OCR RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4761/01**

MATHEMATICS (MEI)

Mechanics 1

WEDNESDAY 10 JANUARY 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
- Answer booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **7** printed pages and **1** blank page.

Section A (36 marks)

1 Fig. 1 is the velocity-time graph for the motion of a body. The velocity of the body is $v \text{ m s}^{-1}$ at time t seconds.

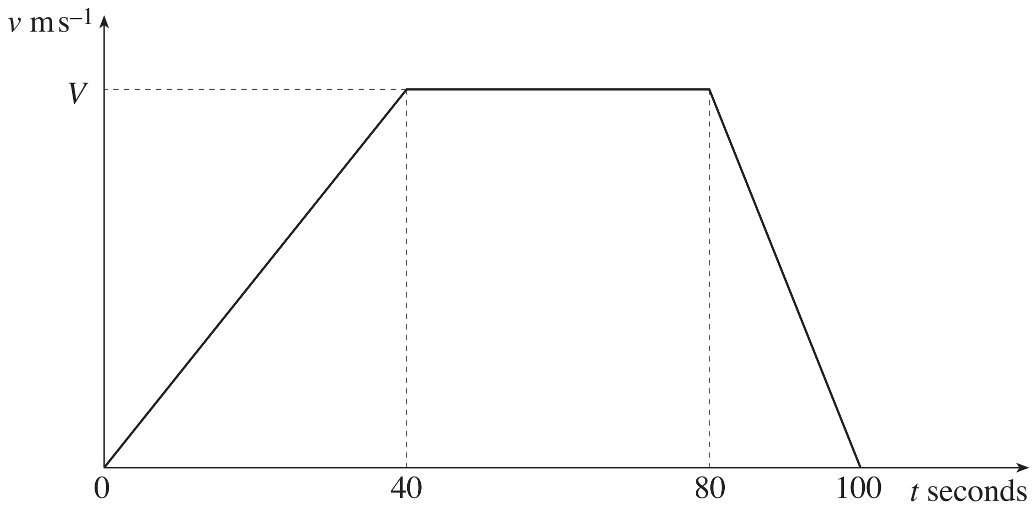

Fig. 1

The displacement of the body from $t = 0$ to $t = 100$ is 1400 m. Find the value of V. [4]

2 A particle moves along a straight line containing a point O. Its displacement, x m, from O at time t seconds is given by

$$x = 12t - t^3, \text{ where } -10 \leqslant t \leqslant 10.$$

Find the values of x for which the velocity of the particle is zero. [5]

3 A box of mass 5 kg is at rest on a rough horizontal floor.

(i) Find the value of the normal reaction of the floor on the box. [1]

The box remains at rest on the floor when a force of 10 N is applied to it at an angle of 40° to the upward vertical, as shown in Fig. 3.

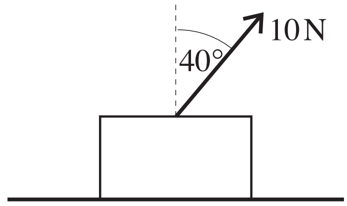

Fig. 3

(ii) Draw a diagram showing all the forces acting on the box. [2]

(iii) Calculate the new value of the normal reaction of the floor on the box and also the frictional force. [4]

4 Fig. 4 shows forces of magnitudes 20 N and 16 N inclined at 60°.

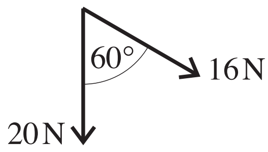

Fig. 4

(i) Calculate the component of the resultant of these two forces in the direction of the 20 N force. [1]

(ii) Calculate the magnitude of the resultant of these two forces. [3]

These are the only forces acting on a particle of mass 2 kg.

(iii) Find the magnitude of the acceleration of the particle and the angle the acceleration makes with the 20 N force. [3]

5 A block of mass 4 kg slides on a horizontal plane against a constant resistance of 14.8 N. A light, inextensible string is attached to the block and, after passing over a smooth pulley, is attached to a freely hanging sphere of mass 2 kg. The part of the string between the block and the pulley is horizontal. This situation is shown in Fig. 5.

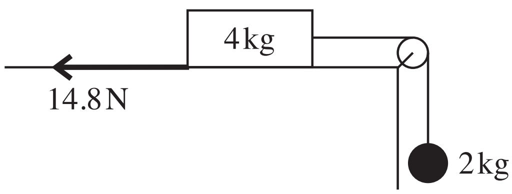

Fig. 5

The tension in the string is T N and the acceleration of the block and of the sphere is a m s^{-2}.

(i) Write down the equation of motion of the block and also the equation of motion of the sphere, each in terms of T and a. [3]

(ii) Find the values of T and a. [3]

6 The velocity of a model boat, $\mathbf{v}$ m s^{-1}, is given by

$$\mathbf{v} = \begin{pmatrix} -5 \\ 10 \end{pmatrix} + t\begin{pmatrix} 6 \\ -8 \end{pmatrix},$$

where t is the time in seconds and the vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are east and north respectively.

(i) Show that when $t = 2.5$ the boat is travelling south-east (i.e. on a bearing of 135°). Calculate its speed at this time. [3]

The boat is at a point O when $t = 0$.

(ii) Calculate the bearing of the boat from O when $t = 2.5$. [4]

Section B (36 marks)

7 A horizontal force of 24 N acts on a block of mass 12 kg on a horizontal plane. The block is initially at rest.

This situation is first modelled assuming the plane is smooth.

(i) Write down the acceleration of the block according to this model. [1]

The situation is now modelled assuming a constant resistance to motion of 15 N.

(ii) Calculate the acceleration of the block according to this new model. How much less distance does the new model predict that the block will travel in the first 4 seconds? [5]

The 24 N force is removed and the block slides *down* a slope at 5° to the horizontal. The speed of the block at the top of the slope is 1.5 m s^{-1}, as shown in Fig. 7. The answers to parts **(iii)** and **(iv)** should be found using the assumption that the resistance to the motion of the block is still a constant 15 N.

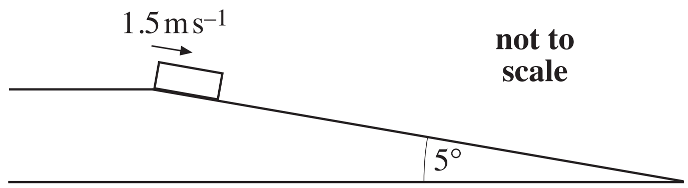

Fig. 7

(iii) Calculate the acceleration of the block in the direction of its motion. [4]

(iv) For how much time does the block slide down the slope before coming to rest and how far does it slide in that time? [4]

Measurements show that the block actually comes to rest in 3.5 seconds.

(v) Assuming that the error in the prediction is due only to the value of the resistance, calculate the true value of the resistance. [4]

[Question 8 is printed overleaf.]

8 In this question the value of g should be taken as $10\,\text{m}\,\text{s}^{-2}$.

As shown in Fig. 8, particles A and B are projected towards one another. Each particle has an initial speed of $10\,\text{m}\,\text{s}^{-1}$ vertically and $20\,\text{m}\,\text{s}^{-1}$ horizontally. Initially A and B are 70 m apart horizontally and B is 15 m higher than A. Both particles are projected over horizontal ground.

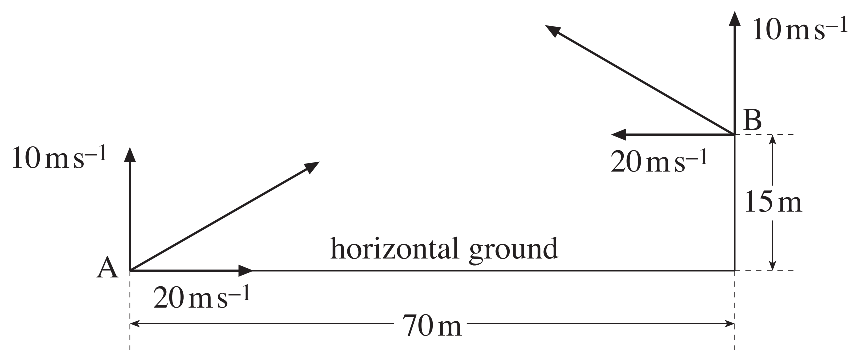

Fig. 8

(i) Show that, t seconds after projection, the height in metres of each particle above its point of projection is $10t - 5t^2$. [1]

(ii) Calculate the horizontal range of A. Deduce that A hits the horizontal ground between the initial positions of A and B. [5]

(iii) Calculate the horizontal distance travelled by B before reaching the ground. [5]

(iv) Show that the paths of the particles cross but that the particles do not collide if they are projected at the same time. [2]

In fact, particle A is projected 2 seconds after particle B.

(v) Verify that the particles collide 0.75 seconds after A is projected. [5]

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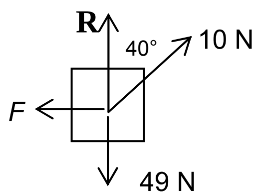
Mark Scheme 4761
January 2007

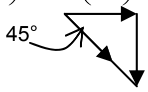
Q 1		mark		sub
	either			
		M1	Attempt at area. If not trapezium method at least one part area correct. Accept equivalent.	
	$70V$ obtained	A1	Or equivalent – need not be evaluated.	
	So $70V = 1400$	M1	Equate **their** $70V$ to 1400. Must have attempt at complete areas or equations.	
	and $V = 20$	A1	cao	
	or	M1	Attempt to find areas in terms of ratios (at least one correct)	
		A1	Correct total ratio – need not be evaluated. (Evidence may be 800 or 400 or 200 seen).	
		M1	Complete method. (Evidence may be 800/40 or 400/20 or 200/10 seen).	
	$V = 20$	A1	cao [Award 3/4 for 20 seen WWW]	
				4

Q 2		mark		sub
	$(v =)12 - 3t^2$	M1	Differentiating	
		A1	Allow confusion of notation, including x =	
	$v = 0 \Rightarrow 12 - 3t^2 = 0$	M1	Dep on 1st M1. Equating to zero.	
	so $t^2 = 4$ and $t = \pm 2$	A1	Accept one answer only but no extra answers. FT only if quadratic or higher degree.	
	$x = \pm 16$	A1	cao. Must have both and no extra answers.	
				5

Q 3		mark		sub
(i)	$R = mg$ so 49 N	B1	Equating to weight. Accept $5g$ (but not mg)	1
(ii)		B1 B1	All except F correct (arrows and labels) (Accept mg, W etc and no angle). Accept cpts instead of 10N. No extra forces. F clearly marked and labelled	2
(iii)	↑ $R + 10\cos 40 - 49 = 0$	M1	Resolve vertically. All forces present and 10N resolved	
		B1	Resolution correct and seen in an equation. (Accept $R = \pm 10\cos 40$ as an equation)	
	R = 41.339… so 41.3 N (3 s. f.)	A1		
	$F = 10\sin 40$ = 6.4278… so 6.43 N (3 s. f.)	B1	Allow –ve if consistent with the diagram.	4
				7

Q 4		mark		sub
(i)	↓ $20+16\cos 60 = 28$	B1		1
(ii)	**either** → $16\sin 60$ Mag $\sqrt{28^2+192} = 31.2409...$ so 31.2 N (3 s.f.) **or** Cos rule $\text{mag}^2 = 16^2+20^2-2\times 16\times 20\times\cos 120$ 31.2 N (3 s. f.)	B1 M1 F1 M1 A1 A1	Any form. May be seen in (i). Accept any appropriate equivalent resolution. Use of Pythag with 2 distinct cpts (but not 16 and ± 20) Allow 34.788… only as FT Must be used with 20 N, 16 N and 60° or 120° Correct substitution	3
(iii)	Magnitude of accn is 15.620… m s^{-2} so 15.6 m s^{-2} (3 s. f.) angle with 20 N force is $\arctan\left(\frac{16\sin 60}{28}\right)$ so 26.3295… so 26.3° (3 s. f.)	B1 M1 A1	Award only for **their** $F \div 2$ Or equiv. May use force or acceleration. Allow use of sine or cosine rules. FT only $s \leftrightarrow c$ and sign errors. Accept reciprocal of the fraction. cao	3
				7

Q 5		mark		sub
(i)	 sphere $19.6 - T = 2a$ block $T - 14.8 = 4a$	M1 A1 A1	N2L. All forces attempted in one equation. Allow sign errors. No extra forces. Don't condone *F* = *mga*. Accept 2*g* for 19.6	3
(ii)	Solving *T* = 18 *a* = 0.8	M1 A1 F1	Attempt to solve. Award only if two equations present both containing *a* and *T*. Either variable eliminated. Either found cao Other value. Allow wrong equation(s) and wrong working for 1st value [If combined equation used award: M1 as in (i) for the equation with mass of 6 kg; A1 for *a* = 0.8; M1 as in (i) for equation in *T* and *a* for either sphere or block; A1 equation correct; F1 for *T*, FT **their** *a*; B1 Second equation in *T* and *a*.]	3
				6

Q 6		mark		sub
(i)	$t = 2.5 \Rightarrow \mathbf{v} = \begin{pmatrix} -5 \\ 10 \end{pmatrix} + 2.5\begin{pmatrix} 6 \\ -8 \end{pmatrix} = \begin{pmatrix} 10 \\ -10 \end{pmatrix}$	B1	Need not be in vector form	
		E1	Accept diag and/or correct derivation of just $\pm 45°$	
	speed is $\sqrt{10^2 + 10^2} = 14.14...$ so 14.1 m s $^{-1}$ (3 s. f.)	F1	FT **their v**	3
(ii)	$\mathbf{s} = 2.5\begin{pmatrix} -5 \\ 10 \end{pmatrix} + \frac{1}{2} \times 2.5^2 \times \begin{pmatrix} 6 \\ -8 \end{pmatrix}$	M1	Consideration of **s** (const accn or integration)	
		A1	Correct sub into *uvast* with **u** and **a.** (If integration used it must be correct but allow no arb constant)	
	$= \begin{pmatrix} 6.25 \\ 0 \end{pmatrix}$	A1		
	so 090°	A1	cao. CWO.	4
				7

Q 7		mark		sub
(i)	acceleration is $\frac{24}{12}$ so 2 m s^{-2}	B1		1
(ii)	$24-15=12a$	M1	Use of N2L. Both forces present. Must be *F* = *ma*. No extra forces.	
	a = 0.75 m s^{-2}	A1		
	1st distance is $0.5\times2\times16=16$	M1	Appropriate *uvast* applied at least once.	
	2nd distance is $0.5\times0.75\times16=6$	A1	Need not evaluate. Both found. May be implied. FT (i)	
	Difference is 10 m	A1	cao	5
(iii)	$12g\sin5-15=12a$	M1	Use of *F* = *ma*, allow 15 N missing *or* weight not resolved. No extra forces. Allow use of $12\sin5$.	
		M1	Attempt at weight cpt. Allow sin $\leftrightarrow$ cos. Accept seen on diagram. Accept the use of 12 instead of 12*g*.	
		A1	Weight cpt correct. Accept seen on diagram. Allow not used.	
	$a=-0.39587...$ so – 0.396 m s^{-2} (3 s. f.)	A1	Correct direction must be made clear	4
(iv)	time $0=1.5+at\Rightarrow t=3.789...$	M1	Correct *uvast* . Use of 0, 1.5 and **their** *a* from (iii) or **their** *s* from (iv). Allow sign errors. Condone $u\leftrightarrow v$.	
	so 3.79 s (3 s. f.)	A1		
	distance $s=0.5\times(1.5+0)\times3.789...$ (or…)	M1	Correct *uvast* . Use of 0, 1.5 and **their** *a* from (iii) or **their** *t* from (iv). Allow sign errors. Condone $u\leftrightarrow v$.	
	giving *s* = 2.8418… so 2.84 m (3 s. f.)	A1	[The first A1 awarded for *t* or *s* has FT **their** *a* if signs correct; the second awarded is cao]	4
(v)	accn is given by $0=1.5+3.5a\Rightarrow a=-\frac{3}{7}=-0.42857...$	M1	Use of 0, 1.5 and 3.5 in correct *uvast.* Condone $u\leftrightarrow v$.	
		A1	Allow ±	
	$12g\sin5-R=12\times-0.42857...$	M1	N2L. Must use **their** *new* accn. Allow only sign errors.	
	so *R* = 15.39… so 15.4 N (3 s. f.)	A1	cao	4
				18

Q 8		mark		sub
(i)	Using $s = ut + 0.5at^2$ with u = 10 and a = -10	E1	Must be clear evidence of derivation of – 5. Accept one calculation and no statement about the other.	1
(ii)	**either** s = 0 gives $10t - 5t^2 = 0$ so $5t(2-t) = 0$ so t = 0 or 2. Clearly need t = 2 **or** Time to highest point is given by 0 = 10 – 10t Time of flight is 2×1 = 2 s horizontal range is 40 m as 40 < 70, hits the ground	 B1 M1 A1 M1 M1 A1 B1 E1	 Factorising Award 3 marks for t = 2 seen WWW Dep on 1st M1. Doubling **their** t. Properly obtained FT 20 $\times$ **their** t Must be clear. FT **their** range.	5
(iii)	need $10t - 5t^2 = -15$ Solving $t^2 - 2t - 3 = 0$ so $(t-3)(t+1) = 0$ and t = 3 range is 60 m	M1 M1 A1 M1 A1	[May divide flight into two parts] Equate s = -15 or equivalent. Allow use of ± 15. Method leading to solution of a quadratic. Equivalent form will do. Obtaining t = 3. Allow no reference to the other root. [Award SC3 if t = 3 seen WWW] Range is 20 $\times$ **their** t (provided t > 0) cao. CWO.	5
(iv)	Using (ii) & (iii), since 40 + 60 > 70, paths cross (For $0 < t \le 2$) both have same vertical motion so B is always 15 m above A	E1 E1	Must be convincing. Accept sketches. Do not accept evaluation at one or more points alone. That B is *always* above A must be clear.	2
(v)	Need x components summing to 70 $20 \times 0.75 + 20 \times 2.75 = 15 + 55 = 70$ so true Need y components the same $10 \times 2.75 - 5 \times 2.75^2 + 15 = 4.6875$ $10 \times 0.75 - 5 \times 0.75^2 = 4.6875$	M1 E1 M1 B1 E1	May be implied. Or correct derivation of 0.75 s or 2.75 s Attempt to use 0.75 and 2.75 in two vertical height equations (accept same one or wrong one) 0.75 and 2.75 each substituted in the appropriate equn Both values correct. [Using cartesian equation: B1, B1 each equation: M1 solving: A1 correct point of intersection: E1 Verify times]	5
				18

4761 - Mechanics 1

General Comments

The majority of the candidates seemed well prepared for this paper and were able to finish or, at least, make substantial progress on every question; there were many beautifully presented and clearly argued solutions. Many of the candidates showed good algebraic and arithmetic skills.

Very few candidates seemed unfamiliar with all of the principles required for the unit but rather more struggled to make much progress. It was notable that this latter group tended not just to have poor general mathematical skills (as well as a lack of specific knowledge about mechanics) but handicapped themselves further by poor presentation. There were many examples of such candidates overlooking some parts of questions and in extreme cases mixing up their attempts to a whole question by not specifying which part was being answered. Candidates should know that if an answer cannot be reasonably associated with a specific question then no credit may be the consequence.

Many candidates did not know how to do Q6 (ii) and not many were able to assemble complete arguments to satisfy the requests in Q8 (iv) & (v). A lot of candidates did not appreciate the help offered through the structure of Q4 but those who saw what was required found it easy to answer. Apart from these questions, most candidates knew what was expected of them at each step and many did much of it very well, especially in Q7 where full marks were common.

Comments on Individual Questions

Section A

1 **Using a velocity – time graph**

This was generally done fairly efficiently for full marks but a few candidates could not get started. Relatively few candidates used the area of a trapezium but instead considered two triangles and a rectangle. Surprisingly, quite a few used an argument based on the fraction of the whole area represented by each of the three sections. The most common error was to attempt to apply a constant acceleration formula once only for the entire 100 second interval.

2 **A kinematics problem involving calculus**

Most candidates realised that they should differentiate to find the velocity, did so accurately and correctly equated their expression to zero. Many forgot the negative root or discarded it despite the time interval clearly indicating the inclusion of negative values. Many candidates forgot that the final answer was to be the displacement not the time.

3 **The static equilibrium of a box on a rough horizontal floor with a force applied at an angle to the floor.**

(i) Most candidates obtained the correct normal reaction.

(ii) Many candidates omitted the frictional force or (fewer) the normal reaction. A common error from generally weaker candidates was to label the normal reaction with the value found in part (i).

(iii) There were pleasingly many correct solutions, the most common error being to transpose sine and cosine. There were also some sign errors, often from candidates who when considering the vertical equilibrium tried to write down an expression with the normal reaction as subject.

4 **The resultant of two forces and the magnitude and direction of the acceleration when they are applied to a particle**

This question was structured to lead candidates through a common method of solution. However, a number struggled to do much at all or used the cosine and sine rules instead.

(i) This part presented unexpected problems of comprehension and many candidates gave the complete resultant vector instead of the required component.

(ii) Perhaps a majority of the candidates used the cosine rule; for these a common error was to take the angle between the forces as 60° instead of 120°. Many of those who used Pythagoras' Theorem (as expected) had a sign error in the calculation of the component in the direction of the 20 N force or failed to use components or used incomplete resolutions.

(iii) Many candidates were able to follow through from part (ii) to obtain the acceleration. Others started from first principles, often making different mistakes with components to those made in part (ii); some included a weight of $2g$ in the vertical component. With wrong components the direction found was wrong and there was limited follow through allowed from earlier parts. The few candidates who used the sine rule did well.

5 **Motion of a block on a horizontal connected by a string over a pulley to a hanging object**

This standard problem was recognised as such by many candidates who then went on to obtain full marks. A few clearly did not know what to do and came up with wrong equations not obtained from any clear method.

(i) A common error from those who broadly knew what to do was to produce two equations with inconsistent signs. Other errors were to include the weight of the block in its equation of motion and the resistance on the block in the equation of motion of the sphere.

6 **The kinematics of particle in a plane with constant acceleration. The direction of motion and of displacement**

Many candidates answered part (i) quite well but very few understood what was required for part (ii), (the worst answered part on the paper). In part (i) they were given a velocity vector in terms of t and they then correctly used its direction when $t = 2.5$ as the direction of the motion at that time (perhaps without thinking). In part (ii) they had to realize that the direction required was that of a displacement and most failed to do so.

(i) This was usually quite well done with the direction established acceptably by many. Quite a few candidates failed to give the speed – possibly they forgot it was required.

(ii) Most candidates either left the part out or considered **v**(0) or Δ **v**. Quite a few who realized they must find a displacement used integration instead of a constant acceleration result in vector form; these were candidates who had shown themselves generally strong and they usually obtained the correct answer.

7 **Newton's second law applied to motion on a horizontal plane and down a slope**

This question was answered well by all but the weakest candidates and even most of those made some progress. Almost all of the stronger candidates scored high or even full marks. It was particularly pleasing to see so many applying Newton's second law correctly to a block on an inclined plane.

(i) Usually done correctly.

(ii) This was done well by most but there were lapses from some weak candidates who used distance = speed $\times$ time as if the acceleration were zero. There were also a surprisingly large number of errors when substituting (for example a formula was written down correctly but the value of t was substituted where t^2 was required).

(iii) This was done pleasingly well by many candidates. The most common errors were to use cosine instead of sine when resolving or to resolve the 15N or even to include a component of the velocity.

(iv) Most candidates managed to find the time and the distance using the value of their acceleration but some found only one of these – the other was, perhaps, forgotten.

(v) Again, the response to this question was very pleasing. Most candidates knew that they must find a new acceleration and did so accurately, the most common error from these being with signs.

8 **Two objects with the same initial horizontal and vertical speeds but different initial heights projected towards one another**

Parts (i) to (iii) seemed to be found straightforward by most candidates but fewer could manage the much more sophisticated arguments required in parts (iv) and (v).

(i) Surprisingly many candidates could not establish this result clearly, leaving the reason for the negative sign ambiguous.

(ii) Most candidates did this well. The most common method was to equate the vertical height expression to zero but some considered twice the time to the highest point and others first found the cartesian equation of the trajectory. A common omission was to show that A landed between the initial positions of A and B. It seemed that many candidates thought this so obvious that they simply repeated the words in the question without saying *why* it was obvious.

(iii) Most candidates also did this part well. The most popular method used was to equate the vertical height expression to – 15; the most common error was to equate to 15 instead. Candidates who used this method mostly solved the quadratic equation accurately. Some candidates split the motion into two parts but these often made sign errors somewhere in this more complicated approach. Some of the weaker candidates produced some very poor attempts where they equated the vertical and horizontal displacements or added distances to speeds etc.

(iv) This part was found difficult by many candidates and was not done particularly well mainly because the candidates did not communicate their arguments sufficiently clearly. When trying to show that A and B do not collide, many argued that the two equations for the vertical motion were inconsistent for the same time and others tried to show that the heights were different when the horizontal positions were the same. Fewer candidates made a good attempt to show that the paths intersect; most who were successful considered the sum of the ranges they had found in the two previous parts. There were a number of well thought out valid arguments presented by strong candidates.

(v) This part was also done in a variety of ways. The most popular was to demonstrate that the vertical displacements were equal at the two times; however, a large number of these candidates omitted to check that the horizontal positions were also the same. A number of candidates attempted to derive one of the times. This was usually unsuccessful due to $(t + 2)$ being used instead of $(t - 2)$ in the motion for A. However, the problem was well within the capabilities of the stronger candidates and a pleasing number of them scored full marks for this part using, as with part (iv), a variety of interesting methods.