MEI CORE1 MAY 2008 ①

1) $3x - 1 > 5 - x$

$3x + x > 5 + 1$

$4x > 6$

$x > \frac{6}{4}$

$x > \frac{3}{2}$

2) $2x + 3y = 12$

i) $x = 0 \Rightarrow 3y = 12$

$y = 4$

$y = 0 \Rightarrow 2x = 12$

$x = 6$

Points of intersection with axes $(0,4)$, $(6,0)$

ii) $3y = -2x + 12$

$y = -\frac{2}{3}x + 4$

Gradient $= -\frac{2}{3}$

3) i) $2x^2 + 3x = 0$

$x(2x+3) = 0$

Either $x = 0$

or $2x + 3 = 0$

$2x = -3$

$x = -\frac{3}{2}$

Answer $x = 0$, $x = -\frac{3}{2}$

3ii) No real roots if $b^2 - 4ac < 0$

$2x^2 + 3x - k = 0$

$b^2 - 4ac = 9 + 8K$

No roots when $9 + 8k < 0$

$8k < -9$

$k < -\frac{9}{8}$

4)

i) $2n+1$ is odd integer TRUE

ii) $3n+1$ is even integer EITHER

iii) n is odd $\Rightarrow n^2$ is odd TRUE

iv) n^2 is odd $\Rightarrow n^3$ is even FALSE

5) $y = \frac{x+3}{x-2}$

$y(x-2) = x+3$

$yx - 2y = x + 3$

$yx - x = 3 + 2y$

$x(y-1) = 3 + 2y$

$x = \frac{3+2y}{y-1}$

6) i) $\left(\frac{1}{25}\right)^{-\frac{1}{2}} = \left(\frac{25}{1}\right)^{\frac{1}{2}} = \sqrt{25} = 5$

ii) $\frac{(2x^2y^3z)^5}{4y^2z} = \frac{32x^{10}y^{15}z^5}{4y^2z} = 8x^{10}y^{13}z^4$

7) i)
$$\frac{1}{5+\sqrt{3}} = \frac{1}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}}$$
$$= \frac{5-\sqrt{3}}{25-3} = \frac{5-\sqrt{3}}{22}$$

ii)
$$(3-2\sqrt{7})^2$$
$$=(3-2\sqrt{7})(3-2\sqrt{7})$$
$$= 9 - 6\sqrt{7} - 6\sqrt{7} + 4\times 7$$
$$= 9 - 12\sqrt{7} + 28$$
$$= 37 - 12\sqrt{7}$$

8)
$$(5-2x)^5$$

Term in x^3 given by
$$^5C_3 (5)^2(-2x)^3$$
$$= \frac{5\times 4}{2\times 1} \times 25 \times (-8x^3)$$
$$= -2000x^3$$

Coeff $= -2000$

9)
$$y^2 - 7y + 12 = 0$$
$$(y-4)(y-3) = 0$$
$$\Rightarrow \quad y=4 \text{ or } y=3$$

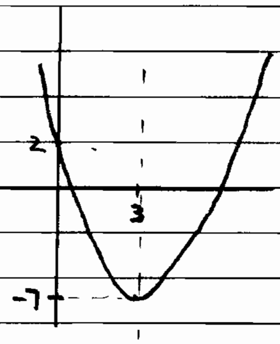
$$x^4 - 7x^2 + 12 = 0$$
$$(x^2-4)(x^2-3) = 0$$
$$\Rightarrow x^2 = 4 \text{ or } x^2 = 3$$
$$\Rightarrow x = \pm 2, \quad x = \pm\sqrt{3}$$

Section B

10) i)
$$x^2 - 6x + 2 = (x-3)^2 + 2 - 9$$
$$= (x-3)^2 - 7$$

ii) Turning point at $(3, -7)$

iii)



iv)
$$y = x^2 - 6x + 2 \quad ①$$
$$y = 2x - 14 \quad ②$$

Subst for y in ①

$$2x - 14 = x^2 - 6x + 2$$
$$0 = x^2 - 6x + 2 - 2x + 14$$
$$0 = x^2 - 8x + 16$$
$$0 = (x-4)(x-4)$$
$$\Rightarrow x = 4$$
$$y = 2\times 4 - 14 = -6$$

Only 1 point of intersection at $(4, -6)$

Repeated root implies line is tangent to curve.

11) $f(x) = 2x^3 + 7x^2 - 7x - 12$

i) $f(-4) = 2(-4)^3 + 7(-4)^2 - 7(-4) - 12$

$= -128 + 112 + 28 - 12 = 0$

$\therefore x = -4$ is a root

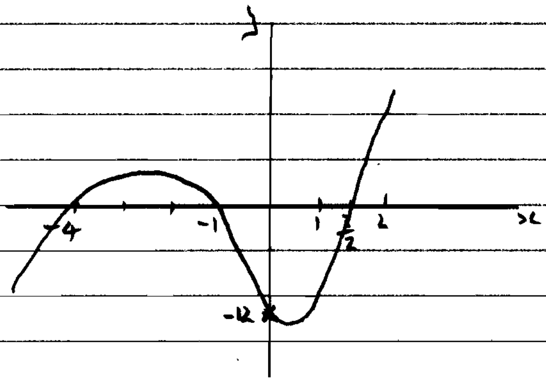
ii)

$$\begin{array}{r|l} & 2x^2 - x - 3 \\ \hline x+4 & 2x^3 + 7x^2 - 7x - 12 \\ & \underline{2x^3 + 8x^2} \\ & \quad -x^2 - 7x \\ & \quad \underline{-x^2 - 4x} \\ & \qquad -3x - 12 \\ & \qquad \underline{-3x - 12} \end{array}$$

$f(x) = (x+4)(2x^2 - x - 3)$

$f(x) = (x+4)(2x-3)(x+1)$

iii)



iv)

$f(x-4) = (x-4+4)(2(x-4)-3)(x-4+1)$

$= x(2x-8-3)(x-3)$

$= x(2x-11)(x-3)$

$= x(2x^2 - 11x - 6x + 33)$

$= x(2x^2 - 17x + 33)$

$= 2x^3 - 17x^2 + 33x$

12) $A(-1, 1)$

i) $B(3, 9)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-1}{9-1} = \frac{x--1}{3--1}$$

$$\frac{y-1}{8} = \frac{x+1}{4}$$

$y - 1 = 2(x+1)$

$y - 1 = 2x + 2$

$y = 2x + 3$

ii)

Gradient of $\perp$ bisector will be $-\frac{1}{2}$

Midpoint of AB $= \left(\frac{-1+3}{2}, \frac{1+9}{2}\right)$

$= (1, 5)$

$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{2}(x - 1)$

$y - 5 = -\frac{1}{2}x + \frac{1}{2}$

$y = -\frac{1}{2}x + \frac{11}{2}$

$2y = -x + 11$

$2y + x = 11$ as required

12iii) $(x-5)^2 + (y-3)^2 = k$

Passes through $A(-1,1)$

$\therefore \quad (-1-5)^2 + (1-3)^2 = k$

$36 + 4 = k$

$k = 40$

$B(3,9)$

$(3-5)^2 + (9-3)^2$

$= \quad 4 + 36 = 40$

$\therefore$ B is on circle

$(x-5)^2 + (y-3)^2 = 40$

12iv)

Crosses x axis when $y = 0$

$(x-5)^2 + (0-3)^2 = 40$

$(x-5)^2 + 9 = 40$

$(x-5)^2 = 31$

$x - 5 = \pm\sqrt{31}$

$x = 5 \pm \sqrt{31}$