MEI CORE 1 JAN 2011

1) Parallel to $y = 5x - 4$

$\therefore \ m = 5$

Passes through $(2, 13)$

$$y - y_1 = m(x - x_1)$$
$$y - 13 = 5(x - 2)$$
$$y - 13 = 5x - 10$$
$$y = 5x + 3$$

2) i) (A) $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

(B) $9^0 = 1$

ii) $\left(\frac{64}{125}\right)^{4/3}$

$$= \left(\sqrt[3]{\frac{64}{125}}\right)^4$$
$$= \left(\frac{4}{5}\right)^4$$
$$= \frac{256}{625}$$

3) $\frac{(3xy^4)^3}{6x^5y^2}$

$$= \frac{27x^3y^{12}}{6x^5y^2}$$
$$= \frac{9y^{10}}{2x^2}$$

4) $5 - 2x < 0$

$$5 < 2x$$
$$\frac{5}{2} < x$$
$$x > \frac{5}{2}$$

5) $V = \frac{1}{3}\pi r^2\sqrt{\ell^2 - r^2}$

$$3V = \pi r^2\sqrt{\ell^2 - r^2}$$
$$\frac{3V}{\pi r^2} = \sqrt{\ell^2 - r^2}$$
$$\frac{9V^2}{\pi^2 r^4} = \ell^2 - r^2$$
$$\frac{9V^2}{\pi^2 r^4} + r^2 = \ell^2$$
$$\ell = \sqrt{\frac{9V^2}{\pi^2 r^4} + r^2}$$

6) $(2 - 3x)^5$

1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$$= 2^5 + 5(2)^4(-3x) + 10(2)^3(-3x)^2 + \ldots$$
$$= 32 - 240x + 720x^2 - \ldots\ldots$$

first 3 terms as required

MEI CORE 1 JAN 2011

7)

i) $\dfrac{81}{\sqrt{3}} = \dfrac{3^4}{3^{1/2}} = 3^{7/2}$

ii) $\dfrac{5+\sqrt{3}}{5-\sqrt{3}}$

$= \dfrac{5+\sqrt{3}}{5-\sqrt{3}} \times \dfrac{5+\sqrt{3}}{5+\sqrt{3}}$

$= \dfrac{25+5\sqrt{3}+5\sqrt{3}+3}{5^2 - \sqrt{3}^2}$

$= \dfrac{28+10\sqrt{3}}{22}$

$= \dfrac{14+5\sqrt{3}}{11}$

8) $x + 2y = 5$ ①

$y = 5x - 1$ ②

Sub for y in ①

$x + 2(5x-1) = 5$

$x + 10x - 2 = 5$

$11x = 7$

$x = \dfrac{7}{11}$

Sub for x in ②

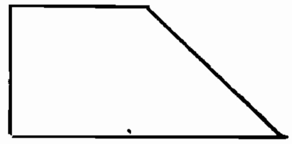
$y = 5 \times \dfrac{7}{11} - 1$

$y = \dfrac{35}{11} - \dfrac{11}{11} = \dfrac{24}{11}$

$x = \dfrac{7}{11}, \quad y = \dfrac{24}{11}$

9)

i)



Area $= \frac{1}{2}(a+b)h$

$= \dfrac{1}{2}\left(3x+6+x+2\right) \times 2x = 140$

$\Rightarrow (4x+8)x = 140$

$4x^2 + 8x - 140 = 0$

$x^2 + 2x - 35 = 0$

ii) $x^2 + 2x - 35 = 0$

$(x+7)(x-5) = 0$

$\Rightarrow \cancel{x = -7}$ or $x = 5$

$|AB| = 3x + 6 = 3(5) + 6$

$= 21$ cm

10)

i) $P \Leftarrow Q$

ii) none of the above

iii) $P \Rightarrow Q$

MEI CORE 1 JAN 2011

11) A(−1, 6)
B(1, 0)
C(13, 4)

i) Gradient AB $= \frac{y_2 - y_1}{x_2 - x_1}$

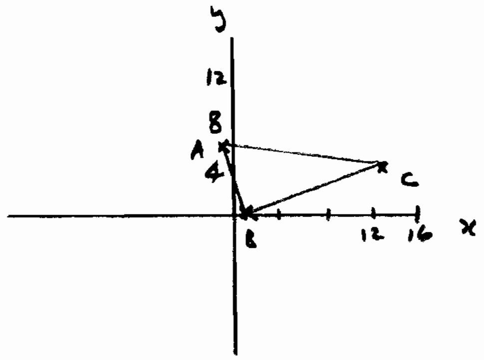
$= \frac{0-6}{1--1} = \frac{-6}{2}$

$= -3$

Gradient BC $= \frac{4-0}{13-1} = \frac{4}{12}$

$= \frac{1}{3}$

$\perp$ since $-3 \times \frac{1}{3} = -1$

ii) 

Area $= \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} \times |BC| \times |AB|$

$|BC| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(13-1)^2+(4-0)^2}$

$= \sqrt{12^2+4^2}$

$= \sqrt{160}$

$|AB| = \sqrt{(1--1)^2+(0-6)^2}$

$= \sqrt{2^2+6^2}$

$= \sqrt{40}$

Area $= \frac{1}{2} \times \sqrt{160} \times \sqrt{40}$

$= \frac{1}{2} \times 4\sqrt{10} \times 2\sqrt{10}$

$= \frac{1}{2} \times 4 \times 2 \times 10$

$= 40$ units^2

iii) $\angle ABC = 90^\circ$ (angle in semicircle)

$\therefore$ AC is a diameter

Centre is midpoint of AC

$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-1+13}{2}, \frac{6+4}{2}\right)$

$= (6, 5)$

radius = distance between (6, 5) and (13, 4)

$= \sqrt{(13-6)^2+(4-5)^2}$

$= \sqrt{7^2+(-1)^2}$

$= \sqrt{50}$

MEI CORE 1 JAN 2011

Eqn of circle

centre $(6,5)$ radius $\sqrt{50}$

$$(x-6)^2+(y-5)^2=50$$

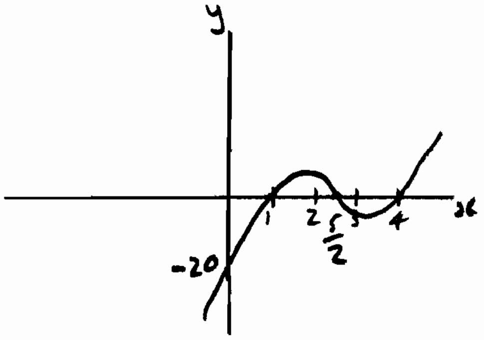
iv) By similar Δs

$B(1,0)$

Centre $(6,5)$

opposite B $(11,10)$

12) i) A) $f(x)=(2x-5)(x-1)(x-4)$



Crosses x axis at $1,\ \frac{5}{2},\ 4$

B) $f(x)=(2x-5)(x^2-x-4x+4)$

$=(2x-5)(x^2-5x+4)$

$= 2x^3-5x^2$
$\quad -10x^2+25x$
$\quad +8x-20$

$=2x^3-15x^2+33x-20$

ii) $g(x)=2x^3-15x^2+33x-40$

A) $g(5)$

$=2(5)^3-15(5)^2+33(5)-40$

$=250-375+165-40$

$=0$

B) Factor theorem $\Rightarrow (x-5)$ a factor

$$\begin{array}{r|l} & 2x^2-5x+8 \\ \hline x-5 & 2x^3-15x^2+33x-40 \\ & 2x^3-10x^2 \\ \hline & \quad -5x^2+33x \\ & \quad -5x^2+25x \\ \hline & \qquad 8x-40 \\ & \qquad 8x-40 \\ \hline \end{array}$$

$g(x)=(x-5)(2x^2-5x+8)$

C) For roots

either $x-5=0 \Rightarrow x=5$

or $2x^2-5x+8=0$

Discriminant b^2-4ac

$=25-4\times2\times8$

$=25-64$

$=-39 < 0$

$\therefore$ no real roots

So $g(x)=0$ has only one root

iii) Translation by $\begin{pmatrix}0\\-20\end{pmatrix}$

MEI CORE 1 JAN 2011

13 i) $y = x^4 - 2$

When $x = 0$, $y = -2$

Cuts y-axis at $(0, -2)$

when $y = 0$

$0 = x^4 - 2$

$2 = x^4$

$\pm 2^{\frac{1}{4}} = x$

Cuts x-axis at $(2^{\frac{1}{4}}, 0)$

and $(-2^{\frac{1}{4}}, 0)$

ii) $y = x^4 - 2$ ①

$y = x^2$ ②

Subst for y in ①

$x^2 = x^4 - 2$

$0 = x^4 - x^2 - 2$

$0 = (x^2 + 1)(x^2 - 2)$

$\Rightarrow x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

When $x = \sqrt{2}$ $y = \sqrt{2}^2 = 2$

when $x = -\sqrt{2}$ $y = (-\sqrt{2})^2 = 2$

$(\sqrt{2}, 2)$ and $(-\sqrt{2}, 2)$

Points of intersection

$(\sqrt{2}, 2)$ and $(-\sqrt{2}, 2)$

(Note: $x^2 + 1 = 0$ does not give any roots)

iii) $y = x^4 - 2$ ①

$y = kx^2$ ②

Sub for y in ①

$kx^2 = x^4 - 2$

$x^4 - kx^2 - 2 = 0$

Discriminant $= b^2 - 4ac$

$= k^2 - 4(1)(-2)$

$= k^2 + 8$

> 0 for all k

$\therefore$ always real roots of this equation, and so always points of intersection