MEI CORE 1 JAN 2005 SECTION A

1) $2(x-3) < 6x + 15$

$2x - 6 < 6x + 15$

$2x - 6x < +15 + 6$

$-4x < 21$

$x > \frac{21}{-4}$

$x > -5\frac{1}{4}$

2) $V = \frac{4}{3}\pi r^3$

$3V = 4\pi r^3$

$\frac{3V}{4\pi} = r^3$

$r = \sqrt[3]{\frac{3V}{4\pi}}$

3) i) P: n an even number
Q: n is a multiple of 4

All multiples of 4 are even
$\therefore Q \Rightarrow P$

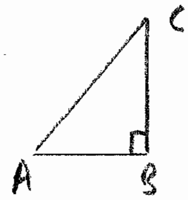
However, 2 is even but not a multiple of 4

$\therefore P \not\Rightarrow Q$

Answer $P \Leftarrow Q$

ii) For $\triangle ABC$

P: $B = 90^\circ$
Q: $AB^2 + BC^2 = AC^2$



Pythagoras Theorem

This is a statement of Pythagoras Theorem for the $\triangle ABC$

$\therefore P \Leftrightarrow Q$

4) $(2+3x)^5$

1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$= 2^5 + 5(2)^4(3x) + 10(2)^3(3x)^2$
$+ 10(2)^2(3x)^3 + 5(2)(3x)^4 + (3x)^5$

Term in x^3 is $10 \times 4 \times 27 x^3$

$= 1080x^3$

$\therefore$ coefficient of $x^3 = 1080$

5) i) $\left(\frac{1}{3}\right)^{-2} = \frac{1}{\left(\frac{1}{3}\right)^2}$

$= \frac{1}{\frac{1}{9}} = 9$

ii) $16^{\frac{3}{4}} = \left(\sqrt[4]{16}\right)^3$

$= 2^3$

$= 8$

6) L parallel to $y=-2x+1$ through $(5,2)$

Gradient will equal -2

Using $y-y_1 = m(x-x_1)$

$$y-2 = -2(x-5)$$

$$y-2 = -2x+10$$

L: $y = -2x+12$

When $x=0$, $y=12$

When $y=0$, $-2x+12=0$

$$12 = 2x$$

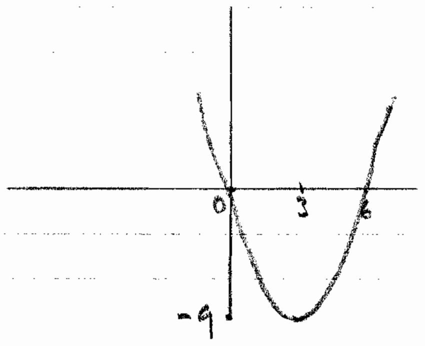
$$6 = x$$

L intersects with axes at

$(0,12)$ and $(6,0)$

7)

$$x^2-6x = (x-3)^2-9$$



Min pt $(3,-9)$

Crosses axes at $(0,0)$ and $(6,0)$

8) Line through $A(3,7)$ and $B(5,-1)$

Using $\dfrac{y-y_1}{y_1-y_2} = \dfrac{x-x_1}{x_1-x_2}$

$$\frac{y-7}{7--1} = \frac{x-3}{3-5}$$

$$\frac{y-7}{8} = \frac{x-3}{-2}$$

$$-2(y-7) = 8(x-3)$$

$$-2y+14 = 8x-24$$

$$-2y = 8x-38$$

$$y = -4x+19$$

Midpoint of AB $= \left(\dfrac{3+5}{2}, \dfrac{7+-1}{2}\right)$

$$= (4,3)$$

Show $(4,3)$ lies on $x+2y=10$

Substituting $4+2\times3 = 10$ ✓

$\therefore$ point $(4,3)$ is on the line

9)

$$(3+\sqrt{2})(3-\sqrt{2})$$

$$= 9 + \cancel{3\sqrt{2}} - \cancel{3\sqrt{2}} - 2 = 7$$

$$\frac{1+\sqrt{2}}{3-\sqrt{2}} = \frac{(1+\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$$

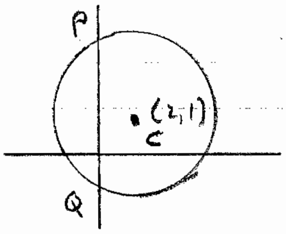
$$= \frac{3+3\sqrt{2}+\sqrt{2}+2}{7}$$

$$= \frac{5+4\sqrt{2}}{7}$$

$$= \frac{5}{7} + \frac{4}{7}\sqrt{2}$$

10)

i)



Eqn of circle centre (2,1) with radius 5 is given by

$$(x-2)^2+(y-1)^2=5^2$$

$$x^2-4x+4+y^2-2y+1=25$$

$$x^2+y^2-4x-2y+4+1-25=0$$

$$x^2+y^2-4x-2y-20=0$$

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ii) Cuts y axis when $x=0$

$$0+y^2-0-2y-20=0$$

$$y^2-2y-20=0$$

Using $y=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$y=\frac{2\pm\sqrt{4+80}}{2}$$

$$y=\frac{2\pm\sqrt{84}}{2}$$

$$y=\frac{2\pm\sqrt{4\times21}}{2}$$

$$y=\frac{2\pm2\sqrt{21}}{2}$$

$$y=1\pm\sqrt{21}$$

$\therefore$ $P(0, 1+\sqrt{21})$

$Q(0, 1-\sqrt{21})$

iii) Verify $A(5,-3)$ on circle

$$x^2+y^2-4x-2y-20=0$$

$$5^2+(-3)^2-4\times5-2(-3)-20$$

$$=25+9-20+6-20=0 \checkmark$$

$\therefore$ $A(5,-3)$ on circle

CA is a radius

$C(2,1)$ $A(5,-3)$

Gradient of CA $=\dfrac{1--3}{2-5}=\dfrac{4}{-3}$

$$=-\frac{4}{3}$$

$\therefore$ gradient of tgt (since $\perp$ to radius) $=+\dfrac{3}{4}$

Using $y-y_1=m(x-x_1)$

$$y--3=\frac{3}{4}(x-5)$$

$$y+3=\frac{3}{4}(x-5)$$

$$4y+12=3(x-5)$$

$$4y+12=3x-15$$

$$4y=3x-27$$

MEI CORE 1 JAN 2005 SECTION B

11) $f(x) = x^3 + x^2 - 10x + 8$

i) $f(1) = 1^3 + 1^2 - 10\times 1 + 8$
$= 1 + 1 - 10 + 8$
$= 0$

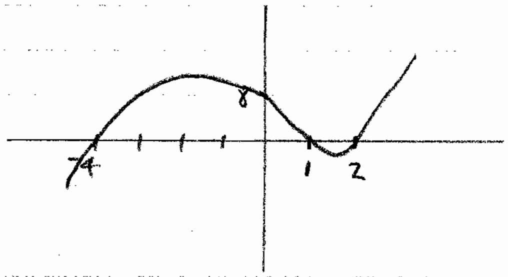
$\therefore$ by factor theorem

$(x-1)$ is a factor of $f(x)$

$$\begin{array}{r}
x^2 + 2x - 8 \\
x - 1 \,\big)\, \overline{x^3 + x^2 - 10x + 8} \\
\underline{x^3 - x^2} \qquad\qquad \\
2x^2 - 10x \qquad \\
\underline{2x^2 - 2x} \qquad \\
-8x + 8 \\
\underline{-8x + 8}
\end{array}$$

$f(x) = (x-1)(x^2 + 2x - 8)$

$= (x-1)(x+4)(x-2)$



ii) $y = f(x)$ translated by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

$y = f(x+3)$

$y = (x+3)^3 + (x+3)^2 - 10(x+3) + 8$

When $x = 0$

$y = (0+3)^3 + (0+3)^2 - 10(0+3) + 8$

$y = 27 + 9 - 30 + 8 = 14$

y intercept $= 14$

12)
i) $y = x^2 - 3x + 11$

$y = \left(x - \frac{3}{2}\right)^2 + 11 - \frac{9}{4}$

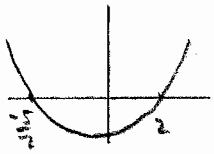
$= \left(x - \frac{3}{2}\right)^2 + \frac{35}{4}$

$\geqslant \frac{35}{4} \quad \forall x \in \mathbb{R}$
(for all values of x)

ii) $y = 2x^2 + x - 10$
$y = (2x+5)(x-2)$

$y = 0$ when $2x + 5 = 0$
$\Rightarrow x = -\frac{5}{2}$

and when $x - 2 = 0$
$\Rightarrow x = 2$



Above x axis when $y > 2$
or when $y < -\frac{5}{2}$

iii) $y = x^2 - 3x + 11$ ①
$y = 2x^2 + x - 10$ ②

Subst for y in ①

$2x^2 + x - 10 = x^2 - 3x + 11$

$2x^2 - x^2 + x + 3x - 10 - 11 = 0$

$x^2 + 4x - 21 = 0$

$(x+7)(x-3) = 0$

$\Rightarrow x = -7$ or $x = +3$

12 iii) cont Subst for $x = -7$ in (1)

$y = (-7)^2 - 3(-7) + 11$

$= 49 + 21 + 11 = 81$

$\therefore (-7, 81)$ is a point of intersection

Subst for $x = +3$ in (1)

$y = 3^2 - 3 \times 3 + 11$
$= 9 - 9 + 11 = 11$

$\therefore (3, 11)$ is a point of intersection

Answer $(-7, 81)$ and $(3, 11)$