RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4761/01**

MATHEMATICS (MEI)

Mechanics 1

FRIDAY 23 MAY 2008 Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

Section A (36 marks)

1 Fig. 1.1 shows a circular cylinder of mass 100 kg being raised by a light, inextensible vertical wire AB. There is negligible air resistance.

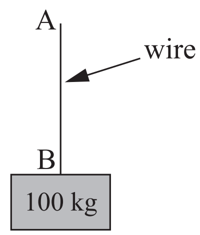

Fig. 1.1

(i) Calculate the acceleration of the cylinder when the tension in the wire is 1000 N. [3]

(ii) Calculate the tension in the wire when the cylinder has an upward acceleration of $0.8\,\mathrm{m\,s^{-2}}$. [2]

The cylinder is now raised inside a fixed smooth vertical tube that prevents horizontal motion but provides negligible resistance to the upward motion of the cylinder. When the wire is inclined at 30° to the vertical, as shown in Fig. 1.2, the cylinder again has an upward acceleration of $0.8\,\mathrm{m\,s^{-2}}$.

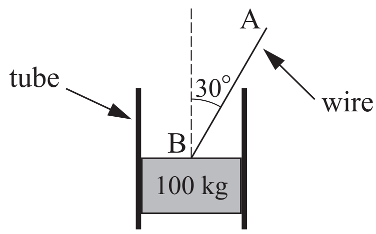

Fig. 1.2

(iii) Calculate the new tension in the wire. [3]

2 A particle has a position vector **r**, where **r** = 4**i** – 5**j** and **i** and **j** are unit vectors in the directions east and north respectively.

(i) Sketch **r** on a diagram showing **i** and **j** and the origin O. [1]

(ii) Calculate the magnitude of **r** and its direction as a bearing. [4]

(iii) Write down the vector that has the same direction as **r** and three times its magnitude. [1]

3 An object of mass 5 kg has a constant acceleration of $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ m s^{-2} for $0 \leqslant t \leqslant 4$, where t is the time in seconds.

(i) Calculate the force acting on the object. [2]

When $t = 0$, the object has position vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ m and velocity $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ m s^{-1}.

(ii) Find the position vector of the object when $t = 4$. [3]

4

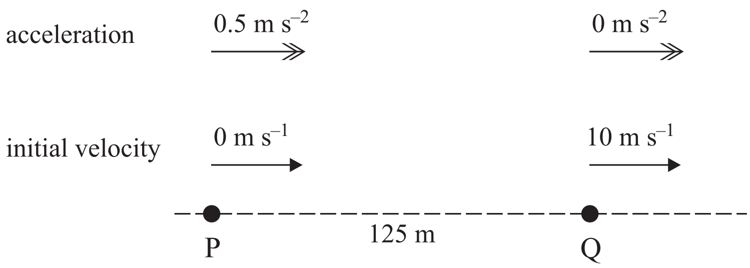

Fig. 4

Particles P and Q move in the same straight line. Particle P starts from rest and has a constant acceleration towards Q of 0.5 m s^{-2}. Particle Q starts 125 m from P at the same time and has a constant speed of 10 m s^{-1} away from P. The initial values are shown in Fig. 4.

(i) Write down expressions for the distances travelled by P and by Q at time t seconds after the start of the motion. [2]

(ii) How much time does it take for P to catch up with Q and how far does P travel in this time? [5]

5 Boxes A and B slide on a smooth, horizontal plane. Box A has a mass of 4 kg and box B a mass of 5 kg. They are connected by a light, inextensible, horizontal wire. Horizontal forces of 9 N and 135 N act on A and B in the directions shown in Fig. 5.

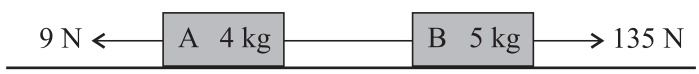

Fig. 5

Calculate the tension in the wire joining the boxes. [4]

6 **In this question take $g = 10$.**

A golf ball is hit from ground level over horizontal ground. The initial velocity of the ball is 40 m s^{-1} at an angle α to the horizontal, where $\sin \alpha = 0.6$ and $\cos \alpha = 0.8$. Air resistance may be neglected.

(i) Find an expression for the height of the ball above the ground t seconds after projection. [2]

(ii) Calculate the horizontal range of the ball. [4]

Section B (36 marks)

7

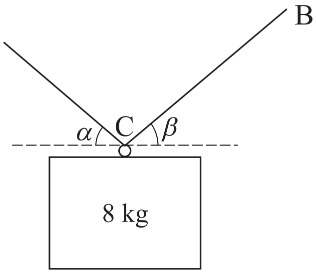

Fig. 7.1

A box of mass 8 kg is supported by a continuous light string ACB that is fixed at A and at B and passes through a smooth ring on the box at C, as shown in Fig. 7.1. The box is in equilibrium and the tension in the string section AC is 60 N.

(i) What information in the question indicates that the tension in the string section CB is also 60 N? [2]

(ii) Show that the string sections AC and CB are equally inclined to the horizontal (so that $\alpha = \beta$ in Fig. 7.1). [2]

(iii) Calculate the angle of the string sections AC and CB to the horizontal. [5]

In a different situation the same box is supported by two separate light strings, PC and QC, that are tied to the box at C. There is also a horizontal force of 10 N acting at C. This force and the angles between these strings and the horizontal are shown in Fig. 7.2. The box is in equilibrium.

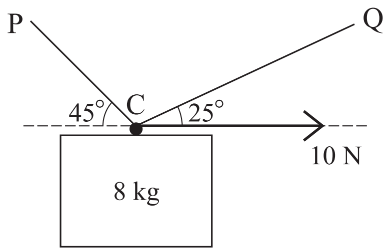

Fig. 7.2

(iv) Calculate the tensions in the two strings. [8]

8 The displacement, x m, from the origin O of a particle on the x-axis is given by

$$x = 10 + 36t + 3t^2 - 2t^3,$$

where t is the time in seconds and $-4 \leqslant t \leqslant 6$.

(i) Write down the displacement of the particle when $t = 0$. [1]

(ii) Find an expression in terms of t for the velocity, v m s^{-1}, of the particle. [2]

(iii) Find an expression in terms of t for the acceleration of the particle. [2]

(iv) Find the maximum value of v in the interval $-4 \leqslant t \leqslant 6$. [3]

(v) Show that $v = 0$ only when $t = -2$ and when $t = 3$. Find the values of x at these times. [5]

(vi) Calculate the *distance* travelled by the particle from $t = 0$ to $t = 4$. [3]

(vii) Determine how many times the particle passes through O in the interval $-4 \leqslant t \leqslant 6$. [3]

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Q 1		mark	comment	sub
(i)	N2L ↑ $1000 - 100 \times 9.8 = 100a$ a = 0.2 so 0.2 m s^{-2} upwards	M1 B1 A1	N2L. Accept $F = mga$ and no weight Weight correct (including sign). Allow if seen. Accept ± 0.2. Ignore units and direction	3
(ii)	$T_{BA} - 980 = 100 \times 0.8$ so tension is 1060 N	M1 A1	N2L. $F = ma$. Weight present, no extras. Accept sign errors.	2
(iii)	$T_{BA} \cos 30 = 1060$ $T_{BA} = 1223.98...$ so 1220 N (3 s. f.)	M1 A1 A1	Attempt to resolve their (ii). Do not award for **their** 1060 resolved unless all forces present and all resolutions needed are attempted. If start again allow no weight. Allow $\sin \leftrightarrow \cos$. No extra forces. Condone sign errors FT **their** 1060 only cao	3
		8		

Q 2		mark	comment	sub
(i)		B1	Sketch. O, **i**, **j** and **r** (only require correct quadrant.) Vectors must have arrows. Need not label **r**.	1
(ii)	$\sqrt{4^2 + (-5)^2}$ = $\sqrt{41}$ or 6.4031… so 6.40 (3 s. f.) Need $180 - \arctan\left(\frac{4}{5}\right)$ 141.340 so 141°	M1 A1 M1 A1	Accept $\sqrt{4^2 - 5^2}$ Or equivalent. Award for $\arctan\left(\pm\frac{4}{5}\right)$ or $\arctan\left(\pm\frac{5}{4}\right)$ or equivalent seen without 180 or 90. cao	4
(iii)	12**i** – 15**j** or $\begin{pmatrix} 12 \\ -15 \end{pmatrix}$	B1	Do not award for magnitude given as the answer. Penalise spurious notation by 1 mark at most once in paper	1
		6		

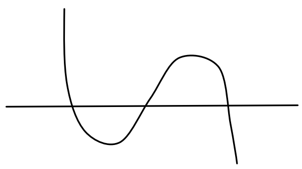
Q 3		mark	comment	sub
			Penalise spurious notation by 1 mark at most once in paper	
(i)	**F** = $5\begin{pmatrix}-1\\2\end{pmatrix}=\begin{pmatrix}-5\\10\end{pmatrix}$ so $\begin{pmatrix}-5\\10\end{pmatrix}$ N	M1	Use of N2L in vector form	
		A1	Ignore units. [Award 2 for answer seen] [SC1 for $\sqrt{125}$ or equiv seen]	2
(ii)	**s** = $\begin{pmatrix}-2\\3\end{pmatrix}+4\begin{pmatrix}4\\5\end{pmatrix}+\frac{1}{2}\times4^2\times\begin{pmatrix}-1\\2\end{pmatrix}$	M1	Use of $\mathbf{s}=t\mathbf{u}+0.5t^2\mathbf{a}$ or integration of **a**. Allow $\mathbf{s}_0$ omitted. If integrated need to consider **v** when t = 0	
		A1	Correctly evaluated; accept $\mathbf{s}_0$ omitted.	
	$\mathbf{s}=\begin{pmatrix}6\\39\end{pmatrix}$ so $\begin{pmatrix}6\\39\end{pmatrix}$ m	B1	Correctly adding $\mathbf{s}_0$ to a vector (FT). Ignore units. [NB $\begin{pmatrix}8\\36\end{pmatrix}$ seen scores M1 A1]	3
		5		

Q 4		mark	comment	sub
(i)	The distance travelled by P is $0.5\times0.5\times t^2$	B1		
	The distance travelled by Q is 10t	B1	Accept 10t + 125 if used correctly below.	2
(ii)	Meet when $0.25t^2=125+10t$	M1	Allow **their** wrong expressions for P and Q distances Allow ± 125 or 125 omitted	
		F1	Award for **their** expressions as long as one is quadratic and one linear. Must have 125 with correct sign.	
	so $t^2-40t-500=0$			
	Solving	M1	Accept any method that yields (smaller) + ve root of their 3 term quadratic	
	t = 50 (or -10)	A1	cao Allow –ve root not mentioned	
	Distance is $0.25\times50^2=625$ m	A1	cao [SC2 400 m seen]	5
		7		

Q 5		mark	comment	sub
	either Overall, N2L $\rightarrow$ $135 - 9 = (5 + 4)a$ $a = 14$ so 14 m s^{-2} For A, N2L $\rightarrow$ $T - 9 = 4 \times 14$ so 65 N **or** $135 - T = 5a$ $T - 9 = 4a$ Solving $T = 65$ so 65 N	 M1 A1 M1 A1 M1 A1 M1 A1	 Use of N2L. Allow *F* = *mga* but no extra forces. Allow 9 omitted. N2L on A or B with correct mass. *F* = *ma*. All relevant forces and no extras. cao * 1 equation in *T* and *a*. Allow sign errors. Allow *F* = *mga* Both equations correct and consistent Dependent on M* solving for *T*. cao.	 4
		4		

Q 6		mark	comment	sub
(i)	$40 \times 0.6t - 5t^2$ $= 24t - 5t^2$	M1 A1	Use of $s = ut + 0.5at^2$ with $a = \pm 9.8,\ \pm 10$. Accept 40 or 40×0.8 for '*u*'. Any form	 2
(ii)	**either** Need zero vertical distance so $24t - 5t^2 = 0$ so *t* = 0 or *t* = 4.8 **or** Time to highest point, *T* 0 = 40 × 0.6 – 10*T* so *T* = 2.4 and time of flight is 4.8 range is $40 \times 0.8 \times 4.8$ = 153.6 so 154 m (3 s. f.)	 M1 A1 M1 A1 M1 A1	 Equate **their** *y* to zero. With fresh start must have correct *y*. Accept no reference to *t* = 0 and the other root in any form. FT **their** *y* if gives *t* > 0 Allow use of *u* = 40 and 40 × 0.8. Award even if half range found. May be awarded for doubling half range later. Horiz cpt. Accept 0.6 instead of 0.8 only if consistent with expression in (i). FT **their** *t*. cao [NB Use of half range or half time to get 76.8… (g = 10) or 78.36… (g = 9.8) scores 2] [If range formula used: M1 sensible attempt at substitution; allow sin2α wrong B1 sin2α correct A1 all correct A1 cao]	 4
		6		

Q 7		mark	comment	sub
(i)	Continuous string: smooth ring: light string	E1 E1	One reason Another reason	2
(ii)	Resolve $\leftarrow$: $60\cos\alpha - 60\cos\beta = 0$ (so $\cos\alpha = \cos\beta$) and so $\alpha = \beta$	M1 E1	[(ii) and (iii) may be argued using Lami or triangle of forces] Resolution and an equation or equivalent. Accept $s \leftrightarrow c$. Accept a *correct* equation seen without method stated. Accept the use of '*T*' instead of '60'. Shown. Must have stated method (allow $\rightarrow$ seen).	2
(iii)	Resolve $\uparrow$ $2\times60\times\sin\alpha - 8g = 0$ so α = 40.7933… so 40.8° (3 s. f.)	M1 B1 B1 A1 A1	Resolution and an equation. Accept $s \leftrightarrow c$. Do not award for resolution that cannot give solution (e.g. horizontal) Both strings used (accept use of half weight), seen in an equation $\sin\alpha$ or equivalent seen in an equation All correct	5
(iv)	Resolve $\rightarrow$ $10 + T_{QC}\cos25 - T_{PC}\cos45 = 0$ Resolve $\uparrow$ $T_{PC}\sin45 + T_{QC}\sin25 - 8g = 0$ Solving $T_{CQ} = 51.4701...$ so 51.5 N (3 s. f.) $T_{CP} = 80.1120...$ so 80.1 N (3 s. f.)	M1 M1 A1 M1 A1 M1 A1 F1	Recognise strings have different tensions. Resolution and an equation. Accept $s \leftrightarrow c$. No extra forces. All forces present. Allow sign errors. Correct. Any form. Resolution and an equation. Accept $s \leftrightarrow c$. No extra forces. All forces present. Allow sign errors. Correct. Any form. * A method that leads to at least one solution of a pair of simultaneous equations. cao either tension other tension. Allow FT only if M1* awarded [Scale drawing: 1st M1 then A1, A1 for answers correct to 2 s.f.]	8
		17		

Q 8		mark	comment	sub
(i)	10	B1		1
(ii)	$v = 36 + 6t - 6t^2$	M1 A1	Attempt at differentiation	2
(iii)	$a = 6 - 12t$	M1 F1	Attempt at differentiation	2
(iv)	Take *a* = 0 so $t = 0.5$ and *v* = 37.5 so 37.5 m s^{-1}	M1 A1 A1	Allow table if maximum indicated or implied FT **their** *a* cao Accept no justification given that this is maximum	3
(v)	**either** Solving $36 + 6t - 6t^2 = 0$ so *t* = -2 or *t* = 3 **or** Sub the values in the expression for *v* Both shown to be zero A quadratic so the only roots **then** *x*(-2) = -34 *x*(3) = 91	 M1 B1 E1 M1 E1 B1 B1 B1	 A method for two roots using **their** *v* Factorization or formula or … of **their** expression Shown Allow just 1 substitution shown Both shown Must be a clear argument cao cao	5
(vi)	$\|x(3) - x(0)\| + \|x(4) - x(3)\|$ = $\|91 - 10\| + \|74 - 91\|$ = 98 so 98 m	M1 A1 A1	Considering two parts Either correct cao [SC 1 for *s*(4) – *s*(0) = 64]	3
(vii)	At the SP of *v* *x*(-2) = -34 i.e. < 0 and *x*(3) = 91 i.e. > 0 Also *x*(-4) = 42 > 0 and *x*(6) = -98 < 0  so three times	M1 B1 B1	Or any other valid argument e.g find all the zeros, sketch, consider sign changes. Must have some working. If only a sketch, must have correct shape. Doing appropriate calculations e.g. find all 3 zeros; sketch cubic reasonably (showing 3 roots); sign changes in range 3 times seen	3
		19		

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General Comments

Although many of the candidates obtained high scores and many scored full marks to several of the questions, there were quite a few who struggled to make much progress with any questions other than Q8 (i), (ii) and (iii), which were done correctly by most. All of the questions were answered well by many of the candidates but Q4, Q5 Q6 and Q7 tended to attract very good or quite poor answers, very often from whole centres.

As in previous sessions, there were quite a few candidates who, when stuck, seemed to be trying almost everything they knew about mechanics instead of trying to identify the principle that might apply to the situation. It is always a pity when this happens as the candidate may well know much more about mechanics than is being passed on to the examiner.

There were two examples this time of questions that many candidates didn't quite finish, presumably because they forgot exactly what they had been asked to do. In Q4 (ii), many candidates found the time taken for the particles to collide but didn't go on to find the distance travelled. In Q8 (v), many candidates showed that the velocity was zero at certain times but did not go on to find the position of the particle at those times. Candidates should always re-read a question when they think they have finished it to be sure they really have done so.

It was a pleasure to see so many well presented and clearly argued solutions to the questions from candidates who had clearly developed a very sound understanding of the principles and techniques required for this unit.

Comments on Individual Questions

Section A

1) **Newton's second law applied to vertical motion**

(i) This was done well by many candidates. The most common error was to omit the weight term. A few candidates used $F = mga$.

(ii) Again, this was done well by many candidates, the most common error being to omit the weight term.

(iii) Quite a few candidates 'started again' instead of using their result from (ii). The most common error, apart from omitting the weight term, was to resolve the acceleration term instead of the tension term.

It was interesting that many candidates omitted the weight term from only one or two of the three parts. Many of those who 'started again' in part (iii) did not include or omit the weight term consistently with their attempt at part (ii).

2) **Magnitude, direction and a scalar multiple of a vector**

(i) Very few candidates scored this mark. The most common mistake(s) were to omit arrows or not to indicate unit vectors **i** and **j**.

(ii) Most candidates correctly calculated the magnitude of the vector. Calculation of the bearing proved more challenging but there were very many correct answers. Relatively few candidates used an incorrect trigonometric ratio but many, working without using a diagram, did not do the right thing with the angle they found; 90 + 38.7, 180 + 38.7,
180 – 51.3 and 360 – 38.7 etc were commonly seen.

(iii) Most candidates wrote down the answer correctly but quite a few spent a long time establishing the answer approximately by working out the components of a vector with three times the magnitude and the same direction as the given position vector.

3) **Newton's second law and kinematics with constant acceleration, both in vector form**

(i) Most candidates applied Newton's second law correctly but quite a few made arithmetic slips and many gave the magnitude of the force instead of the vector as their answer.

(ii) There were many correct answers but also a lot of mistakes were seen. Some candidates did not use an appropriate constant acceleration formula to find a displacement and of those that did, many then forgot to take account of the initial position. Many made arithmetical errors in evaluating their expressions. The candidates who used a calculus approach very often started off integrating the velocity at t = 0 once instead of the acceleration twice.

4) **A kinematics problem in a line where one particle catches up with another**

(i) Most candidates managed to obtain expressions for the distances travelled by P and Q but some clearly didn't understand what is meant by 'an expression …… at time t seconds'. Many candidates gave the position of Q instead the distance travelled.

(ii) Most candidates did attempt an equation connecting the distances travelled but many omitted the original separation of 125 m. Many candidates found the roots of a three term quadratic accurately. A common mistake was to give the time at which the particles collide as the answer instead of the distance travelled by P. Some candidates did not know what to do and quite a few of them wasted a *lot* of time on an iterative approach. They first found out how long it took P to get to the starting point of Q and then calculated how much further Q had gone in that time. They then repeated the process until they gave up.

5) **Tension in the coupling between two accelerating connected boxes**

This question was not generally done well although many perfect solutions were seen. Most of the candidates who knew what to do first found the acceleration by considering the overall motion and then applied Newton's second law again to the motion of one of the boxes. A smaller number set up the equations of motion for each of the boxes in terms of the common acceleration and the tension in the coupling and then solved their equations simultaneously; with this approach there were sometimes inconsistencies with signs or incorrect solution of the equations. With either approach there were misuses of Newton's second law, the most common being to write it as $F - mg = ma$, a formula presumably learned to solve a problem involving vertical motion and misapplied here. As always with connected body problems, quite a few candidates do not realize that Newton's second law is involved and do complicated things with the weights and the resistances.

6) **The range of a projectile**

(i) Most candidates made a good attempt at this part. The most common mistakes were to take the initial vertical component of velocity to be 40 instead of $40 \times 0.6 = 24$. Few candidates confused sine and cosine. The definition of the angle as sin□= 0.6,
cos□= 0.8 seemed unfamiliar to some candidates.

(ii) Most candidates had a suitable plan. Those who tried to find when $y = 0$ were usually accurate. Most of those found the time to the highest point did this accurately but many then forgot to double this time to get the time of flight. Many candidates who tried to use the formula for the range either misremembered it or struggled to find 2□or wrongly used the form for the maximum horizontal range. Candidates are usually better advised to work the given problem than to try to apply formulae of this sort.

Section B

7) **The static equilibrium of a box supported by two strings**

It is pleasing to be able to report that there were many complete solutions to this question. The candidates from many centres maintained a high level of accuracy and, generally, there were fewer instances of sine and cosine being used wrongly in resolution.

(i) Most candidates found at least two of: 'light string', 'continuous string' and 'smooth pulley'. The most common error was to think that the system being in equilibrium was a reason.

(ii) This part was often not done as well as part (iii). The most common errors came from candidates producing circular arguments that started by assuming the symmetry that they were trying to establish. The most common method was to resolve horizontally but many candidates failed to say where their equation had come from. There were a few nice arguments based on the triangle of forces.

(iii) This was done accurately and efficiently by many candidates. Quite a common mistake was to consider only one string and then try to fudge finding an angle that has a sine greater than 1. A few obtained the complementary angle.

(iv) Most candidates recognised that in this situation there could be different tensions in the two strings. Quite a few candidates did not understand how to set up the equations for horizontal and vertical equilibrium and instead tried to write down expressions for the tension in one string that did not involve the tension in the other (these expressions were based on no stated principle). Some candidates tried to resolve in directions that included only one string; most of these seemed to think that this was achieved by resolving parallel to each string instead of perpendicular to each string.

However, it was a pleasure to see very many candidates writing down two correct equations from resolution and then solving them correctly. There were many more who obtained the correct equations but could not solve them correctly; many of these had not spotted or used the simplification that cos 45° = sin 45°.

8) **The kinematics of a particle travelling along a straight line with non-constant acceleration.**

Most candidates realized that calculus was required and did well on parts (i), (ii) and (iii). As expected, only the stronger candidates could see how they might apply the answers to the earlier parts to help them with parts (vi) and (vii).

(i) This was answered correctly by almost every candidate.

(ii) Most candidates differentiated and did so correctly.

(iii) Most candidates differentiated but a surprising number obtained $a = 6 - 6t$ or $a = 6 - 16t$ instead of $a = 6 - 12t$. Some candidates who had correctly differentiated x to get v for part (ii) now integrated x in this part.

(iv) Although many candidates recognized this as a standard problem others did not and simply produced a table of some values of v in the interval; the conclusion of these candidates was almost always that the maximum value was 36 m s^{-1}, the value at $t = 0$ and $t = 1$. Quite a few candidates only gave the time at which the maximum value of v is obtained, not the value.

(v) Most candidates knew what to do and many did it correctly. Quite a few candidates who used the factorization method made sign errors; very few of those who used the substitution method established that there are *only* two times at which $t = 0$. A surprisingly large number could not correctly evaluate x when $t = -2$. Quite a few candidates did not make an attempt to evaluate x at the given times – presumably they forgot that they had been asked to do so.

(vi) Very few candidates realized that they were looking for a *distance travelled* in a situation where the particle changed direction and so many found the displacement instead. Quite a few candidates did not seem to have a plan.

(vii) A few candidates produced complete and well argued solutions. However, many candidates did not make it clear what they were trying to do; those who considered a sketch graph usually made at least some progress. Some candidates tried to solve the cubic equation $x = 0$ with false methods. Others used information obtained earlier in the question and looked for sign changes; many of these were successful. Many candidates thought that the answer must be three because x against t is a cubic curve. Some thought that the answer is zero because the graph of x against t does not pass through the origin.