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1) (A) False - if it were a distance time graph it would indicate he finished where he started. However, it is a speed time graph

(B) True

(C) True

(D) False - travels

$\frac{1}{2} \times 10 \times 8 + 45 \times 8 + \frac{1}{2} \times 5 \times 8$

$= 40 + 360 + 20 = 420\text{m}$

2) i) $a = 6t - 12$

At $t = 0$, $v = 9\text{ ms}^{-1}$
$s = -2\text{m}$

$v = \int a\,dt = \int (6t - 12)\,dt$

$v = 3t^2 - 12t + c$

Sub for $t = 0, v = 9$

$9 = 0 - 0 + c$

$9 = c$

$\Rightarrow v = 3t^2 - 12t + 9$

When $t = 3$

$v = 3(3)^2 - 12(3) + 9$

$= 27 - 36 + 9$

$= 0\text{ ms}^{-1}$ so stationary

2 ii) $s = \int v\,dt$

$s = \int (3t^2 - 12t + 9)\,dt$

$s = t^3 - 6t^2 + 9t + k$

Sub $t = 0$, $s = -2$

$-2 = k$

$\therefore s = t^3 - 6t^2 + 9t - 2$

When $t = 2$

$s = 2^3 - 6(2)^2 + 9(2) - 2$

$= 8 - 24 + 18 - 2$

$= 0$

Particle is at origin

3) $\underline{P} = 5\underline{i} + 4\underline{j}$

$\underline{Q} = 3\underline{i} - 5\underline{j}$

$\underline{R} = -8\underline{i} + \underline{j}$

i) $\underline{P} + \underline{Q} + \underline{R} = \underline{0}$

ii) A) Resultant force $\underline{0}$

Particle at rest or has constant velocity (if no other forces)

B) Hiker ends walk at position he started

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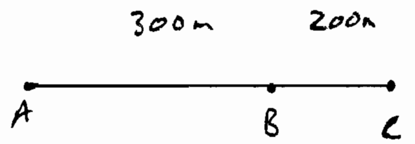
4) i)



using $s = ut + \frac{1}{2}at^2$

$$s - ut = \frac{1}{2}at^2$$

$$\frac{2(s-ut)}{t^2} = a$$

$$a = \frac{2(500 - 5\times 20)}{20^2}$$

$$a = 2 \text{ ms}^{-2}$$

ii) using $v^2 = u^2 + 2as$

$$v^2 = 5^2 + 2\times 2\times 300$$

$$v^2 = 1225$$

$$v = 35 \text{ ms}^{-1}$$

using $v = u + at$

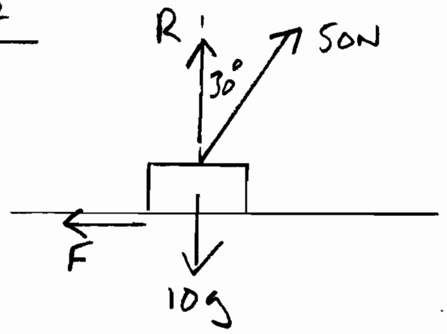
$$v - u = at$$

$$\frac{v-u}{a} = t$$

$$t = \frac{35-5}{2}$$

$$t = 15 \text{ s}$$

5) i)



ii) Resolve $\leftrightarrow$

$$F = 50 \sin 30$$

$$F = 25 \text{ N}$$

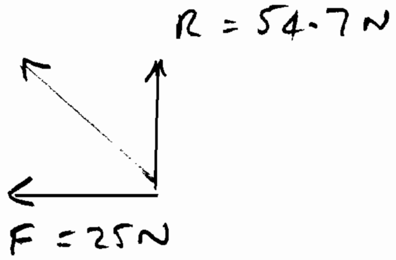
iii) Resolve $\updownarrow$

$$R + 50\cos 30° = 10g$$

$$R = 10\times 9.8 - 50\cos 30°$$

$$R = 54.7 \text{ N}$$

iv)



Magnitude of total force exerted by floor

$$= \sqrt{25^2 + 54.7^2}$$

$$= 60.1 \text{ N}$$

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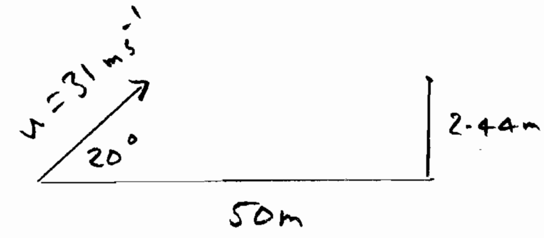
6) i)



Horizontal Motion

$$u_x = 31\cos 20$$

Time t to travel 50m

$$t = \frac{50}{31\cos 20°} = 1.716\text{ s}$$

Vertical Motion

$$y = u_y t + \tfrac{1}{2}at^2$$

$$y = 31\sin 20 \times t - 4.9t^2$$

$$y = 31\sin 20 \times 1.716 - 4.9 \times 1.716^2$$

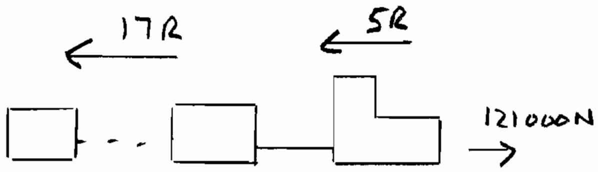
$$y = 3.77\text{ m}$$

Height of ball is 3.77m when it reaches goal so yes it goes over crossbar.

ii) Assumed no air resistance

Section B

7) i)



Trucks
mass $17 \times 40 \times 1000$
680 000 kg

Loco
120000 kg

Using $F = ma$

$$121000 - 22R = (680000 + 120000) \times 0.11$$

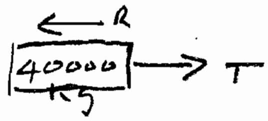
$$121000 - 800000 \times 0.11 = 22R$$

$$R = \frac{121000 - 88000}{22}$$

$$R = 1500\text{ N}$$

7ii) Last truck

A)



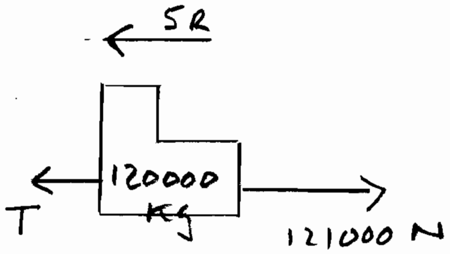
$$F = ma$$

$$T - R = 40000 \times 0.11$$

$$T = 40000 \times 0.11 + 1500$$

$$T = 5900\text{ N}$$

B) First truck and loco



Using $F = ma$

$$121000 - T - 5R = 120000 \times 0.11$$

$$121000 - 5 \times 1500 - 13200 = T$$

$$121000 - 7500 - 13200 = T$$

$$T = 100{,}300\text{ N}$$

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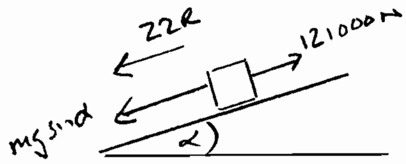
7 iii)



$\sin\alpha = \frac{1}{80}$

Using $F = ma$ (up slope)

$121000 - mg\sin\alpha - 22R = ma$

$121000 - 800000 \times 9.8 \times \frac{1}{80} - 22 \times 1500$

$= 800000a$

$-10000 = 800000a$

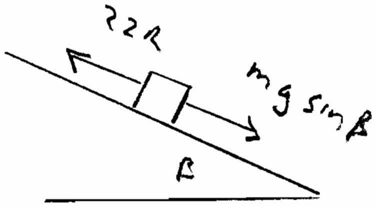
$-0.0125 = a$

$a = -0.0125 \text{ ms}^{-2}$

so train is decelerating

ie acceleration is down slope

iv)



Constant speed so

$22R = mg\sin\beta$

$\frac{22 \times 1500}{800000 \times 9.8} = \sin\beta$

$\sin\beta = 0.0042$

$\beta = 0.24^\circ$

8) Rosemary

i) $\underline{r} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 8 \end{pmatrix} t$

At A, $t = 0$

$\underline{r} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 8 \end{pmatrix} 0$

$\therefore A(3, 2)$

At B, $t = 2$

$\underline{r} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 8 \end{pmatrix} 2$

$\underline{r} = \begin{pmatrix} 15 \\ 18 \end{pmatrix}$

$\therefore B(15, 18)$

$|AB| = \sqrt{(15-3)^2 + (18-2)^2}$

$|AB| = 20 \text{ Km}$

ii) For Rosemary

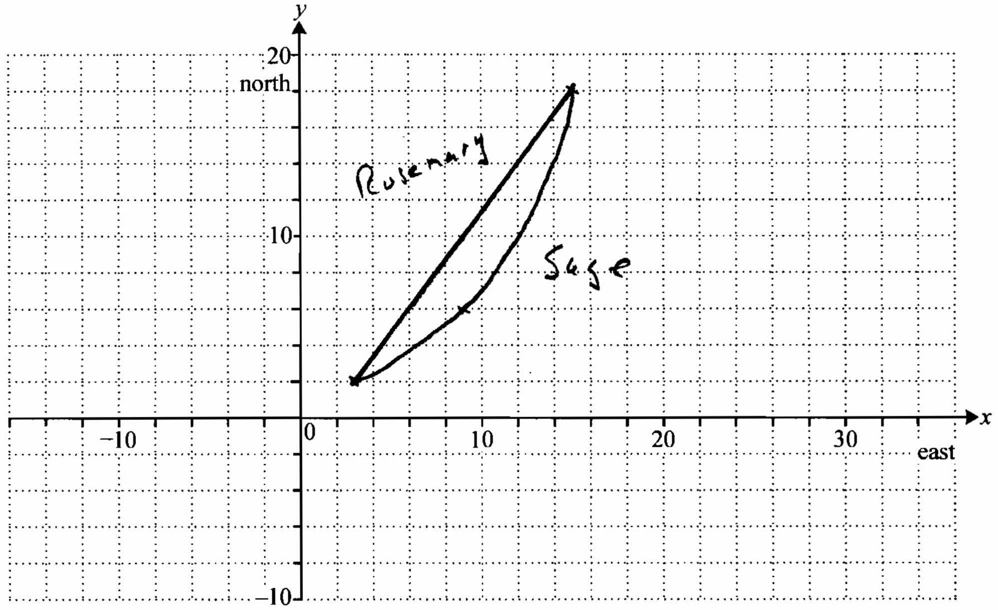
$\underline{v} = \frac{dr}{dt} = \frac{d}{dt}\left(\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 8 \end{pmatrix} t\right)$

$= \begin{pmatrix} 6 \\ 8 \end{pmatrix} \text{ Kmh}^{-1}$

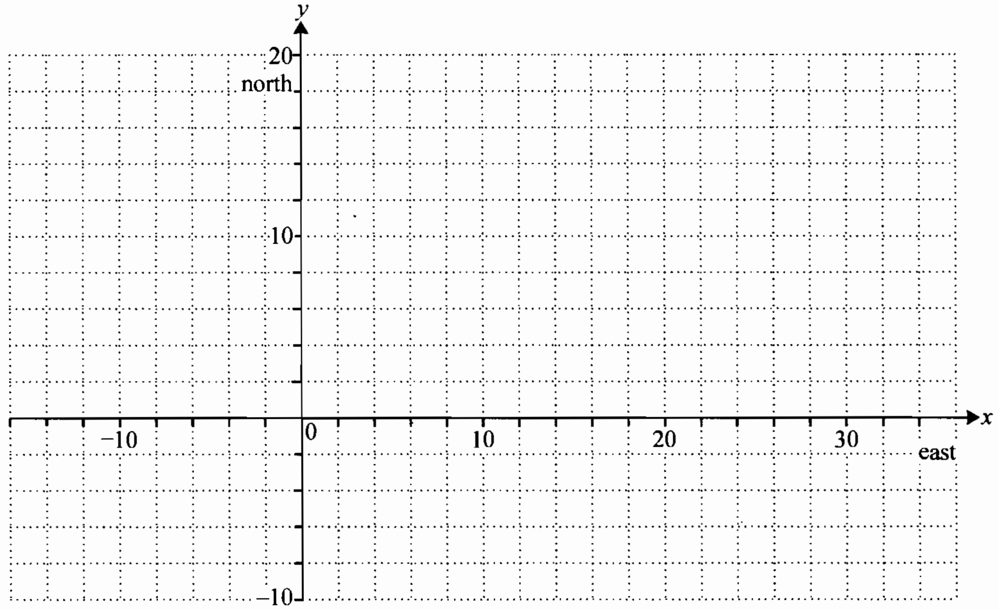
which is constant

8 iii) and iv) on next sheet

8 (iii)



Spare copy of grid for 8(iii)



8 (iv)

When $t = 1$ Sage $\underline{r} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$ when $t = 2$ $\underline{r} = \begin{pmatrix} 15 \\ 18 \end{pmatrix}$

$\underline{r} = \begin{pmatrix} 3(2t+1) \\ 2(2t^2+1) \end{pmatrix}$

Result is a draw as they both arrive at B at $t = 2$

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8 v) Sage

$$\underline{r} = \begin{pmatrix} 6t+3 \\ 4t^2+2 \end{pmatrix}$$

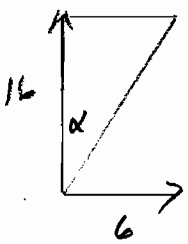
$$\underline{v} = \frac{dr}{dt} = \begin{pmatrix} 6 \\ 8t \end{pmatrix}$$

When $t=2$

$$\underline{v} = \begin{pmatrix} 6 \\ 16 \end{pmatrix}$$

$$\text{Speed} = \sqrt{6^2+16^2}$$

$$= 17.1 \text{ kmh}^{-1}$$



$$\alpha = \tan^{-1}\frac{6}{16} = 20.6°$$

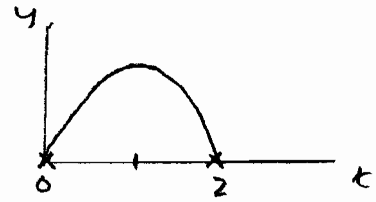
Bearing $021°$ to nearest degree

vi) Displacement of Rosemary from Sage $= \underline{r}_R - \underline{r}_S$

$$= \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 8 \end{pmatrix}t - \begin{pmatrix} 6t+3 \\ 4t^2+2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 8t-4t^2 \end{pmatrix}$$

$$y = 8t - 4t^2$$



Negative parabola – by symmetry max when $t=1$

$$8(1) - 4(1)^2 = 4$$

Max displacement of R from S during race

$$\begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

∴ max distance between Rosemary and Sage $= 4$ km

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