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1) Through $(2,6)$, gradient -4

$$y - y_1 = m(x - x_1)$$

$$y - 6 = -4(x - 2)$$

$$y - 6 = -4x + 8$$

$$y = -4x + 14$$

When $x = 0$, $y = 14$

when $y = 0$, $0 = -4x + 14$

$$4x = 14$$

$$x = \frac{14}{4} = \frac{7}{2}$$

Line intersects axes at

$(0, 14)$ and $\left(\frac{7}{2}, 0\right)$

2)

$$s = ut + \tfrac{1}{2}at^2$$

$$s - ut = \tfrac{1}{2}at^2$$

$$\frac{2(s-ut)}{t^2} = a$$

$$a = \frac{2(s-ut)}{t^2}$$

3) Let $f(x) = x^3 - kx + 4$

$$f(3) = 3^3 - 3k + 4 = 31 - 3k$$

By remainder theorem

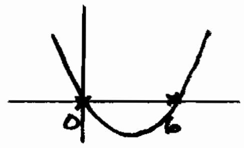
$f(3) = $ remainder

$$31 - 3k = 1$$

$$31 - 1 = 3k$$

$$k = 10$$

4) $x(x-6) > 0$



Either $x < 0$ or $x > 6$

5) i) $${}^5C_3 = \frac{5!}{2!\,3!} = \frac{5.4}{2.1}$$

$${}^5C_3 = 10$$

ii) Find coeff of x^3 in

$$(1+2x)^5$$

Term in x^3 is ${}^5C_3\,(1)^2(2x)^3$

$$= 10 \times 1 \times 8x^3$$

$$= 80x^3$$

Coeff $= 80$

6) Given n is an integer

$$n^3 - n = n(n^2 - 1)$$

$$= n(n+1)(n-1)$$

$$= (n-1)n(n+1)$$

These are 3 consecutive integers, at least one of which must be even.
2 is a factor of such an even number so 2 is a factor of the above product.

The product is therefore even

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7) i) $5^2 \times 5^{-2} = 5^0 = 1$

ii) $100^{3/2} = \left(\sqrt[2]{100}\right)^3 = 10^3 = 1000$

8) i) $\dfrac{\sqrt{48}}{2\sqrt{27}} = \dfrac{\sqrt{16\times 3}}{2\sqrt{9\times 3}}$

$= \dfrac{4\sqrt{3}}{2\times 3\sqrt{3}}$

$= \dfrac{2}{3}$

ii) $(5-3\sqrt{2})^2$

$= (5-3\sqrt{2})(5-3\sqrt{2})$

$= 25 - 15\sqrt{2} - 15\sqrt{2} + 18$

$= 43 - 30\sqrt{2}$

9) $x^2 + 6x + 5$

i) $= (x+3)^2 + 5 - 9$

$= (x+3)^2 - 4$

ii) Min point $(-3, -4)$

10) $x^4 - 5x^2 - 36 = 0$

$(x^2+4)(x^2-9) = 0$

$\Rightarrow x^2 = -4$ no real roots

or $x^2 = 9 \Rightarrow x = \pm 3$

Answer $x = 3$
$x = -3$

Section B

11) i) $A(0,3)$ $B(6,1)$

Gradient of AB $= \dfrac{3-1}{0-6} = \dfrac{2}{-6} = -\dfrac{1}{3}$

$\therefore$ gradient of $\perp$ line $= +3$

Line through origin with gradient $+3$ is given by $y = 3x$

ii) Find eqn of AB

$\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$

$\dfrac{y-3}{1-3} = \dfrac{x-0}{6-0}$

$\dfrac{y-3}{-2} = \dfrac{x}{6}$

$y - 3 = \dfrac{-2x}{6} = -\dfrac{x}{3}$

AB $y = -\dfrac{x}{3} + 3$

Solve $\begin{cases} y = -\dfrac{x}{3} + 3 & (1) \\ y = 3x & (2) \end{cases}$

Subst for y in (1)

$3x = -\dfrac{x}{3} + 3$

$9x = -x + 9$

$10x = 9 \Rightarrow x = \dfrac{9}{10}$

11 ii) cont)

Subst for x in (2)

$$y = 3\left(\frac{9}{10}\right) = \frac{27}{10}$$

Point of intersection with AB

$$(0.9, 2.7)$$

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11 iii) $\perp$ distance is distance between $(0,0)$ and $\left(\frac{9}{10}, \frac{27}{10}\right)$

$$= \sqrt{\left(\frac{9}{10}-0\right)^2 + \left(\frac{27}{10}-0\right)^2}$$

$$= \sqrt{\frac{81}{100} + \frac{729}{100}}$$

$$= \sqrt{\frac{810}{100}}$$

$$= \sqrt{\frac{81 \times 10}{100}}$$

$$= \frac{9\sqrt{10}}{10}$$

$$\begin{array}{r} 27 \\ 27 \\ \hline 189 \\ 540 \\ \hline 729 \end{array}$$

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11 iv)

$$|AB| = \sqrt{(6-0)^2 + (1-3)^2}$$

$$= \sqrt{36+4} = \sqrt{40}$$

$$= \sqrt{4 \times 10}$$

$$= 2\sqrt{10}$$

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11 v) Area $= \frac{1}{2}$ base $\times$ perp height

$= \frac{1}{2}|AB| \times |\text{answer to iii}|$

$$= \frac{1}{2}\left(2\sqrt{10} \times \frac{9\sqrt{10}}{10}\right)$$

$$= \frac{1}{2}\left(\frac{18 \times 10}{10}\right) = 9 \text{ units}^2$$

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12) $f(x) = (x+1)(x-2)(x-4)$

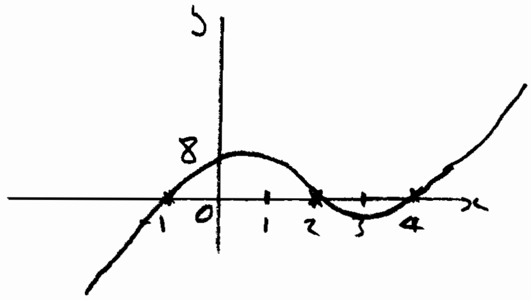
i)

A) $f(x) = (x^2 - x - 2)(x-4)$

$$f(x) = x^3 - x^2 - 2x - 4x^2 + 4x + 8$$

$$f(x) = x^3 - 5x^2 + 2x + 8$$

B)



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c) Let new function be $g(x)$

$$g(x) = (x-3)^3 - 5(x-3)^2 + 2(x-3) + 8$$

or $g(x) = (x-3+1)(x-3-2)(x-3-4)$

$$g(x) = (x-2)(x-5)(x-7)$$

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ii)

$$x^3 - 5x^2 + 2x + 8 = -4 \quad (*)$$

$$3^3 - 5(3)^2 + 2(3) + 8$$

$$= 27 - 45 + 6 + 8 = -4$$

$\therefore x = 3$ is a root of $(*)$

Rearrange eqn

$$x^3 - 5x^2 + 2x + 12 = 0$$

12ii) cont)

$$\begin{array}{r}
x^2 - 2x - 4 \\
x-3 \overline{\big)\, x^3 - 5x^2 + 2x + 12} \\
x^3 - 3x^2 \\
\hline
-2x^2 + 2x \\
-2x^2 + 6x \\
\hline
-4x + 12 \\
-4x + 12 \\
\hline
\end{array}$$

Eqn becomes

$$(x-3)(x^2-2x-4)=0$$

$x=3$ or $x = \dfrac{2 \pm \sqrt{4+16}}{2}$

$$x = \frac{2 \pm 2\sqrt{5}}{2}$$

$x=3$ or $x = 1 \pm \sqrt{5}$

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13) i) $(x-5)^2 + (y-2)^2 = 20$

Centre $(5, 2)$

Radius $= \sqrt{20}$

ii) No because centre is 5 units away from y axis and radius is $\sqrt{20}$ which is less than 5.

iii) Line parallel to $y=2x$ has gradient 2

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 2(x-5)$$

$$y - 2 = 2x - 10$$

$$y = 2x - 8$$

iv) Solve

$$\begin{cases} (x-5)^2 + (y-2)^2 = 20 & (1) \\ y = 2x + 2 & (2) \end{cases}$$

Subst for y in (1)

$$(x-5)^2 + (2x+2-2)^2 = 20$$

$$x^2 - 10x + 25 + 4x^2 = 20$$

$$5x^2 - 10x + 5 = 0$$

$$5(x^2 - 2x + 1) = 0$$

$$5(x-1)^2 = 0$$

$\Rightarrow$ only one root $x=1$

When $x=1$, $y = 2(1)+2 = 4$

Only one point of intersection at $(1, 4)$. $\therefore$ line is a tangent to circle.