

1) i) ${}^8C_4 = 70$

ii) 4 tracks can be arranged in $4!$ ways = 24

2)

i)

£ spent	frequency
$0 < x \leq 20$	800
$20 < x \leq 50$	480
$50 < x \leq 100$	400
$100 < x \leq 200$	200

40 squares \times 20

24 squares \times 20

20 squares \times 20

10 squares \times 20

ii) Estimate of total using midpoints of intervals

$$10 \times 800 + 35 \times 480 + 75 \times 400 + 150 \times 200 = \pounds 84,800$$

3)

$$n = 56, \quad \Sigma x = 3026, \quad \Sigma x^2 = 178,890$$

i) $\bar{x} = \frac{\Sigma x}{n} = \frac{3026}{56} = 54.0357 = 54.0$ to 3 s.f.

$$\text{s.d.} = \sqrt{\frac{\Sigma x^2 - n\bar{x}^2}{n-1}} = \sqrt{\frac{178,890 - 56 \times 54.0357^2}{55}}$$

$$\text{s.d.} = 16.72 = 16.7 \text{ to 3 s.f.}$$

ii)

Outlier if more than $2 \times$ s.d. above mean

$$54.0 + 2 \times 16.7 = 87.4 \quad \therefore 93 \text{ is an outlier}$$

iii)

$$y = 1.2x - 10 \quad \bar{y} = 1.2\bar{x} - 10 = 1.2 \times 54.0 - 10 = 54.8$$

$$\text{s.d.}_y = 1.2 \times \text{s.d.}_x = 1.2 \times 16.72 = 20.064$$

$$= 20.1 \text{ to 3 s.f.}$$

4)

i) A) $P(\text{At Least 1}) = \frac{36}{50} = \frac{18}{25}$ $\frac{\text{Everything in circles}}{\text{Total}}$

B) $P(\text{Exactly 1}) = \frac{9+6+5}{50} = \frac{20}{50} = \frac{2}{5}$

ii) $P(\text{Does not recycle paper / recycles aluminium})$

$$= \frac{9+4}{9+4+3+8} = \frac{13}{24}$$

iii)

$$P(\text{person recycles kitchen waste}) = \frac{4+8+1+5}{50} = \frac{18}{50}$$

From 2 people chosen

$P(\text{Exactly one recycles kitchen waste})$

$$= P(\text{First does}) \times P(\text{Second doesn't}) + P(\text{First doesn't}) \times P(\text{Second does})$$

$$= \frac{18}{50} \times \frac{32}{49} + \frac{32}{50} \times \frac{18}{49} = \frac{1152}{2450} = 0.4702$$

5) i)

Days Rain	Freq	Cum freq
0	2	2
1	1	3
2	4	7
3	3	10
4	5	15
5	3	18
6	2	20
7	2	22

$$\text{Median is item } \frac{22+1}{2}$$

$$= 11.5^{\text{th}}$$

$$\text{Median} = 4$$

since 11th and 12th items are both 4

$$LQ = \text{item } \frac{11}{2} = 5.5^{\text{th}} \quad UQ = \text{item } 17.5^{\text{th}}$$

$$5 \text{ cont}) \quad \therefore LQ = 2 \quad \text{and} \quad UQ = 5$$

$$IQR = UQ - LQ = 5 - 2 = 3$$

5ii)

Not valid: any two valid reasons such as:

1. sample is only for 2 years, which may not be representative.
2. data refer to local area, not all of Britain
3. even if decreasing, it may not be due to global warming.
4. more days with rain does not imply more rainfall.
5. a 5 year timescale may not be enough to show a trend

6)

Number of ways of choosing 4 from 7 is ${}^7C_4 = 35$

i) Only 1 of these correct so

$$P(\text{correct}) = \frac{1}{35}$$

6ii)

r	0	1	2	3	4
r^2	0	1	4	9	16
$P(X=r)$	0	$\frac{4}{35}$	$\frac{18}{35}$	$\frac{12}{35}$	$\frac{1}{35}$

$$E(X) = \frac{1}{35} \left[0 \times 0 + 4 \times 1 + 18 \times 2 + 12 \times 3 + 1 \times 4 \right]$$

$$E(X) = \frac{80}{35} = \frac{16}{7} \quad \text{or} \quad 2.286$$

$$E(X^2) = \frac{1}{35} \left[0 \times 0 + 4 \times 1 + 18 \times 4 + 12 \times 9 + 1 \times 16 \right]$$

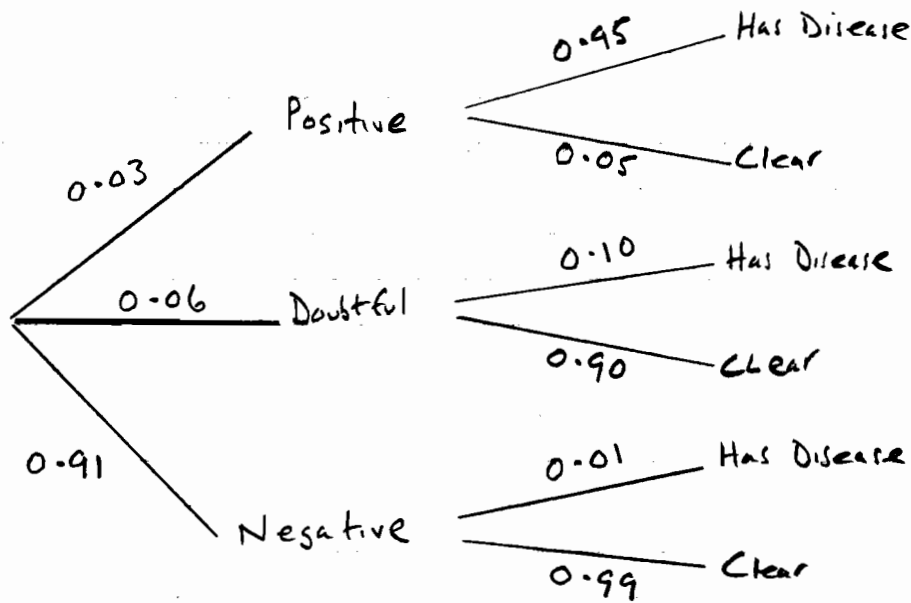
$$= \frac{200}{35}$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$= \frac{200}{35} - \left(\frac{16}{7}\right)^2 = 0.4898$$

$$= 0.490 \quad \text{to 3 s.f.}$$

7) i)



ii)
$$P(\text{Negative} \cap \text{Clear}) = 0.91 \times 0.99 = 0.9009$$

iii)
$$P(\text{Has Disease}) = 0.03 \times 0.95 + 0.06 \times 0.10 + 0.91 \times 0.01$$

$$= 0.0436$$

iv)
$$P(\text{Negative} \mid \text{Has Disease}) = \frac{P(\text{Negative} \cap \text{Has Disease})}{P(\text{Has Disease})}$$

$$= \frac{0.91 \times 0.01}{0.0436} = 0.2087$$

v) Approximately 1 in 5 of people with the disease will be given the all clear by this screening process. The test is therefore not very reliable

vi)
$$P(\text{Negative}) + P(\text{Doubtful} \cap \text{Has Disease} \cap \text{Cleared}) + P(\text{Doubtful} \cap \text{Clear})$$

$$= 0.91 + 0.06 \times 0.10 \times 0.02 + 0.06 \times 0.90$$

$$= 0.9642$$

$$8) X \sim B(17, 0.2)$$

$$i) P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.5489 = 0.4511$$

$$ii) E(X) = np = \frac{17 \times 0.2}{1} = 3.4$$

iii) Modal value will be next to 3.4

$$P(X=2) = {}^{17}C_2 \times 0.2^2 \times 0.8^{15} = 0.1914$$

$$P(X=3) = {}^{17}C_3 \times 0.2^3 \times 0.8^{14} = 0.2393 \quad \text{modal value}$$

$$P(X=4) = {}^{17}C_4 \times 0.2^4 \times 0.8^{13} = 0.2093$$

Most likely number is $X=3$

iv)

$$H_0: p = 0.2$$

$$A) H_1: p > 0.2$$

where p is the prob a
maths graduate is accepted

B) $H_1: p > 0.2$ because it is suggested maths graduates more successful

$$v) P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.8943 = 0.1057$$

$$P(X \geq 7) = 1 - P(X \leq 6) = 1 - 0.9623 = 0.0377 < 5\%$$

Critical region $\{7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17\}$

vi)

Unaltered because $P(X \geq 6) = 0.1057 > 10\%$

so still need $X \geq 7$ to be in critical region $< 10\%$