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1) i) $\left(\frac{5}{3}\right)^{-2} = \left(\frac{3}{5}\right)^{2} = \frac{9}{25}$

ii) $81^{3/4} = \left(\sqrt[4]{81}\right)^{3}$

$= 3^{3} = 27$

2) $\frac{(4x^5y)^3}{(2xy^2)\times(8x^{10}y^4)}$

$= \frac{64x^{15}y^3}{16x^{11}y^6}$

$= \frac{4x^4}{y^3}$ or $4x^4y^{-3}$

3) $C = 2\pi r$

$d = 2r$

$A = \pi r^2$

$Cd = 2\pi r \times 2r = 4\pi r^2$

$= 4A$

so if $Cd = kA$

then $k = 4$

4) $5x^2 - 28x - 12 \leq 0$

$5\times -12 = -60$

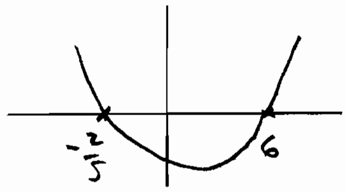
factors $+2, -30$

$5x^2 + 2x - 30x - 12 \leq 0$

$x(5x+2) - 6(5x+2) \leq 0$

$(x-6)(5x+2) \leq 0$

Sketch $y = (x-6)(5x+2)$



$x - 6 = 0 \Rightarrow x = 6$

$5x + 2 = 0 \Rightarrow 5x = -2 \Rightarrow x = -\frac{2}{5}$

Solution:

$-\frac{2}{5} \leq x \leq 6$

5) $f(x) = x^2 + kx + c$

$f(2) = 2^2 + 2k + c = 0$

$4 + 2k + c = 0$ (1)

$f(-3) = (-3)^2 - 3k + c = 35$

$9 - 3k + c = 35$ (2)

From (1) $2k + c = -4$ (3)

From (2) $-3k + c = 26$ (4)

(3) - (4) $5k = -30$

$\Rightarrow k = -6$

Sub for k in (3) $-12 + c = -4$

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5 cont)

$$c = -4 + 12$$

$$c = 8$$

Solution: $c = 8$, $k = -6$

6) Constant term

$${}^6C_3\,(2x)^3\left(\frac{5}{x}\right)^3$$

$$= \frac{6.5.4}{1.2.3} \times 8x^3 \times \frac{125}{x^3}$$

$$= 20 \times 8 \times 125$$

$$= 20{,}000$$

7) $\sqrt{48} + \sqrt{75}$

i) $= \sqrt{16\times 3} + \sqrt{25\times 3}$

$$= 4\sqrt{3} + 5\sqrt{3} = 9\sqrt{3}$$

ii)

$$\frac{7+2\sqrt{5}}{7+\sqrt{5}} = \frac{7+2\sqrt{5}}{7+\sqrt{5}} \times \frac{7-\sqrt{5}}{7-\sqrt{5}}$$

$$= \frac{(7+2\sqrt{5})(7-\sqrt{5})}{7^2 - \sqrt{5}^2}$$

$$= \frac{49 + 14\sqrt{5} - 7\sqrt{5} - 10}{49 - 5}$$

$$= \frac{39 + 7\sqrt{5}}{44}$$

8) $5c + 9t = a(2c + t)$

$$5c + 9t = 2ac + at$$

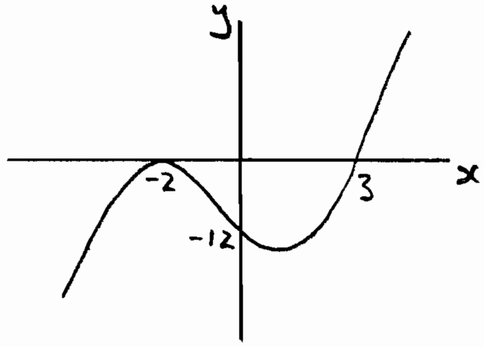
$$5c - 2ac = at - 9t$$

$$c(5 - 2a) = t(a - 9)$$

$$c = \frac{t(a-9)}{5-2a} \text{ or } \frac{ta - 9t}{5-2a}$$

9) $f(x) = (x+2)^2(x-3)$

i) $f(0) = 2^2(-3) = -12$



ii) $f(x+3)$ obtained from $f(x)$ by translation $\begin{pmatrix}-3\\0\end{pmatrix}$

ie 3 units left

Roots of $f(x+3) = 0$

$$x = -2 - 3 = -5$$

and $x = 3 - 3 = 0$

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10) A $(-2,1)$
B $(3,4)$

i) Midpoint $\left(\frac{-2+3}{2}, \frac{1+4}{2}\right)$

$= \left(\frac{1}{2}, \frac{5}{2}\right)$

Gradient AB $= \frac{y_2-y_1}{x_2-x_1} = \frac{4-1}{3--2}$

$= \frac{3}{5}$

$\therefore$ gradient of $\perp$ bisector

$= -\frac{5}{3}$

Using $y - y_1 = m(x - x_1)$

$y - \frac{5}{2} = -\frac{5}{3}\left(x - \frac{1}{2}\right)$

$\Rightarrow \quad 6y - 15 = -10\left(x - \frac{1}{2}\right)$

(multiplying eqn by 6)

$6y - 15 = -10x + 5$

$10x + 6y = 20$

$\Rightarrow \quad 5x + 3y = 10$

10ii) $C(-5,4)$
$D(3,6)$

Solve $\begin{cases} 5x + 3y = 10 & (1) \\ 4y = x + 21 & (2) \end{cases}$

from (2) $\quad x = 4y - 21 \quad (3)$

Sub for x in (1)

$5(4y-21) + 3y = 10$

$20y - 105 + 3y = 10$

$23y = 115$

$y = \frac{115}{23} = 5$

Sub for y in (3)

$x = 4(5) - 21$

$x = -1$

Point of intersection:

$E(-1, 5)$

10iii) $|AE| = \sqrt{(-2--1)^2 + (1-5)^2}$

$= \sqrt{1+16} = \sqrt{17}$

$|BE| = \sqrt{(3--1)^2 + (4-5)^2}$

$= \sqrt{16+1} = \sqrt{17}$

$\therefore$ both A and B are $\sqrt{17}$ units from E, so both lie on circle

$(x+1)^2 + (y-5)^2 = 17$

Show C, D on this circle

$C(-5,4) \quad (-5+1)^2 + (4-5)^2$
$= 4^2 + 1^2 = 17$ ✓

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10iii) cont) $D(3,6)$

$$(3+1)^2+(6-5)^2$$

$$= 4^2+1^2 = 17 \checkmark$$

so C, D on circle

$$|CD| = \sqrt{(-5-3)^2+(4-6)^2}$$

$$= \sqrt{64+4} = \sqrt{68}$$

$$= \sqrt{4\times17} = 2\sqrt{17}$$

$\therefore$ C and D are on circle and distance between them is $2 \times$ radius

$\therefore$ CD is a diameter

11) i)

$$x^2-5x+6$$

$$= \left(x-\tfrac{5}{2}\right)^2+6-\tfrac{25}{4}$$

$$= \left(x-\tfrac{5}{2}\right)^2-\tfrac{1}{4}$$

Turning point $\left(\frac{5}{2}, -\frac{1}{4}\right)$

11ii) $y = x^2-5x+6$

when $x=0$, $y=6$

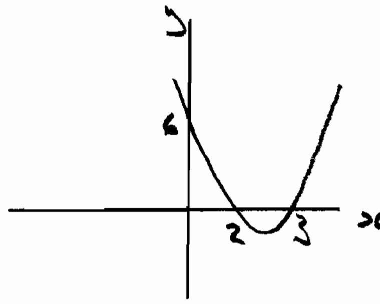
Intersects y-axis at $(0,6)$

when $y=0$, $x^2-5x+6=0$

$$(x-3)(x-2)=0$$

$$\Rightarrow x=2 \text{ or } x=3$$

Intersects x-axis at $(2,0)$ and $(3,0)$



11iii) $y = x^2-5x+6$ ①

$x+y=2$ ②

From ② $y = 2-x$

Sub for y in ①

$$2-x = x^2-5x+6$$

$$0 = x^2-5x+6+x-2$$

$$0 = x^2-4x+4$$

$$0 = (x-2)(x-2)$$

$\Rightarrow x=2$ double root

when $x=2$, $y=2-2=0$

Single point of intersection at $(2,0)$

$\therefore x+y=2$ is a tangent to curve at $(2,0)$, which is one of points curve cuts x-axis as shown in part (ii)

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12) i) $f(x) = x^4 - x^3 + x^2 + 9x - 10$

$f(1) = 1^4 - 1^3 + 1^2 + 9(1) - 10$

$= 1 - 1 + 1 + 9 - 10$

$= 11 - 11 = 0$

$\therefore x = 1$ is a root of $f(x) = 0$

$$
\begin{array}{r|l}
 & x^3 + x + 10 \\
\hline
x-1 & x^4 - x^3 + x^2 + 9x - 10 \\
 & x^4 - x^3 \\
\hline
 & 0 + x^2 + 9x \\
 & + x^2 - x \\
\hline
 & 10x - 10 \\
 & 10x - 10 \\
\hline
\end{array}
$$

$f(x) = (x-1)(x^3 + x + 10)$

12 ii) By inspection

$(-2)^3 + (-2) + 10$

$= -8 - 2 + 10 = 0$

So $x = -2$ is a root

12iii)

$$
\begin{array}{r|l}
 & x^2 - 2x + 5 \\
\hline
x+2 & x^3 + x + 10 \\
 & x^3 + 2x^2 \\
\hline
 & -2x^2 + x \\
 & -2x^2 - 4x \\
\hline
 & 5x + 10 \\
 & 5x + 10 \\
\hline
\end{array}
$$

$f(x) = (x-1)(x+2)(x^2 - 2x + 5)$

So two linear factors $(x-1)$ and $(x+2)$

However, for $(x^2 - 2x + 5) = 0$

$b^2 - 4ac = 4 - 20 = -16 < 0$

$\therefore$ no real roots

$\therefore$ only two linear factors and only two real roots

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