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2) i) Meteors are seen randomly and independently

There is a uniform mean rate of occurrence of meteor sightings

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ii) $X \sim \text{Poisson}(1.3)$

A) $P(X=1) = e^{-1.3} \times 1.3 = 0.3543$

B) $P(X \geq 4) = 1 - P(X \leq 3)$

$= 1 - 0.9569$

$= 0.0431$

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iii) $X \sim \text{Poisson}(13)$ for 10 min intervals

$P(X=10) = e^{-13} \times \frac{13^{10}}{10!} = 0.0859$

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iv) For periods of 1 hour $X \sim \text{Poisson}(60 \times 1.3)$

$X \sim \text{Poisson}(78)$

Approximate with

$X \sim N(78, \sqrt{78}^2)$

$P(X > 99.5) = 1 - P(X < 99.5)$

$z = \frac{x - \mu}{\sigma} = \frac{99.5 - 78}{\sqrt{78}} = 2.434$

$P(Z < 2.434) = 0.9926$

$P(X > 99.5) = 1 - 0.9926 = 0.0074$

Prob sees at least 100 meteors in an hour $= 0.0074$

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2v) $P(X \geq 1) \geq 0.99 \Rightarrow P(X=0) \leq 0.01$

Trial and error

If $t = 10$ mins $X \sim \text{Poisson}(10 \times 1.3)$

$X \sim \text{Poisson}(13)$

$P(X=0) = e^{-13} = 0.000002 < 0.01$

If $t = 8$ mins $X \sim \text{Poisson}(8 \times 1.3)$

$X \sim \text{Poisson}(10.4)$

$P(X=0) = e^{-10.4} = 0.00003 < 0.01$

If $t = 4$ mins $X \sim \text{Poisson}(4 \times 1.3)$

$X \sim \text{Poisson}(5.2)$

$P(X=0) = e^{-5.2} = 0.0052 < 0.01$

If $t = 3$ mins $X \sim \text{Poisson}(3 \times 1.3)$

$X \sim \text{Poisson}(3.9)$

$P(X=0) = e^{-3.9} = 0.0202 > 0.01$

$\therefore$ smallest integer value of t is $t = 4$ mins