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2 i) $X \sim \text{Poisson}(0.37)$ for 5 ml samples

A) $P(X=2) = \dfrac{e^{-0.37} \times 0.37^2}{2!} = 0.0473$

B) $P(X>2) = 1 - P(X \leq 2)$

$= 1 - \left(P(X=2) + P(X=1) + P(X=0) \right)$

$= 1 - \left(0.0473 + e^{-0.37} \times 0.37 + e^{-0.37} \right)$

$= 1 - \left(0.0473 + 0.2556 + 0.6907 \right)$

$= 0.0064$

ii) Prob (not more than 2 bacteria in sample) $= 1 - 0.0064$

$= 0.9936$

Prob (at most 1 day when more than 2 bacteria)

$= P(0 \text{ days with } > 2) + P(1 \text{ day in } 30 \text{ with } > 2)$

$= 0.9936^{30} + {}^{30}C_1 \times 0.0064 \times 0.9936^{29}$

$= 0.9842$

iii) For 50 ml sample $X \sim \text{Poisson}(0.37 \times 10)$

$X \sim \text{Poisson}(3.7)$

$P(X>8) = 1 - P(X \leq 8)$

$= 1 - 0.9863 = 0.0137$

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iv) For 1000 ml sample $X \sim \text{Poisson}(3.7 \times 20)$

$X \sim \text{Poisson}(74)$

Approximate with

$$X \sim N(\underset{\mu}{74}, \underset{\sigma^2}{\sqrt{74}^2})$$

Find $P(X > 90.5)$

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{90.5 - 74}{\sqrt{74}}$$

$$z = 1.918$$

$P(z > 1.918) = 1 - P(z < 1.918)$

$= 1 - 0.9724$

$= 0.0276$

v) P(supply declared questionable)

= P(>2 in 5ml sample) × P(>8 in 50ml sample) × P(>90 in 1000ml)

$= 0.0064 \times 0.0137 \times 0.0276$

$= 0.00000242$