MEI CORE 1 MAY 2005

1) Let $f(x) = x^3 + 2x^2 - 5$

$f(3) = 3^3 + 2 \times 3^2 - 5$
$= 27 + 18 - 5$
$= 40$

By remainder theorem remainder $= 40$

2) $3x - 5y = y - mx$

$3x + mx = y + 5y$

$x(3+m) = 6y$

$x = \dfrac{6y}{3+m}$

3)
$n, \; n+1, \; n+2$
Let above be any 3 consecutive integers
Sum is $3n+3$
$= 3(n+1)$

$\therefore$ divisible by 3 since 3 is a factor

4)
$3x + 5y = 12$

$x = 0 \Rightarrow y = \frac{12}{5}$

$y = 0 \Rightarrow x = 4$

Crosses axes at
$(0, \frac{12}{5})$ and $(4, 0)$

$5y = 12 - 3x$

$y = -\frac{3}{5}x + \frac{12}{5}$

Gradient $= -\dfrac{3}{5}$

5) $(2-x)^3$

$= 2^3 + 3(2)^2(-x) + 3(2)(-x)^2 + (-x)^3$

$= 8 - 12x + 6x^2 - x^3$

6)
i) $a^0 = 1$

ii) $a^6 \div a^{-2} = a^8$

iii) $(9a^6b^2)^{-\frac{1}{2}}$

$= \dfrac{1}{\sqrt{9a^6b^2}} = \dfrac{1}{3a^3b}$

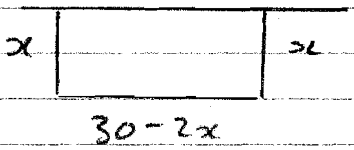
7) i)
$\sqrt{24} + \sqrt{6}$
$= \sqrt{4 \times 6} + \sqrt{6}$
$= 2\sqrt{6} + \sqrt{6} \qquad = 3\sqrt{6}$

ii) $\dfrac{36}{5-\sqrt{7}} = \dfrac{36}{(5-\sqrt{7})} \times \dfrac{(5+\sqrt{7})}{(5+\sqrt{7})}$

$= \dfrac{36(5+\sqrt{7})}{25-7} = \dfrac{36(5+\sqrt{7})}{18}$

$= 2(5+\sqrt{7})$

$= 10 + 2\sqrt{7}$

8)


Length $= 30 - 2x$
Area $= (30-2x)x = 112$

$30x - 2x^2 = 112$

$15x - x^2 = 56$
$x^2 - 15x + 56 = 0$

8 (cont)

$$x^2 - 15x + 56 = 0$$
$$(x-8)(x-7) = 0$$
$$\Rightarrow x = 8 \text{ or } x = 7$$

Possible dimensions

$14\text{m} \times 8\text{m}$

or $16\text{m} \times 7\text{m}$

9)

$$y = 3x + 2$$
$$y = 3x^2 - 7x + 1$$

Subst for y

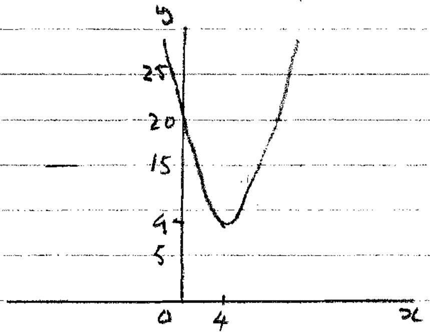
$$3x + 2 = 3x^2 - 7x + 1$$
$$3x^2 - 10x - 1 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{10 \pm \sqrt{100 + 12}}{6}$$
$$x = \frac{10 \pm \sqrt{16 \times 7}}{6}$$
$$x = \frac{10 \pm 4\sqrt{7}}{6}$$
$$x = \frac{5 + 2\sqrt{7}}{3} \text{ or } x = \frac{5 - 2\sqrt{7}}{3}$$

SECTION B

10) i)

$$x^2 - 8x + 25$$
$$= (x-4)^2 + 25 - 16$$
$$= (x-4)^2 + 9$$

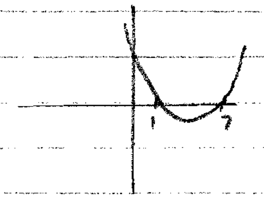
ii) Min point is $(4, 9)$



iii)

$$x^2 - 8x + 25 > 18$$
$$x^2 - 8x + 7 > 0$$
$$(x-1)(x-7) > 0$$

Graph of $y = x^2 - 8x + 7$



Either $x < 1$ or $x > 7$

iv)

$$y = x^2 - 8x + 25$$

translated by $\begin{pmatrix} 0 \\ -20 \end{pmatrix}$

giving graph

$$y = x^2 - 8x + 5$$

11) i) $A(0,2)$ $B(7,9)$ $C(6,10)$

$$|AC| = \sqrt{(6-0)^2 + (10-2)^2}$$
$$= \sqrt{36 + 64}$$
$$= 10$$

Gradient of AB $= \frac{9-2}{7-0} = \frac{7}{7} = 1$

Gradient of BC $= \frac{10-9}{6-7} = \frac{1}{-1} = -1$

$\therefore$ AB and BC are $\perp$

$\therefore \angle ABC = 90^\circ$

11 ii)

Since $\angle ABC = 90°$

AC is a diameter

Centre is $\left(\frac{0+6}{2}, \frac{2+10}{2}\right)$

$= (3, 6)$

$|AC| = 10$ from part (i)

$\therefore$ radius $= \frac{1}{2} \times$ diameter $= 5$

Circle is

$$(x-3)^2 + (y-6)^2 = 25$$

11 iii) Tgt $\perp$ to AC

Gradient of AC $= \frac{10-2}{6-0} = \frac{8}{6}$

$\therefore$ gradient of tgt $= -\frac{6}{8} = -\frac{3}{4}$

Eqn of line thro $(6, 10)$ with gradient $-3/4$

Using $y - y_1 = m(x - x_1)$

$$y - 10 = -\frac{3}{4}(x - 6)$$

$$y - 10 = -\frac{3}{4}x + \frac{9}{2}$$

$$y = -\frac{3}{4}x + \frac{29}{2}$$

When $x = 0$, $y = \frac{29}{2}$

When $y = 0$ $\quad \frac{3}{4}x = \frac{29}{2}$

$6x = 116$

$x = 19\frac{1}{3}$

Crosses axes at $(0, 14\frac{1}{2})$

and $(19\frac{1}{3}, 0)$

12) Roots $f(x) = 0$

are $-1, 2, 5$

$$f(x) = (x+1)(x-2)(x-5)$$

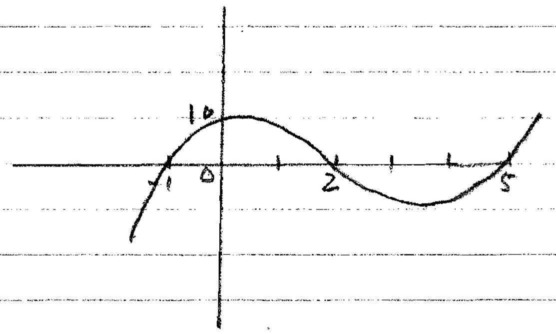
$$= (x^2 + x - 2x - 2)(x-5)$$

$$= (x^2 - x - 2)(x - 5)$$

$$= x^3 - x^2 - 2x - 5x^2 + 5x + 10$$

$$= x^3 - 6x^2 + 3x + 10$$

ii)



iii)

$f(x) + 10$

When $x = 4$

$f(4) + 10 = 4^3 - 6 \times 4^2 + 3 \times 4 + 10 + 10$

$= 64 - 96 + 12 + 20 = 0$

$\therefore$ $x = 4$ is a root of $f(x) + 10 = 0$

$$\begin{array}{r|l} & x^2 - 2x - 5 \\ \hline x - 4 & x^3 - 6x^2 + 3x + 20 \\ & x^3 - 4x^2 \\ \hline & -2x^2 + 3x \\ & -2x^2 + 8x \\ \hline & -5x + 20 \\ & -5x + 20 \\ \hline \end{array}$$

$$f(x) + 10 = (x-4)(x^2 - 2x - 5)$$

$x^2 - 2x - 5 = 0$ will be satisfied by other two roots