

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education  
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

4761

Mechanics 1

Friday

14 JANUARY 2005

Morning

1 hour 30 minutes

Additional materials:

Answer booklet

Graph paper

MEI Examination Formulae and Tables (MF2)

**TIME** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .
- The total number of marks for this paper is 72.

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**This question paper consists of 5 printed pages and 3 blank pages.**

## Section A (36 marks)

- 1 The position vector,  $\mathbf{r}$ , of a particle of mass 4 kg at time  $t$  is given by

$$\mathbf{r} = t^2 \mathbf{i} + (5t - 2t^2) \mathbf{j},$$

where  $\mathbf{i}$  and  $\mathbf{j}$  are the standard unit vectors, lengths are in metres and time is in seconds.

- (i) Find an expression for the acceleration of the particle. [4]

The particle is subject to a force  $\mathbf{F}$  and a force  $12\mathbf{j}$  N.

- (ii) Find  $\mathbf{F}$ . [3]

- 2 Particles of mass 2 kg and 4 kg are attached to the ends X and Y of a light, inextensible string. The string passes round fixed, smooth pulleys at P, Q and R, as shown in Fig. 2. The system is released from rest with the string taut.

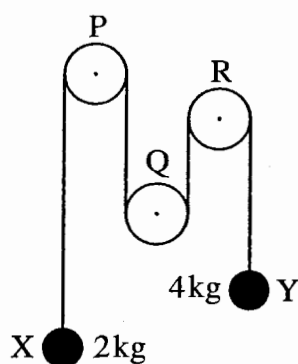


Fig. 2

- (i) State what information in the question tells you that
- (A) the tension is the same throughout the string,
- (B) the magnitudes of the accelerations of the particles at X and Y are the same. [2]

The tension in the string is  $T$  N and the magnitude of the acceleration of the particles is  $a$  m s<sup>-2</sup>.

- (ii) Draw a diagram showing the forces acting at X and a diagram showing the forces acting at Y. [1]
- (iii) Write down equations of motion for the particles at X and at Y. Hence calculate the values of  $T$  and  $a$ . [5]

- 3 A particle is in equilibrium when acted on by the forces  $\begin{pmatrix} x \\ -7 \\ z \end{pmatrix}$ ,  $\begin{pmatrix} 4 \\ y \\ -5 \end{pmatrix}$  and  $\begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix}$ , where the units are newtons.

(i) Find the values of  $x$ ,  $y$  and  $z$ . [4]

(ii) Calculate the magnitude of  $\begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix}$ . [2]

- 4 A particle is projected vertically upwards from a point O at  $21 \text{ ms}^{-1}$ .

(i) Calculate the greatest height reached by the particle. [2]

When this particle is at its highest point, a second particle is projected vertically upwards from O at  $15 \text{ ms}^{-1}$ .

(ii) Show that the particles collide 1.5 seconds later and determine the height above O at which the collision takes place. [6]

- 5 A small box B of weight  $400 \text{ N}$  is held in equilibrium by two light strings AB and BC. The string BC is fixed at C. The end A of string AB is fixed so that AB is at an angle  $\alpha$  to the vertical where  $\alpha < 60^\circ$ . String BC is at  $60^\circ$  to the vertical. This information is shown in Fig. 5.

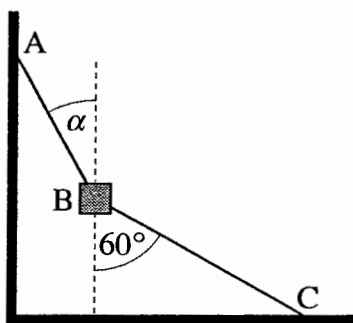


Fig. 5

(i) Draw a labelled diagram showing all the forces acting on the box. [1]

(ii) In one situation string AB is fixed so that  $\alpha = 30^\circ$ .

By drawing a triangle of forces, or otherwise, calculate the tension in the string BC and the tension in the string AB. [4]

(iii) Show carefully, but briefly, that the box cannot be in equilibrium if  $\alpha = 60^\circ$  and BC remains at  $60^\circ$  to the vertical. [2]

- 7 The trajectory ABCD of a small stone moving with negligible air resistance is shown in Fig. 7. AD is horizontal and BC is parallel to AD.

The stone is projected from A with speed  $40 \text{ m s}^{-1}$  at  $50^\circ$  to the horizontal.

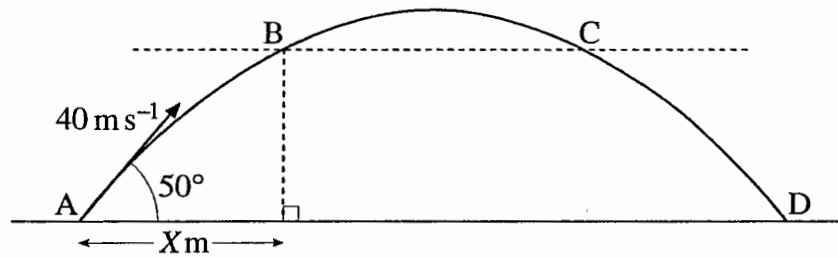


Fig. 7

- (i) Write down an expression for the horizontal displacement from A of the stone  $t$  seconds after projection. Write down also an expression for the vertical displacement at time  $t$ . [3]
- (ii) Show that the stone takes 6.253 seconds (to three decimal places) to travel from A to D. Calculate the range of the stone. [5]

You are given that  $X = 30$ .

- (iii) Calculate the time it takes the stone to reach B. Hence determine the time for it to travel from A to C. [4]
- (iv) Calculate the direction of the motion of the stone at C. [5]

## Section B (36 marks)

6 In this question take  $g$  as  $10 \text{ m s}^{-2}$ .

A small ball is released from rest. It falls for 2 seconds and is then brought to rest over the next 5 seconds. This motion is modelled in the speed-time graph Fig. 6.

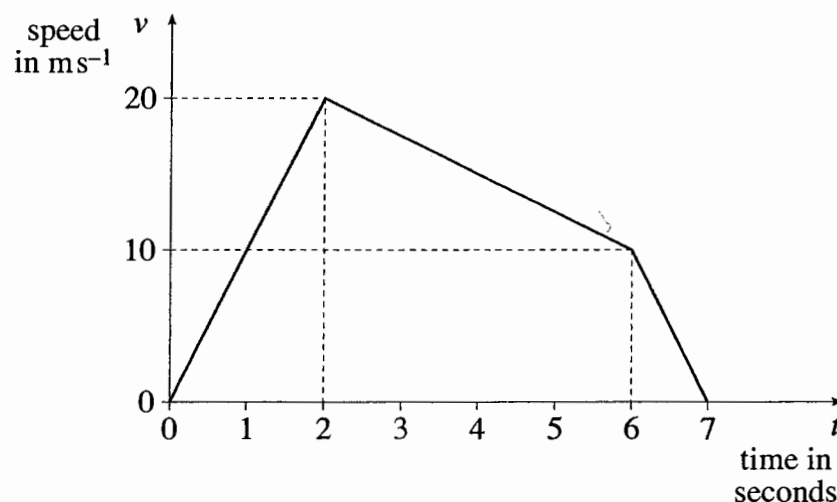


Fig. 6

For this model,

- (i) calculate the distance fallen from  $t = 0$  to  $t = 7$ , [3]
- (ii) find the acceleration of the ball from  $t = 2$  to  $t = 6$ , specifying the direction, [3]
- (iii) obtain an expression in terms of  $t$  for the downward speed of the ball from  $t = 2$  to  $t = 6$ , [3]
- (iv) state the assumption that has been made about the resistance to motion from  $t = 0$  to  $t = 2$ . [1]

The part of the motion from  $t = 2$  to  $t = 7$  is now modelled by  $v = -\frac{3}{2}t^2 + \frac{19}{2}t + 7$ .

- (v) Verify that  $v$  agrees with the values given in Fig. 6 at  $t = 2$ ,  $t = 6$  and  $t = 7$ . [2]
- (vi) Calculate the distance fallen from  $t = 2$  to  $t = 7$  according to this model. [7]

# Mark Scheme

**Solutions and mark scheme**

Q 1		mark		
(i)	Differentiate $\mathbf{v} = 2t \mathbf{i} + (5 - 4t) \mathbf{j}$  Differentiate $\mathbf{a} = 2 \mathbf{i} - 4 \mathbf{j}$	M1 A1  M1 F1	At least 1 cpt correct Award for RHS seen  Do not award if $\mathbf{i}$ and $\mathbf{j}$ lost in $\mathbf{v}$ . At least 1 cpt correct. FT FT from <b>their</b> 2 component $\mathbf{v}$	4
(ii)	$\mathbf{F} + 12 \mathbf{j} = 4(2 \mathbf{i} - 4 \mathbf{j})$  $\mathbf{F} = 8 \mathbf{i} - 28 \mathbf{j}$	M1 A1  A1	N2L. Allow $\mathbf{F} = mg \mathbf{a}$ . No extra forces. Allow $12\mathbf{j}$ omitted Allow wrong signs otherwise correct with <b>their</b> vector $\mathbf{a}$ .  cao	3
	total	7		

Q 2		mark		
(i) (A) (B)	the pulleys are smooth and the string is light the string is inextensible	E1 E1	Accept only 'the pulley is smooth'.	2
(ii)	Diagrams	B1	All forces present with labels and arrows. Acc not reqd.	1
	For X, N2L upwards $T - 2g = 2a$  For Y, N2L downwards $4g - T = 4a$ Solve for $a$ and $T$ $a = \frac{g}{3}$ (3.27 (3 s. f.)) $T = \frac{8}{3}g$ (26.1 (3 s. f.))	M1 A1 A1 A1 F1	N2L. Allow $F = mga$ . All forces present Award for equation for X or Y or combined Any form  Any form  FT second answer	5
	total	8		

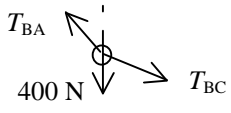
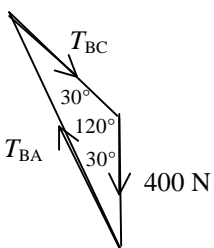
**Solutions and mark scheme**

Q3		mark	
(i)	$\begin{pmatrix} x \\ -7 \\ z \end{pmatrix} + \begin{pmatrix} 4 \\ y \\ -5 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ Equating components gives $x = -9, y = 3, z = 12$	M1 A1 A1 A1	[Allow SC 2/4 if 9, -3, -12 obtained] 4
(ii)	We need $\sqrt{5^2 + 4^2 + (-7)^2}$ $= \sqrt{90}$ or 9.48683... so 9.49 (3 s. f.)	M1 A1	Any reasonable accuracy 2
total		6	

Q4		mark	
(i)	Height reached by first particle is given by $0 = 21^2 - 2 \times 9.8 \times s$ so $s = 22.5$ so 22.5 m	M1 A1	Other methods must be complete. Allow $g = \pm 9.8, \pm 10$ Accept with consistent signs 2
(ii)	<b>Sol (1)</b> $t$ seconds after second particle projected its height is $15t - 4.9t^2$ and the first particle has height $22.5 - 4.9t^2$ (or $21t - 4.9t^2$ )  <b>either</b> Sub $t = 1.5$ to show both have same value State height as 11.475 m  <b>or</b> $15t - 4.9t^2 = 22.5 - 4.9t^2$ giving $t = 1.5$ and height as 11.475 m	M1 A1 M1 A1 E1 A1 M1 A1	Allow $g = \pm 9.8, \pm 10$ Allow $g = \pm 9.8, \pm 10$ Award only if used correctly (or sub $t = 3.64$ into $21t - 4.9t^2$ for 1 <sup>st</sup> & $t = 1.5$ for 2 <sup>nd</sup> ) cao. Accept any reasonable accuracy. Don't award if only one correctly used equation obtained. Both. $t$ shown. Ht cao (to any reasonable accuracy)
	<b>Sol (2)</b> $t$ seconds after second particle projected its height is $15t - 4.9t^2$ and the first particle has fallen $4.9t^2$  Collide when $15T - 4.9T^2 + 4.9T^2 = 22.5$ so $T = 1.5$ $H = 22.5 - 4.9 \times 1.5^2 = 11.475$ m	M1 A1 B1 M1 E1 A1	Allow $g = \pm 9.8, \pm 10$ Or other correct method cao. Accept any reasonable accuracy. Don't award if only one correctly used equation obtained. 6
total		8	



**Solutions and mark scheme**

Q5		mark	
(i)		B1	Different labels. All forces present with arrows in correct directions. Condone no angles.
(ii)	<p>Using triangle of forces</p>  <p>Triangle isosceles so tension in BC is 400 N  Tension in BA is <math>2 \times 400 \times \cos 30 = 400\sqrt{3}</math> N  (693 N, (3 s. f.))</p>	M1  B1  A1 F1	<p>Attempt at triangle of forces. Ignore angles and arrows. Accept 90, 60, 30 triangle.</p> <p>Triangle, arrows, labels and angles correct</p> <p>cao  FT BC only</p> <p>[If resolution used, M1 for 1 equn; M1 for 2<sup>nd</sup> equn + attempt to elim; A1; F1. For M marks all forces present but allow <math>s \leftrightarrow c</math> and sign errors. No extra forces.  If Lami used:  M1 first pair of equations in correct format, condone wrong angles. A1. M1 second pair in correct format, with <b>correct</b> angles. F1 FT their first answer if necessary.]</p>
(iii)	<p>Resolve at B perpendicular to the line ABC</p> <p>Weight has unbalanced component in this direction</p>	E1  E1	<p>Attempt to argue unbalanced force</p> <p>Complete, convincing argument.</p> <p>[<b>or</b> Resolve horiz and establish tensions equal E1  Resolve vert to show inconsistency. E1]</p>
total		7	1  4  2

### Solutions and mark scheme

Q 6		mark		
(i)	Area under curve  $0.5 \times 2 \times 20 + 0.5 \times (20 + 10) \times 4 + 0.5 \times 10 \times 1$  $= 85 \text{ m}$	M1 B1 A1	Attempt to find any area under curve or use const accn results Any area correct (Accept 20 or 60 or 5 without explanation) cao	3
(ii)	$\frac{20 - 10}{4} = 2.5$ upwards	M1 A1 B1	$\Delta v / \Delta t$ accept $\pm 2.5$ Accept $-2.5$ downwards (allow direction specified by diagram etc). Accept 'opposite direction to motion'.	3
(iii)	$v = -2.5t + c$ $v = 20$ when $t = 2$ $v = -2.5t + 25$	M1 M1 A1	Allow <b>their</b> $a$ in the form $v = \pm at + c$ or $v = \pm a(t - 2) + c$ cao [Allow $v = 20 - 2.5(t - 2)$ ] [Allow 2/3 for different variable to $t$ used, e.g. $x$ . Allow any variable name for speed]	3
(iv)	Falling with negligible resistance	E1	Accept 'zero resistance', or 'no resistance' seen.	1
(v)	$-1.5 \times 4 + 9.5 \times 2 + 7 = 20$ $-1.5 \times 36 + 9.5 \times 6 + 7 = 10$ $-1.5 \times 49 + 9.5 \times 7 + 7 = 0$	E1 E1	One of the results shown All three shown. Be generous about the 'show'.	2
(vi)	$\int_2^7 (-1.5t^2 + 9.5t + 7) dt$ $= \left[ -0.5t^3 + 4.75t^2 + 7t \right]_2^7$  $= \left( -\frac{343}{2} + \frac{19 \times 49}{4} + 49 \right) - (-4 + 19 + 14)$  $= 81.25 \text{ m}$	M1 A1 A1 A1 M1 A1 A1	Limits not required A1 for each term. Limits not required. Condone $+ c$ Attempt to use both limits on an integrated expression Correct substitution in <b>their</b> expression including subtraction ( may be left as an expression). cao.	7
	total	19		

**Solutions and mark scheme**

Q7		mark		
(i)	<p>Horiz <math>(40 \cos 50)t</math></p> <p>Vert <math>(40 \sin 50)t - 4.9t^2</math></p>	<p>B1</p> <p>M1</p> <p>A1</p>	<p>Use of <math>s = ut + 0.5at^2</math> with <math>a = \pm 9.8</math> or <math>\pm 10</math>.</p> <p>Allow <math>u = 40</math>. Condone <math>s \leftrightarrow c</math>.</p> <p>Any form</p>	3
(ii)	<p>Need <math>(40 \sin 50)t - 4.9t^2 = 0</math></p> <p>so <math>t = \frac{40 \sin 50}{4.9}</math></p> <p>= 6.2534... so 6.253 s (3 d. p.)</p> <p>Range is <math>(40 \cos 50) \times 6.2534...</math></p> <p>= 160.78... so 161 m (3 s. f.)</p>	<p>M1</p> <p>M1</p> <p>E1</p> <p>M1</p> <p>A1</p>	<p>Equating <b>their</b> <math>y</math> to zero. Allow quadratic <math>y</math> only</p> <p>Dep on 1<sup>st</sup> M1. Attempt to solve.</p> <p>Clearly shown [or M1 (allow <math>u = 40</math> and <math>s \leftrightarrow c</math>) A1 time to greatest height; E1]</p> <p>Use of <b>their</b> horiz expression</p> <p>Any reasonable accuracy</p>	5
(iii)	<p>Time AB is given by</p> <p><math>(40 \cos 50)T = 30</math> so <math>T = 1.16679...</math> so 1.17 s</p> <p><b>then</b></p> <p><b>either</b></p> <p>By symmetry, time AC is time AD – time AB</p> <p>so time AC is <math>6.2534... - \frac{30}{40 \cos 50}</math></p> <p>= 5.086.... so 5.09 s (3 s. f.)</p> <p><b>or</b></p> <p>height is <math>(40 \sin 50)T - 4.9T^2</math></p> <p>and we need</p> <p><math>(40 \sin 50)t - 4.9t^2 = (40 \sin 50)T - 4.9T^2</math></p> <p>solved for larger root</p> <p>i.e. solve <math>4.9t^2 - (40 \sin 50)t + 29.08712... = 0</math></p> <p>for larger root giving 5.086...</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Equating <b>their</b> linear <math>x</math> to 30.</p> <p>Symmetry need not be explicit. Method may be implied. Any valid method using symmetry.</p> <p>cao</p> <p>Complete method to find time to second occasion at that height</p> <p>cao</p>	4
(iv)	<p><math>\mathcal{H} = 40 \cos 50</math></p> <p><math>\mathcal{H} = 40 \sin 50 - 9.8 \times 5.086...</math></p> <p>Need <math>\arctan \frac{\mathcal{H}}{\mathcal{H}}</math></p> <p>So <math>-36.761...^\circ</math></p> <p>so <math>36.8^\circ</math> below horizontal (3 s.f.)</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Must be part of a method using velocities.</p> <p>Use of vert cpt of vel Allow only sign error.</p> <p>FT use of <b>their</b> 5.086..</p> <p>May be implied. Accept <math>\arctan \frac{\mathcal{H}}{\mathcal{H}}</math> but not use of <math>\frac{\mathcal{H}}{\mathcal{H}}</math>.</p> <p>Accept <math>\pm 36.8</math> or equivalent. Condone direction not clear.</p>	5
	total	17		

# Examiner's Report