pi}{12}$$

$$= -\frac{\pi}{3}$$

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2 ii)

$$\frac{z}{w} = 1e^{-j\frac{\pi}{3}} = 1\left(\cos\left(-\frac{\pi}{3}\right) + j\sin\left(-\frac{\pi}{3}\right)\right)$$

$$= \frac{1}{2} - \frac{\sqrt{3}}{2}j$$

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2 iii) $z = \sqrt{32}(1+j) = 8e^{j\frac{\pi}{4}}$

Cube roots given by $8^{\frac{1}{3}}e^{j\left(\frac{\pi}{12}+\frac{2n\pi}{3}\right)}$ for $n = 0, 1, 2$

$= 2e^{j\frac{\pi}{12}}, \quad 2e^{j\frac{3\pi}{4}}, \quad 2e^{-j\frac{7\pi}{12}}$

2 iv) $w = 8e^{j\frac{7\pi}{12}} \Rightarrow w^* = 8e^{-j\frac{7\pi}{12}}$

$\therefore 2e^{-j\frac{7\pi}{12}} = \frac{1}{4}w^*$

$z = 8e^{j\frac{\pi}{4}} \Rightarrow z^* = 8e^{-j\frac{\pi}{4}}$

$\therefore 2e^{j\frac{3\pi}{4}} = -2e^{-j\frac{\pi}{4}} = -\frac{1}{4}z^*$

$w = 8e^{j\frac{7\pi}{12}} \Rightarrow jw = 8e^{j\left(\frac{7\pi}{12}+\frac{\pi}{2}\right)} = 8e^{-j\frac{11\pi}{12}}$

$\therefore 2e^{j\frac{\pi}{12}} = -2e^{-j\frac{11\pi}{12}} = -\frac{1}{4}jw$