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1. $m = -2, \ (x_1, y_1) = (3, 1)$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -2(x - 3)$$
$$y - 1 = -2x + 6$$
$$y = -2x + 7$$

2 $$a = \frac{2}{3}b^2c$$
$$3a = 2b^2c$$
$$\frac{3a}{2c} = b^2$$
$$b = \pm\sqrt{\frac{3a}{2c}}$$

3. i) $$\left(\frac{1}{5}\right)^{-2} = \left(\frac{5}{1}\right)^2 = 25$$

ii) $$\left(\frac{8}{27}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{8}{27}}\right)^2$$
$$= \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

4. $$\frac{x^2 - 9}{x^2 + 5x + 6}$$
$$= \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(x+2)}$$
$$= \frac{x-3}{x+2}$$

5. i) $$\frac{10(\sqrt{6})^3}{\sqrt{24}} = \frac{10 \times 6\sqrt{6}}{\sqrt{4} \times \sqrt{6}}$$
$$= \frac{60\sqrt{6}}{2\sqrt{6}} = 30$$

5 ii) $$\frac{1}{4 - \sqrt{5}} + \frac{1}{4 + \sqrt{5}}$$
$$= \frac{4 + \sqrt{5} + 4 - \sqrt{5}}{(4 - \sqrt{5})(4 + \sqrt{5})}$$
$$= \frac{8}{4^2 - \sqrt{5}^2} = \frac{8}{16 - 5}$$
$$= \frac{8}{11}$$

6. i) $${}^5C_3 = \frac{5!}{3!\,2!}$$
$$= \frac{5 \times 4}{2 \times 1} = 10$$

6 ii) Term in x^3 given by

$${}^5C_3\,(3)^2(-2x)^3$$
$$= 10 \times 9 \times (-8x^3)$$
$$= -720x^3$$

Coefficient $= -720$

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7. Does not intersect x-axis when $x^2+2kx+5=0$ has no real roots

ie when $b^2-4ac<0$

$$(2k)^2-20<0$$

$$4k^2<20$$

$$k^2<5$$

$$\Rightarrow \quad -\sqrt{5}<k<\sqrt{5}$$

8. $f(x) = x^4+bx+c$

$f(2)=0$

$$\Rightarrow \quad 2^4+2b+c=0$$

$$16+2b+c=0$$

$$2b+c=-16 \quad (1)$$

By remainder theorem

$$f(-3)=85$$

$$\Rightarrow \quad (-3)^4-3b+c=85$$

$$81-3b+c=85$$

$$-3b+c=4 \quad (2)$$

$(1)-(2)$ $\quad 5b=-20$

$$b=-4$$

Subst for b in (1)

$$-8+c=-16$$

$$c=-16+8$$

$$c=-8$$

Answer $b=-4$, $c=-8$

9. $(n+3)^2-n^2$

$$=(n+3+n)(n+3-n)$$

$$=3(2n+3) \quad =6n+9$$

$6n$ is even for all n since it has a factor of 2

An even plus an odd is odd

so $6n+9$ is always odd for any integer n

If $(n+3)^2-n^2$ is divisible by 9 then $6n+9$ is divisible by 9

$\Rightarrow$ $6n$ is divisible by 9

$\Rightarrow$ n is divisible by 3

Section B

10 i $\quad A(1,5)$
$B(-1,1)$
$C(3,-1)$
$D(11,5)$

$$|AB| = \sqrt{(1--1)^2+(5-1)^2}$$

$$=\sqrt{4+16} \quad =\sqrt{20}$$

$$|BC| = \sqrt{(-1-3)^2+(1--1)^2}$$

$$=\sqrt{16+4} \quad =\sqrt{20}$$

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10i cont

$\therefore \quad AB = BC$

10ii

gradient $AC = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5 - -1}{1 - 3}$

$= \dfrac{6}{-2} = -3$

gradient $BD = \dfrac{1-5}{-1-11} = \dfrac{-4}{-12}$

$= \dfrac{1}{3}$

$\perp$ since $-3 \times \dfrac{1}{3} = -1$

10iii

midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{1+3}{2}, \dfrac{5-1}{2}\right)$

$= (2, 2)$

Show $(2,2)$ is on BD

Line BD

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-1}{5-1} = \frac{x - -1}{11 - -1}$$

$$\frac{y-1}{4} = \frac{x+1}{12}$$

$$y - 1 = \frac{4}{12}(x+1)$$

$$y - 1 = \frac{1}{3}x + \frac{1}{3}$$

$$y = \frac{1}{3}x + \frac{4}{3}$$

When $x = 2$, $y = \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2$

$\therefore (2,2)$ is on BD and so BD bisects AC

However, midpoint of BD is

$$\left(\frac{11 + -1}{2}, \frac{5+1}{2}\right)$$

$= (5, 3)$ <u>not</u> $(2,2)$

so AC does not bisect BD

11. i) $(x + \frac{1}{2}), (x+2), (x-5)$

$y = k(x + \frac{1}{2})(x+2)(x-5)$

If $y = 2x^3 + ax^2 + bx + c$

then $k = 2$

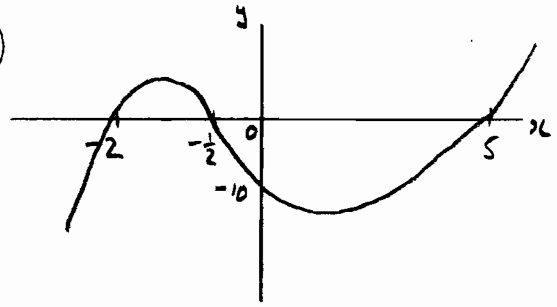
$y = (2x+1)(x+2)(x-5)$

$= (2x+1)(x^2 - 3x - 10)$

$= 2x^3 + x^2$
$\quad - 6x^2 - 3x$
$\quad - 20x - 10$

$y = 2x^3 - 5x^2 - 23x - 10$

11 ii)



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11 iii) Translation by $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$ lowers curve by 8 units so crosses y-axis at $(0, -18)$

11 iv) Translation by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ moves curve 3 units to right

So $f(x) = 2(x+\frac{1}{2})(x+2)(x-5)$

becomes

$$g(x) = 2\left(x - \frac{5}{2}\right)(x-1)(x-8)$$

$$g(x) = (2x-5)(x-1)(x-8)$$

(roots have all been increased by 3)

Cuts y-axis at y-coord

$$-5 \times -1 \times -8 = -40$$

So cuts at $(0, -40)$

12 i) See insert

12 ii)

$$y = \frac{1}{x-3} \quad ①$$

$$y = x^2 - 4x + 1 \quad ②$$

Subst for y in ②

$$\frac{1}{x-3} = x^2 - 4x + 1$$

$$1 = (x^2 - 4x + 1)(x-3)$$

$$1 = x^3 - 4x^2 + x - 3x^2 + 12x - 3$$

$$1 = x^3 - 7x^2 + 13x - 3$$

$$0 = x^3 - 7x^2 + 13x - 4$$

12 iii)

```
              x² - 3x + 1
        ________________________
x - 4 | x³ - 7x² + 13x - 4
        x³ - 4x²
        ---------
            -3x² + 13x
            -3x² + 12x
            ----------
                      x - 4
                      x - 4
                      -----
```

$$x^3 - 7x^2 + 13x - 4 = 0$$

$$\Rightarrow (x-4)(x^2 - 3x + 1) = 0$$

Other roots when $x^2 - 3x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

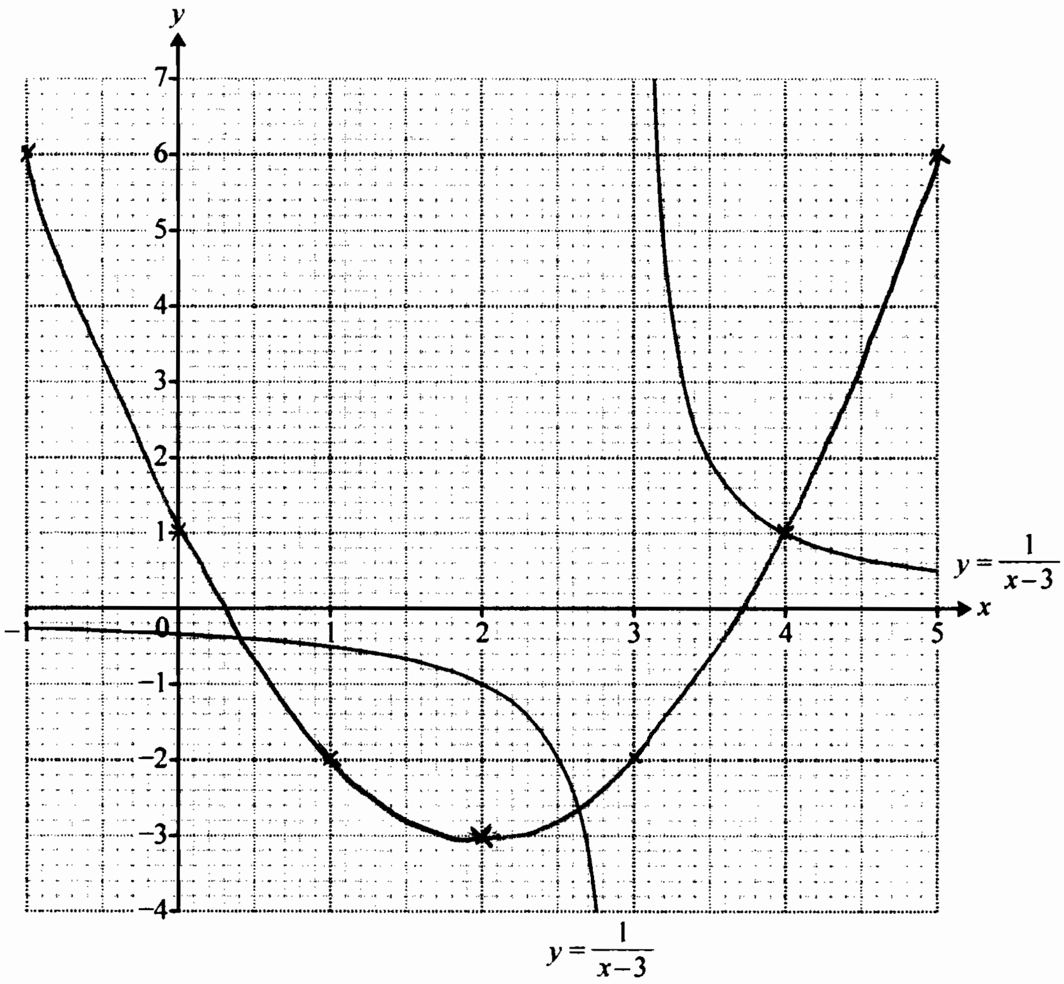
$$x = \frac{3 \pm \sqrt{9-4}}{2}$$

$$x = \frac{3 \pm \sqrt{5}}{2}$$

$$x = \frac{3+\sqrt{5}}{2} \quad \text{and} \quad x = \frac{3-\sqrt{5}}{2}$$

//

Spare copy of Fig. 12 for question 12 (i).



x	−1	0	1	2	3	4	5
x^2	1	0	1	4	9	16	25
$-4x$	+4	0	−4	−8	−12	−16	−20
$+1$	+1	+1	+1	+1	+1	+1	+1
y	6	1	−2	−3	−2	1	6

i) Intersections at
$(4,1)$ $(0.4,-0.4)$
$(2.6,-2.6)$

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