OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4761

Mechanics 1

Tuesday **10 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

Section A (36 marks)

1 A particle travels in a straight line during the time interval $0 \leqslant t \leqslant 12$, where t is the time in seconds. Fig. 1 is the velocity-time graph for the motion.

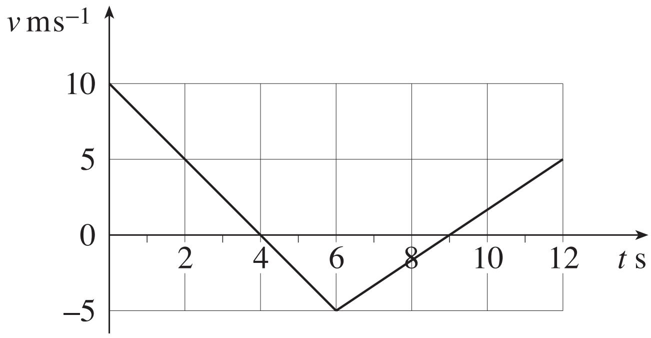

Fig. 1

(i) Calculate the acceleration of the particle in the interval $0 < t < 6$. [2]

(ii) Calculate the distance travelled by the particle from $t = 0$ to $t = 4$. [2]

(iii) When $t = 0$ the particle is at A. Calculate how close the particle gets to A during the interval $4 \leqslant t \leqslant 12$. [2]

2 Fig. 2 shows a light string with an object of mass 4 kg attached at end A. The string passes over a smooth pulley and its other end B is attached to two light strings BC and BD of the same length. The strings BC and BD are attached to horizontal ground and are each inclined at 20° to the vertical.

The system is in equilibrium.

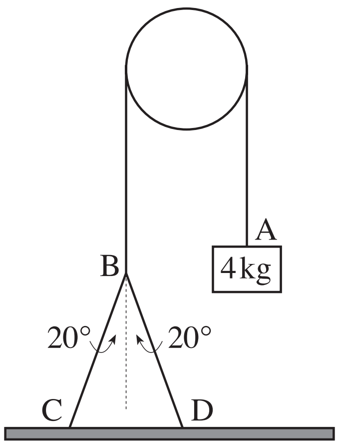

Fig. 2

(i) What information in the question tells you that the tension is the same throughout the string AB? [1]

(ii) What is the tension in the string AB? [1]

(iii) Calculate the tension in the strings BC and BD. [3]

3 A force $\mathbf{F}$ is given by $\mathbf{F} = (3.5\mathbf{i} + 12\mathbf{j})$ N, where $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors east and north respectively.

(i) Calculate the magnitude of $\mathbf{F}$ and also its direction as a bearing. [3]

(ii) $\mathbf{G}$ is the force $(7\mathbf{i} + 24\mathbf{j})$ N. Show that $\mathbf{G}$ and $\mathbf{F}$ are in the same direction and compare their magnitudes. [2]

(iii) Force $\mathbf{F}_1$ is $(9\mathbf{i} - 18\mathbf{j})$ N and force $\mathbf{F}_2$ is $(12\mathbf{i} + q\mathbf{j})$ N. Find q so that the sum $\mathbf{F}_1 + \mathbf{F}_2$ is in the direction of $\mathbf{F}$. [2]

4 A car and its trailer travel along a straight, horizontal road. The coupling between them is light and horizontal. The car has mass 900 kg and resistance to motion 100 N, the trailer has mass 700 kg and resistance to motion 300 N, as shown in Fig. 4. The car and trailer have an acceleration of $1.5\ \mathrm{m\,s^{-2}}$.

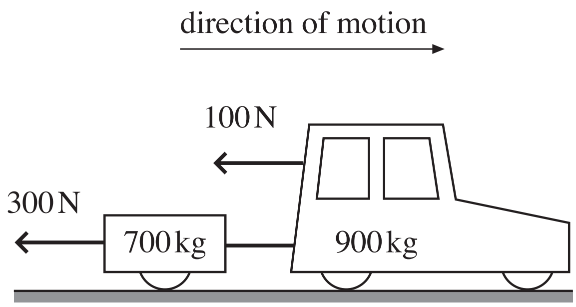

Fig. 4

(i) Calculate the driving force of the car. [3]

(ii) Calculate the force in the coupling. [2]

5 The acceleration of a particle of mass 4 kg is given by $\mathbf{a} = (9\mathbf{i} - 4t\mathbf{j})\ \mathrm{m\,s^{-2}}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors and t is the time in seconds.

(i) Find the acceleration of the particle when $t = 0$ and also when $t = 3$. [1]

(ii) Calculate the force acting on the particle when $t = 3$. [1]

The particle has velocity $(4\mathbf{i} + 2\mathbf{j})\ \mathrm{m\,s^{-1}}$ when $t = 1$.

(iii) Find an expression for the velocity of the particle at time t. [4]

[Turn over

6 A car is driven with constant acceleration, $a\,\mathrm{m\,s^{-2}}$, along a straight road. Its speed when it passes a road sign is $u\,\mathrm{m\,s^{-1}}$. The car travels 14 m in the 2 seconds after passing the sign; 5 seconds after passing the sign it has a speed of $19\,\mathrm{m\,s^{-1}}$.

(i) Write down two equations connecting a and u. Hence find the values of a and u. [5]

(ii) What distance does the car travel in the 5 seconds after passing the road sign? [2]

Section B (36 marks)

7 Clive and Ken are trying to move a box of mass 50 kg on a rough, horizontal floor. As shown in Fig. 7, Clive always pushes horizontally and Ken always pulls at an angle of 30° to the horizontal. Each of them applies forces to the box in the same vertical plane as described below.

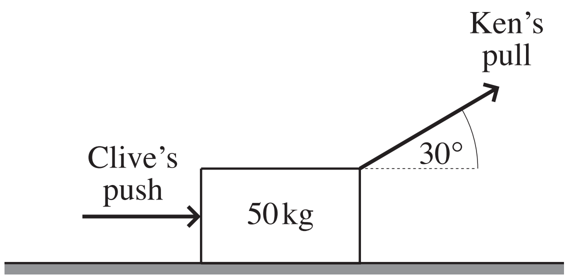

Fig. 7

Initially, the box is in equilibrium with Clive pushing with a force of 60 N and Ken not pulling at all.

(i) What is the resistance to motion of the box? [1]

Ken now adds a pull of 70 N to Clive's push of 60 N. The box remains in equilibrium.

(ii) What now is the resistance to motion of the box? [2]

(iii) Calculate the normal reaction of the floor on the box. [3]

The frictional resistance to sliding of the box is 125 N.

Clive now pushes with a force of 160 N but Ken does not pull at all.

(iv) Calculate the acceleration of the box. [2]

Clive stops pushing when the box has a speed of $1.5\,\mathrm{m\,s^{-1}}$.

(v) How far does the box then slide before coming to rest? [4]

Ken and Clive now try again. Ken pulls with a force of Q N and Clive pushes with a force of 160 N. The frictional resistance to sliding of the box is now 115 N and the acceleration of the box is $3\,\mathrm{m\,s^{-2}}$.

(vi) Calculate the value of Q. [4]

8 A girl throws a small stone with initial speed $14\,\text{m}\,\text{s}^{-1}$ at an angle of 60° to the horizontal from a point 1 m above the ground. She throws the stone directly towards a vertical wall of height 6 m standing on horizontal ground. The point O is on the ground directly below the point of projection, as shown in Fig. 8. Air resistance is negligible.

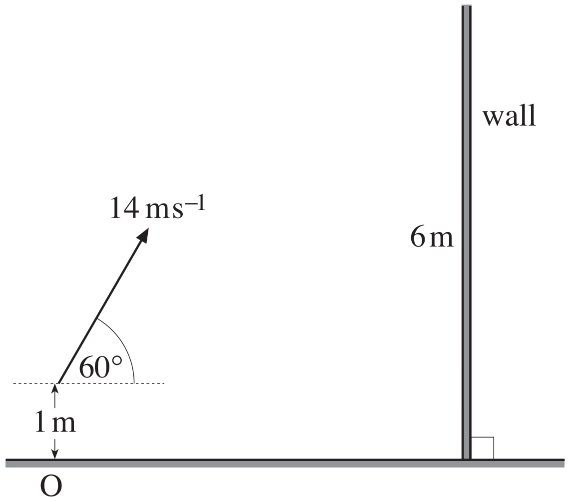

Fig. 8

(i) Write down an expression in terms of t for the horizontal displacement of the stone from O, t seconds after projection. Find also an expression for the height of the stone above O at this time. [5]

The stone is at the top of its trajectory when it passes over the wall.

(ii) (*A*) Find the time it takes for the stone to reach its highest point. [2]

(*B*) Calculate the distance of O from the base of the wall. [2]

(*C*) Show that the stone passes over the wall with 2.5 m clearance. [4]

(iii) Find the cartesian equation of the trajectory of the stone referred to the horizontal and vertical axes, Ox and Oy. There is no need to simplify your answer. [2]

The girl now moves away a further distance d m from the wall. She throws a stone as before and it just passes over the wall.

(iv) Calculate d. [5]

Mark Scheme 4761
January 2006

Section A

Q 1		mark		**Sub**
(i)	$\frac{-15}{6} = -2.5$ so -2.5 m s^{-2}	M1 A1	Use of $\Delta v / \Delta t$. Condone use of *v*/*t*. Must have - ve sign. Accept no units.	2
(ii)	$\frac{1}{2} \times 10 \times 4 = 20$ m	M1 A1	Attempt at area or equivalent	2
(iii)	Area under graph is $\frac{1}{2} \times 5 \times 5 = 12.5$ (and -ve) closest is $20 - 12.5 = 7.5$ m	M1 A1	May be implied. Area from 4 to 9 attempted. Condone missing –ve sign. Do not award if area beyond 9 is used (as well). cao	2
				6

Q 2		mark		**Sub**
(i)	Pulley is smooth (and the string is light)	E1	Only require pulley is smooth. Do not accept only ‘string is light’.	1
(ii)	4*g* = 39.2 N	B1	Accept either	1
(iii)	Let tension in each string be *T* $39.2 = 2T\cos 20$ *T* = 20.85788… so 20.9 N (3 s.f.)	M1 B1 F1	Equating 39.2 to attempt at tensions in **both** BC and BD. Tensions need not be equal. No extra forces. Must attempt resolution. Condone $\sin \leftrightarrow \cos$. For one occurrence of $T\cos 20$ in any equation. Accept reference to only one string. FT **their** 4*g* If Lami’s Theorem used: M1 correct format B1 equation correct. FT **their** 4*g* F1 FT **their** 4*g* If Triangle of Forces used: M1 triangle with **their** 4*g* labelled **and** an	

			attempt to use this triangle. Ignore arrows. B1 for correct equation. FT **their** 4*g*. F1 FT **their** 4*g*.	3
				5

Q 3		mark		**Sub**
(i)	$\lvert\mathbf{F}\rvert = 12.5$ so 12.5 N bearing is $90 - \arctan\frac{12}{3.5}$ = (0)16.260… so (0)16.3° (3 s. f.)	B1 M1 A1	 Use of arctan with 3.5 and 12 or equiv May be obtained directly as $\arctan\frac{3.5}{12}$	3
(ii)	24/7 = 12/3.5 or …. **G** = 2**F** so $\lvert\mathbf{G}\rvert = 2\lvert\mathbf{F}\rvert$	E1 B1	Accept **statement** following **G** = 2**F** shown. Accept equivalent in words.	2
(iii)	$\frac{9+12}{3.5} = \frac{-18+q}{12}$ so $q = 6\times12+18 = 90$	M1 A1	Or equivalent or in scalar equations. Accept $\frac{21}{q-18}$ or $\frac{q-18}{21}$ = tan (i) or tan(90 - (i)) Accept 90**j**	2
				7

Q 4		mark		**Sub**
(i)	N2L in direction of motion $D-(100+300) = (900+700)\times1.5$ D = 2800 so 2800 N	M1 A1 A1	Apply N2L. Allow 1 resistance omitted and sign error but total mass must be used. Condone use of $F = mga$. No extra forces. All correct cao	3
(ii)	N2L on trailer $T-300 = 700\times1.5$ T =1350 so 1350 N	M1 A1	Use either car or trailer. All forces present. No extras. Correct mass and *a* Allow sign error. Must use $F = ma$. cao	2
				5

Q 5		mark		Sub
(i)	$9\mathbf{i}$ m s^{-2}; $(9\mathbf{i} - 12\mathbf{j})$ m s^{-2}	B1	Award for either. Accept no units. (isw e.g. finding magnitudes)	1
(ii)	N2L $\mathbf{F} = 4(9\mathbf{i} - 12\mathbf{j}) = (36\mathbf{i} - 48\mathbf{j})$ N	B1	Accept factored form. isw. FT **a**(3). Accept 60 N or **their** $4\lvert\mathbf{a}\rvert$	1
(iii)	$\mathbf{v} = \int \begin{pmatrix} 9 \\ -4t \end{pmatrix} dt = \begin{pmatrix} 9t + C \\ -2t^2 + D \end{pmatrix}$ Using $\mathbf{v} = 4\mathbf{i} + 2\mathbf{j}$ when $t = 1$ $\begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 + C \\ -2 + D \end{pmatrix}$ $\Rightarrow C = -5, D = 4$ so $\mathbf{v} = (9t - 5)\mathbf{i} + (4 - 2t^2)\mathbf{j}$	M1 A1 M1 A1	Integration. At least one term correct. Neglect arbitrary constant(s) Sub at $t = 1$ to find arb const(s) Any form	4
				6

Q 6		mark		Sub
(i)	$14 = 2u + 0.5a \times 4$ $19 = u + 5a$ Solving gives $u = 4$ and $a = 3$	M1 A1 A1 M1 F1	Use of appropriate *uvast* for either equn Any form Any form Attempt at solution of 2 equns in 2 unknowns. At least one value found . Must have complete correct solution to **their** equns. .	5
(ii)	$19^2 = 4^2 + 2 \times 3 \times s$ or $s = 4 \times 5 + 0.5 \times 3 \times 25$ $s =$ 57.5 so 57.5 m	M1 A1	Use of appropriate *uvast* and **their** *u*, *a* & $t = 5$. cao [Accept 50 if $t = 7$ instead of $t = 5$ in (i) for 2/2]	2
				7

Section B

Q 7		mark		Sub
(i)	60 N	B1		1
(ii)	$60 + 70\cos 30 = 120.62...$ so 121 N (3 s. f.)	M1 A1	70 cos30 or 70 sin 30 used only with 60N. Accept sign errors. cao. Any reasonable accuracy	2
(iii)	resolve ↑ $R + 70\sin 30 - 50g = 0$ R = 455 so 455 N	M1 A1 A1	Resolve ↑ All forces present. No extras. Allow sign errors and $\sin \leftrightarrow \cos$. All correct. cao	3
(iv)	N2L → $160 - 125 = 50a$ a = 0.7 so 0.7 m s $^{-2}$	M1 A1	N2l. No extra forces. Accept 125 N omitted but not use of $F = mga$	2
(v)	N2L → $-125 = 50a$ $a = -2.5$ $0 = 1.5^2 + 2 \times -2.5 \times s$ s = 0.45 so 0.45 m	M1 A1 M1 A1	N2L to find new accn. Accept +125 but not $F = mga$. May be implied. Accept +2.5 Appropriate (sequence of) *uvast* using a new value for acceln. Allow use of ± **their new** a cao. Signs must be justified.	4
(vi)	N2L → $160 + Q\cos 30 - 115 = 50 \times 3$ Q = 121.24… so 121 (3 s. f.)	M1 B1 A1 A1	Use of N2L with cpt of Q attempted. Accept 115 omitted or taken to be 125 and a wrong. Do not allow $F = mga$. Qcos30 seen in any equn. All correct cao	4
				16

Q 8		mark		Sub
(i)	$x = 14\cos 60t$	M1	Consider motion in *x* direction. Need not resolve. Allow $\sin \leftrightarrow \cos$. Condone +1 seen.	
	so $x = 7t$	A1	Need not be simplified.	
	$y = 14\sin 60t - 4.9t^2 + 1$	M1	Suitable *uvast* used for *y* with *g* $= \pm 9.8, \pm 10, \pm 9.81$ soi Need not resolve. Allow $\sin \leftrightarrow \cos$.	
		A1	Allow + 1omitted. Any form and 2 s. f. Need not be simplified	
	$y = 7\sqrt{3}t - 4.9t^2 + 1$ ($y = 12.124...t - 4.9t^2 + 1$)	A1	All correct. +1 need not be justified. Accept any form and 2 s. f. Need not be simplified.	5
(ii) (A)	time taken to reach highest point $0 = 7\sqrt{3} - 9.8T$	M1	Appropriate *uvast* . Accept *u* = 14 and $\sin \leftrightarrow \cos$ and $u \leftrightarrow v$. Require *v* = 0 or equivalent. *g* $= \pm 9.8, \pm 10, \pm 9.81$ soi.	
	so $\frac{5\sqrt{3}}{7}$ s (1.23717…. = 1.24 s (3 s. f.))	A1	cao [If time of flight attempted, do not award M1 if twice interval obtained]	2
(B)	distance from base is $7 \times \frac{5\sqrt{3}}{7} = 5\sqrt{3}$ m	M1	Use of **their** *x* = 7*t* with **their** *T*	
	(= 8.66025… so 8.66 m (3 s. f.))	B1	FT **their** *T* only in *x* = 7*t*. Accept values rounding to 8.6 and 8.7.	2
(C)	**either** Height at this time is $H = 7\sqrt{3} \times \frac{5\sqrt{3}}{7} - 4.9 \times \left(\frac{5\sqrt{3}}{7}\right)^2 + 1$	M1	Subst in **their quadratic** *y* with **their** *T*.	
		A1	Correct subst of **their** *T* in **their** *y* which has attempts at all 3 terms. Do not accept *u* = 14.	
	= 8.5	A1		

	clearance is $8.5-6=2.5$ m	E1	Clearly shown.	
	or for height above pt of projection $0=\left(7\sqrt{3}\right)^2+2\times-9.8\times s$	M1	Appropriate *uvast* . Accept *u* = 14. $g=\pm9.8,\pm10,\pm9.81$ soi	
		A1	Attempt at vert cpt accept $\sin\leftrightarrow\cos$ .Accept sign errors but not *u* = 14.	
	s = 7.5	A1		
	so clearance is $7.5-5=2.5$ m	E1	Clearly shown.	4
(iii)	See over			

Q 8	**continued**	**mark**		**sub**
(iii)	Elim *t* between $y=7\sqrt{3}t-4.9t^2+1$ and $x=7t$	M1	Must see **their** $t=x/7$ fully substituted in **their** quadratic *y* (accept bracket errors)	
	so $y=7\sqrt{3}\frac{x}{7}-4.9\left(\frac{x}{7}\right)^2+1$ so $y=\sqrt{3}x-0.1x^2+1$	F1	Accept any form correctly written. FT **their** *x* and 3 term quadratic *y* (neither using *u* = 14)	2
(iv)	**either** need $6=7\sqrt{3}t-4.9t^2+1$	M1	**their** quadratic *y* from (i) = 6, or equivalent.	
	so $4.9t^2-7\sqrt{3}t+5=0$	M1	Dep. Attempt to solve this 3 term quadratic. (Allow *u* = 14).	
	$t=\frac{5\left(\sqrt{3}\pm1\right)}{7}$ (0.52289…. or 1.95146…)	A1	for either root	
	moves by $\left(\frac{5(\sqrt{3}+1)}{7}-\frac{5\sqrt{3}}{7}\right)\times7$ [(1.95146.. – 1.23717…)$\times7$]	M1	Moves by $\lvert$**their** root - **their** (ii)(A)$\rvert\times7$ or equivalent. Award this for recognition of correct dist (no calc)	
	= 5 m	A1	cao [If new distance to wall found must have larger of 2 +ve roots for 3rd M and award max 4/5 for 13.66]	
	or using equation of trajectory with *y* = 6			

	$6=\sqrt{3}x-0.1x^2+1$ Solving $x^2-10\sqrt{3}x+50=0$ $x=5\left(\sqrt{3}\pm1\right)$ (13.660… or 3.6602….) distance is $5\left(\sqrt{3}+1\right)-5\sqrt{3}$ = 5 m	M1 M1 A1 M1 A1	Equating **their** quadratic trajectory equn to 6 Dep. Attempt to solve this 3 term quadratic. (Allow u = 14). for either root distance is $\lvert$**their** root − **their**(ii)(B)$\rvert$ Award this for recognition of correct dist (no calc) Cao [If new distance to wall found must have larger of 2 + ve roots for 3rd M and award max 4/5 for 13.66]	5
				20

4761: Mechanics 1 (Written Examination)

General Comments

Most of the candidates seemed to find a lot that they could do on this paper. There were few candidates with very low scores and many who achieved full marks or nearly full marks.

It was pleasing that fewer candidates than in the past made elementary mistakes with the inappropriate use of constant acceleration results, and the determination of forces in a coupling and of a normal reaction; it was also saddening that there are still so many who seem unaware of the correct methods.

On the whole, poor presentation of scripts seemed less of a problem than in some recent sessions but many candidates manage to confuse themselves even if they cannot quite manage to confuse the examiner.

Comments on Individual Questions

Section A

1) The use of an acceleration-time graph.

(i) This was usually done correctly but some candidates inverted the fraction and others omitted the negative sign.

(ii) Answered correctly by the majority of the candidates.

(iii) This was one of the least well done parts of the paper. Quite a few candidates did not seem to realize that the displacement from $t = 4$ to $t = 9$ was negative. Many thought that the particle started getting further away from A when the *gradient* of the graph became positive and others assumed that the displacement at $t = 12$ was required. Of course, many others efficiently found the required closest approach.

2) Equilibrium of an object involving a smooth pulley.

(i) Many correct answers were seen. The most common errors were to think that the string being light and/or the system being in equilibrium were enough to ensure the same tension throughout.

(ii) Almost everyone found the tension correctly as the weight of the object.

(iii) There were many correct solutions to this. The most common error was to equate the tension in the string round the pulley to the component of tension in *only one* of BC and BD. A few candidates did not resolve at all and quite a few attempted to resolve in the direction of BC or BD obtaining $T = 39.2 \cos 20°$.

3) The magnitude and direction of a vector. Vectors in the same direction.

(i) This was usually done well. The most common mistake was not to give the direction as a bearing.

(ii) This was usually done well.

(iii) The majority of the candidate knew how to do this but many failed to carry through their plan accurately. The most common errors were to invert one of the ratios, to ‘drop’ the negative sign in front of 18 or to think that $-18 + q = 72$ gives $q = 54$.

4) **Newton's second law applied to a car and trailer accelerating on a horizontal road.**

There were many very good, concise responses to this question but the usual misunderstandings about the application of Newton's second law to connected particles were seen.

(i) Many of those candidates who first considered only the car omitted the tension and so did not see the need for a second equation. Quite a few elementary arithmetic mistakes were seen.

(ii) Often done better than part (i). Quite a few candidates used wrong exotic methods to find the tension.

5) **The dynamics and kinematics of a particle with acceleration given in vector form.**

Many candidates scored marks only in parts (i) and (ii) as they wrongly applied the constant acceleration results in part (iii)

(i) In this part and part (ii), some candidates 'lost' the vector notation.

(ii) Most candidates knew they should apply Newton's second law; the chief error was to give only the magnitude of the force and this was not penalised.

(iii) The use of constant acceleration results was particularly disappointing as part (i) had been deliberately set to draw the attention of the candidates to the acceleration having different values at different times. Quite a few candidates did not involve an arbitrary constant, others assumed it was (vector) zero and yet others gave it wrongly without working shown as –5 **i**.

6) **A kinematics problem involving constant acceleration and simultaneous equations**

Many candidates obtained full marks efficiently. A common mistake was to misread the question as saying that the given speed was achieved 7 seconds instead of 5 seconds after passing the sign.

(i) The chief common mistake was to argue that the car is travelling at 7 m s^{-1} after the first 2 seconds (i.e. assuming that the acceleration is zero); this trivialised the problem. Other errors usually involved inconsistencies in the values taken for u, v and t .

(ii) This was usually done well but many candidates missed it out – perhaps they overlooked it.

Section B

7) **Statics and kinematics of a box being pulled by a string**

Many candidates scored full marks on this question and it was pleasing to see a better standard this session on questions such as parts (iii) and (v). Some errors to very simple arithmetic were seen in the answers.

(i) Most candidates answered this correctly.

(ii) Most candidates knew what to do but some exchanged sine and cosine.

(iii) There were many good answers but, as in previous sessions, many candidates seem to believe that the normal reaction is the component of the weight acting perpendicular to the surface and so they omit the component of the tension in the string.

(iv) This was answered well by most candidates. A common error was to omit the 125 N force.

(v) Quite a few candidates continued to use the 'old' acceleration. Those who realized that the acceleration had changed usually found it correctly and went on to obtain the correct distance.

(vi) It was pleasing to see the large number of correct answers to this part. The most common errors were to fail to resolve Q or, less frequent, to exchange sine and cosine.

8) **A stone thrown over a wall**

Many candidates answered much of the question well but many found some of the parts challenging. The lack of given answers did not seem to be a problem – indeed it seems that many candidates are more comfortable getting the right answer than explaining why a given answer is right.

(i) This was generally done well except by candidates who had struggled elsewhere on the paper. Confusion of sine and cosine was less common than elsewhere in the paper, perhaps because the results were so familiar. The most common error was to omit the +1 in the expression for the vertical height above the ground.

(ii)

(A) Most candidates knew what to do and did it well. The most common errors were: to find the time for the stone to return to its height when projected and not halve this value: to assume that the greatest height reached by the stone is 6 m.

(B) Again, there were many correct answers (or, at least, answers that followed from (A)). Common mistakes were to start again and now get the time wrong or to incorrectly apply the formula for the range.

(C) Again, most candidates knew what to do. Errors usually came from starting again and this time forgetting the 1 m above the ground or from using values t that were insufficiently accurate to establish the value of 2.5 to 2 significant figures.

(iii) Many candidates clearly did not know what this meant and some of these left out the part. Some thought it was a request to write the equation in terms of t in a vector form.

(iv) Many candidates knew that they should equate an expression for y to 6. Most chose an expression for y in terms of t but some (sensibly) used their trajectory equation. Errors often came because candidates 'started again' and this time omitted the 1 m above the ground. Quite a few candidates made errors in the solution of their quadratic equation and others gave the solutions to insufficient accuracy. At the end, many candidates gave the new distance to the wall (13.66 m) instead of the distance moved (13.66 – 8.66 = 5 m). There was only a 1 mark penalty for this.