MEI STATISTICS 2 JANUARY 2006 Q1 POISSON

1 i) Faults are detected randomly and independently B1

Uniform (mean) rate of occurrence B1

1 ii) $X \sim \text{Poisson}(0.15)$ $P(X=r) = \dfrac{e^{-\lambda}\lambda^r}{r!}$

A) $P(X=0) = e^{-0.15} = 0.8607$

B)
$$\begin{aligned} P(X \geq 2) &= 1 - P(X \leq 1) \\ &= 1 - \left(P(X=0) + P(X=1)\right) \\ &= 1 - \left(0.8607 + e^{-0.15} \times 0.15\right) \\ &= 0.0102 \end{aligned}$$

1 iii) For 30 days

$X \sim \text{Poisson}(0.15 \times 30)$

$X \sim \text{Poisson}(4.5)$

$P(X \leq 3) = 0.3423$

1 iv) Total faults for 10 days

$X \sim \text{Poisson}((0.15 + 0.05) \times 10)$

$X \sim \text{Poisson}(2)$

$$P(X=5) = \frac{e^{-2} \times 2^5}{5!}$$

$$= 0.0361$$

1v) For 200 days $X \sim \text{Poisson}(2 \times 20)$

$X \sim \text{Poisson}(40)$

Approximate with $X \sim N(\overset{\mu}{40}, \overset{\sigma^2}{\sqrt{40}^2})$

Find $P(X \geqslant 49.5)$

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{49.5 - 40}{\sqrt{40}}$$

$$z = 1.502$$

$$P(Z \geqslant 1.502) = 1 - P(Z < 1.502)$$

$$= 1 - 0.9334$$

$$= 0.0666$$