MEI STATISTICS 1 — JUNE 2005

Section A

1)
i) $\bar{x} = \frac{\Sigma x}{n} = \frac{657}{20} = 32.85$

$s^2 = \frac{\Sigma x^2 - n\bar{x}^2}{n-1}$

$= \frac{22839 - 20 \times 32.85^2}{19}$

$s^2 = 66.1342$

ii $sd = \sqrt{66.1342}$

$sd = 8.13$

$\bar{x} + 2 \times s.d. = 32.85 + 2 \times 8.13$

$= 49.11$

The two 49s are border line but need not be considered as outliers

2) Results from graph

ii Median = 1·7

$Q_3 = 3{\cdot}1$

$Q_1 = 0{\cdot}7$

$IQR = Q_3 - Q_1 = 2.4$

iii) Positive skew

3)
i) $P(X=4) = \frac{1}{40} \times 4 \times 5$

$= \frac{20}{40} = \frac{1}{2}$

ii)

r =	1	2	3	4
$P(X=r)$ =	$\frac{2}{40}$	$\frac{6}{40}$	$\frac{12}{40}$	$\frac{20}{40}$

$E(X) = \frac{1}{40}(2\times1 + 6\times2 + 12\times3 + 20\times4)$

$= \frac{130}{40} = 3.25$

$Var(X) = E(X^2) - (E(X))^2$

$E(X^2) = \frac{1}{40}(2\times1 + 6\times4 + 12\times9 + 20\times16)$

$= \frac{454}{40} = 11.35$

$Var(X) = 11.35 - 3.25^2$

$= 0.7875$

iii) $P(X=2) = \frac{6}{40}$

Expected number of weeks

$= 45 \times \frac{6}{40}$

$= 6.75$

4)
i) $6C3 = 20$

ii) $6C3 \times 7C4 \times 8C5 = 39{,}200$

iii) Ways at random $= 21C12$

$= 293930$

Prob from correct sections

$= \frac{39{,}200}{293{,}930} = 0.133$

5 i)

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	6	4	10	6
3	3	6	3	12	15	6
4	4	4	12	4	20	12
5	5	10	15	20	5	30
6	6	6	6	12	30	6

ii) A) $P(\text{LCM} > 6) = \frac{12}{36} = \frac{1}{3}$

B) $P(\text{LCM a multiple of } 5) = \frac{11}{36}$

C) $P(\text{LCM} > 6 \text{ and a multiple of } 5) = \frac{8}{36} = \frac{2}{9}$

iii)

$\frac{1}{3} \times \frac{11}{36} = \frac{11}{108}$

$\frac{2}{9} = \frac{24}{108}$ $\qquad \frac{11}{108} \neq \frac{24}{108}$

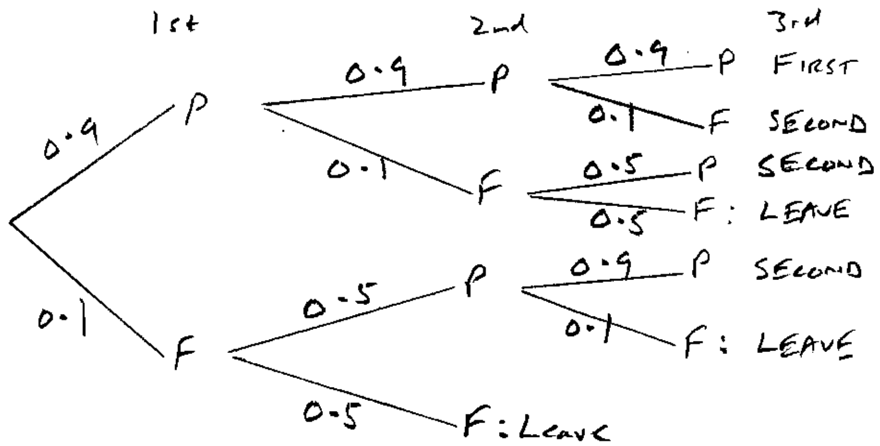
Since $P(\text{LCM} > 6) \times P(\text{LCM a multiple of } 5)$

$\neq P(\text{LCM} > 6 \text{ and LCM a multiple of } 5)$

Events are not independent

SECTION B

6 i)



6 ii) A) $P(\text{FIRST}) = 0.9^3 = 0.729$

B) $P(\text{SECOND})$

$= 0.9^2 \times 0.1 + 0.9 \times 0.1 \times 0.5$
$+ 0.1 \times 0.5 \times 0.9$

$= 0.171$

6 iii)

$P(\text{Leave}) = 1 - 0.729 - 0.171$

$= 0.1$

6 iv)

$P(\text{Leave after 2 games} / \text{Leave})$

$= \dfrac{0.1 \times 0.5}{0.1} = 0.5$

6 v)

P(At least one leaves)

$= 1 - P(\text{All stay})$

$= 1 - 0.9^3 = 0.271$

6 vi) 7 passes required

Possibilities

A	B	S
3	3	1
3	1	3
1	3	3
3	2	2
2	3	2
2	2	3

For any individual

$P(\text{Pass } 3) = 0.729$
$P(\text{Pass } 2) = 0.171$
$P(\text{Pass } 1) = 0.9 \times 0.1 \times 0.5$
$+ 0.1 \times 0.5 \times 0.1$
$= 0.05$

$P(\text{Pass 7 games}) =$

$3 \times .729 \times .729 \times .05$

$+ 3 \times .729 \times .171 \times .171$

$= 0.144$

7) i) $P(\text{No sixes}) = \left(\frac{5}{6}\right)^{15}$

$= 0.0649$

ii) $P(4 \text{ sixes}) = {}^{15}C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^{11}$

$= 0.1418$

iii) Let X be number of sixes

$X \sim B\left(15, \frac{1}{6}\right)$

$P(X > 3) = 1 - P(X \leq 3)$

From tables $= 1 - 0.7685$

$= 0.2315$

iv) A) David

$X \sim B\left(15, \frac{1}{6}\right)$

$H_0 : p = \frac{1}{6}$ $H_1 : p < \frac{1}{6}$

$P(X \leq 0) = 0.0649$

Since $P(X \leq 0) < 10\%$
we reject H_0 and accept
$H_1 : p < \frac{1}{6}$

B) Esme

$X \sim B\left(15, \frac{1}{6}\right)$

$H_0 : p = \frac{1}{6}$ $H_1 : p > \frac{1}{6}$

$P(X \geq 5) = 1 - P(X \leq 4)$
$= 1 - 0.9102$
$= 0.0898$

Since $P(X \geq 5) < 10\%$
we reject H_0 and accept H_1
$p > \frac{1}{6}$

v) David's test concludes $p < \frac{1}{6}$

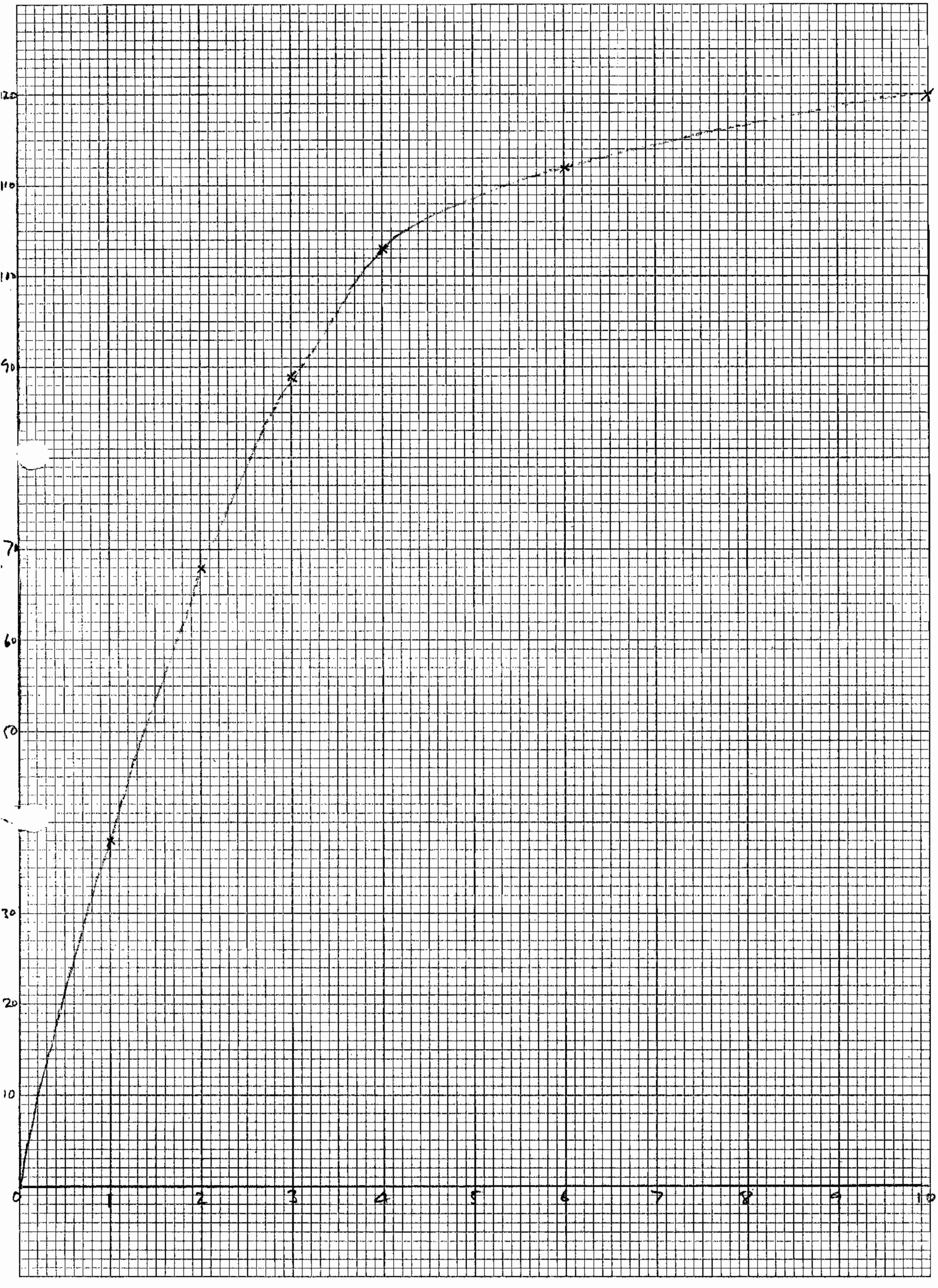
Esme's test concludes $p > \frac{1}{6}$

Hypothesis testing does not guarantee a correct conclusion

Perhaps dice are fair and $p = \frac{1}{6}$

If so there would have been a 6.5% chance of David's result and a 9% chance of Esme's

Tests at the 5% level would both have accepted
$H_0 : p = \frac{1}{6}$


120
110
100
90
70
60
50
30
20
10
0
1
2
3
4
5
6
7
8
9
10