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1) $y = 2x - 5$

gradient $= 2$

$\perp$ gradient $= -\frac{1}{2}$

passes through $(4, 1)$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{2}(x - 4)$$

$$y - 1 = -\frac{1}{2}x + 2$$

$$y = -\frac{1}{2}x + 3$$

2)

$$y = 3x - 2 \quad (1)$$

$$x + 3y = 1 \quad (2)$$

Sub for y in (2)

$$x + 3(3x - 2) = 1$$

$$x + 9x - 6 = 1$$

$$10x = 7$$

$$x = \frac{7}{10} \text{ or } 0.7$$

Sub for x in (1)

$$y = 3 \times 0.7 - 2$$

$$y = 2.1 - 2 = 0.1$$

Solution:

$x = 0.7$

$y = 0.1$

3) i) $(0.2)^{-2} = \frac{1}{0.2^2}$

$$= \frac{1}{0.04}$$

$$= \frac{100}{4} = 25$$

or $(0.2)^{-2} = \left(\frac{1}{5}\right)^{-2} = \left(\frac{5}{1}\right)^2$

$$= 25$$

3 ii) $\left(16a^{12}\right)^{3/4}$

$$= \left(\sqrt[4]{16a^{12}}\right)^3$$

$$= \left(2a^3\right)^3$$

$$= 8a^9$$

4) $V = \frac{1}{3}\pi r^2(a+b)$

$$3V = \pi r^2(a+b)$$

$$\frac{3V}{\pi(a+b)} = r^2$$

$$r = \sqrt{\frac{3V}{\pi(a+b)}}$$

5) $f(x) = x^5 + kx - 20$

Remainder when dividing by $(x-2)$ is given by $f(2)$

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5 cont) $f(2) = 2^5 + k \times 2 - 20 = 18$

$\Rightarrow 32 + 2k - 20 = 18$

$12 + 2k = 18$

$2k = 18 - 12$

$2k = 6$

$k = 3$

6) $(2-4x)^5$

Term in x^3 given by

${}^5C_3 (2)^2(-4x)^3$

$= \frac{5.4}{2.1} \times 4 \times (-64x^3)$

$= -40 \times 64x^3$

$= -2560x^3$

Coefficient $= -2560$

7) $125\sqrt{5} = 5^3 \times 5^{\frac{1}{2}}$

i) $= 5^{7/2}$

ii) $10 + 7\sqrt{5} + \frac{38}{1-2\sqrt{5}}$

$= 10 + 7\sqrt{5} + \frac{38(1+2\sqrt{5})}{(1-2\sqrt{5})(1+2\sqrt{5})}$

$= 10 + 7\sqrt{5} + \frac{38(1+2\sqrt{5})}{1-20}$

$= 10 + 7\sqrt{5} + \frac{38(1+2\sqrt{5})}{-19}$

$= 10 + 7\sqrt{5} - 2(1+2\sqrt{5})$

$= 10 + 7\sqrt{5} - 2 - 4\sqrt{5}$

$= 8 + 3\sqrt{5}$

8) $3x^2 - 12x + 5$

$= 3\left[x^2 - 4x + \frac{5}{3}\right]$

$= 3\left[(x-2)^2 + \frac{5}{3} - 4\right]$

$= 3(x-2)^2 + 5 - 12$

$= 3(x-2)^2 - 7$

Minimum value of

$y = 3x^2 - 12x + 5$

$y = 3(x-2)^2 - 7$

is given by $y = -7$

9) i) $n-1, \ n, \ n+1$

i) $n - 1 + n + (n+1)$

$= 3n$

which is divisible by 3 since 3 is a factor.

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9 ii) $(n-1)^2 + n^2 + (n+1)^2$

$= n^2 - 2n + 1 + n^2 + n^2 + 2n + 1$

$= 3n^2 + 2$

Never divisibly by 3 since 3 is not a factor of this expression.

10) $(x-3)^2 + (y-2)^2 = 20$

i) Centre C is $(3, 2)$

Radius $= \sqrt{20}$

ii) On y-axis x coord is 0

$(0-3)^2 + (y-2)^2 = 20$

$9 + (y-2)^2 = 20$

$(y-2)^2 = 20 - 9 = 11$

$y - 2 = \pm\sqrt{11}$

$y = 2 \pm \sqrt{11}$

Intersects with y-axis at

$(0, 2+\sqrt{11})$

and $(0, 2-\sqrt{11})$

On x-axis y coord is 0

$(x-3)^2 + (0-2)^2 = 20$

$(x-3)^2 + 4 = 20$

$(x-3)^2 = 20 - 4 = 16$

$x - 3 = \pm 4$

$x = \pm 4 + 3$

$x = 7$ or $x = -1$

Intersects with x-axis at

$(7, 0)$ and $(-1, 0)$

10iii) $A(1, 6)$ $B(7, 4)$

Subst in eqn of circle

$(1-3)^2 + (6-2)^2$

$= 4 + 16 = 20$ ✓ A on circle

$(7-3)^2 + (4-2)^2$

$= 16 + 4 = 20$ ✓ B on circle

Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{1+7}{2}, \frac{6+4}{2}\right)$

$= (4, 5)$

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10 iii cont) Distance is measured from midpoint of chord to centre

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Centre $(3, 2)$
Midpoint of chord $(4, 5)$

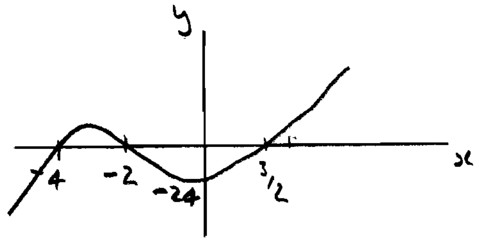
$$d = \sqrt{(4-3)^2 + (5-2)^2}$$

$$d = \sqrt{1 + 9} = \sqrt{10}$$

distance $= \sqrt{10}$ units

11) $f(x) = (2x-3)(x+2)(x+4)$

i)



Intercepts $(-4, 0)$, $(-2, 0)$, $(3/2, 0)$ and $(0, -24)$

(Just in case graph not clear)

ii) $f(x-2)$ is a translation of $f(x)$ by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

roots move 2 to right

roots $-2, \ 0, \ \frac{7}{2}$

iii) $g(x) = f(x) + 15$

A)
$$g(x) = (2x-3)(x+2)(x+4) + 15$$
$$= (2x-3)(x^2+6x+8) + 15$$
$$= 2x^3 - 3x^2 + 12x^2 - 18x + 16x - 24 + 15$$
$$= 2x^3 + 9x^2 - 2x - 9$$

B) $g(1) = 2(1)^3 + 9(1)^2 - 2(1) - 9$

$= 2 + 9 - 2 - 9 = 0$

$\therefore (x-1)$ is a factor of $g(x)$

$$\begin{array}{r|l} & 2x^2 + 11x + 9 \\ \hline x - 1 & 2x^3 + 9x^2 - 2x - 9 \\ & \underline{2x^3 - 2x^2} \\ & \quad 11x^2 - 2x \\ & \quad \underline{11x^2 - 11x} \\ & \qquad 9x - 9 \\ & \qquad \underline{9x - 9} \end{array}$$

$$g(x) = (x-1)(2x^2 + 11x + 9)$$

$$g(x) = (x-1)(2x+9)(x+1)$$

12) $y = 2x + 3$

$x = 0, \quad y = 3$

$x = 2, \quad y = 7$

$x = -2, \quad y = -1$

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12)

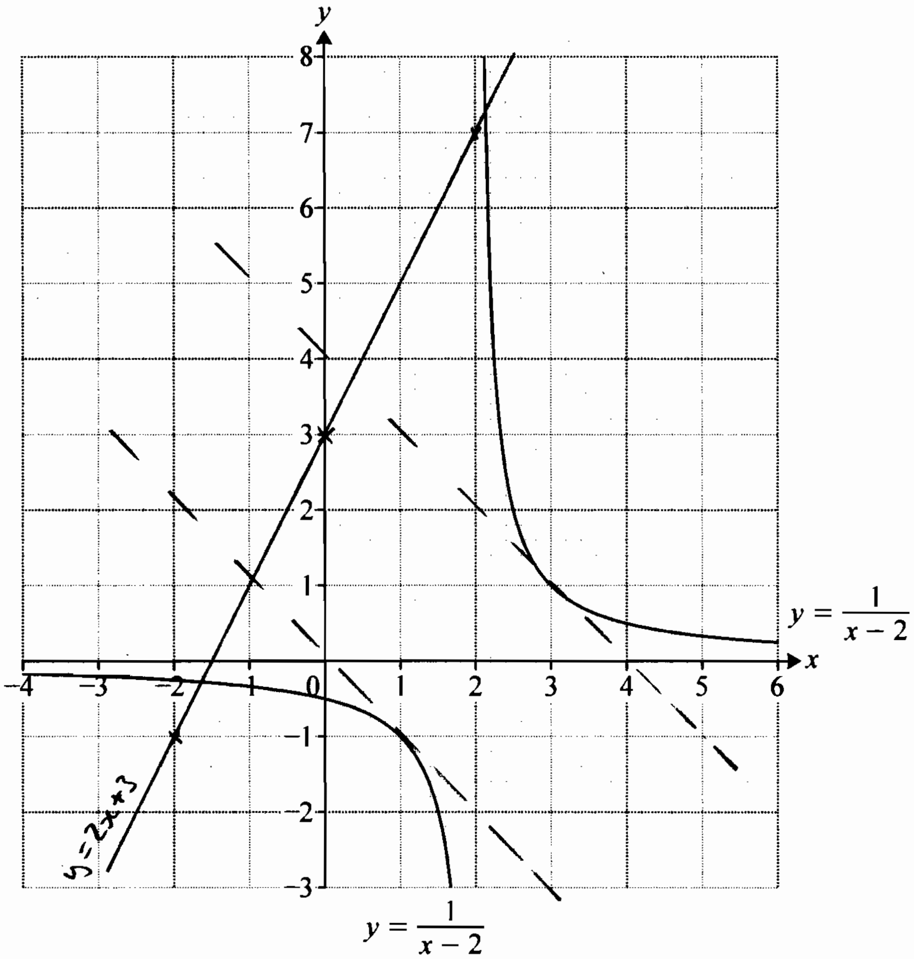

Fig. 12

i) Intersection at $(2.1, 7.3)$ and $(-1.7, -0.3)$

ii) $y = 2x+3$ ①

$y = \frac{1}{x-2}$ ②

Sub for y in ② $2x+3 = \frac{1}{x-2}$

$(2x+3)(x-2) = 1$

$2x^2 + 3x - 4x - 6 = 1$

$2x^2 - x - 7 = 0$

$x = \frac{1 \pm \sqrt{1^2 + 4\times2\times8}}{4}$

$x = \frac{1 \pm \sqrt{29}}{4}$

iii) $y = \frac{1}{x-2}$ $y = -x+k$

$\Rightarrow 1 = (x-2)(-x+k)$

$1 = -x^2 + 2x + kx - 2k$

$x^2 - (2+k)x + 2k+1 = 0$

For double root $b^2 = 4ac$

$(2+k)^2 = 4\times1\times(2k+1)$

$k^2 + 4k + 4 = 8k+4$

$k^2 - 4k = 0$

$(k-4)k = 0$

$\Rightarrow k = 4$ or $k = 0$

(Have shown on graph but not required)