

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education  
Advanced General Certificate of Education**

**MEI STRUCTURED MATHEMATICS**

**4751**

**Introduction to Advanced Mathematics (C1)**

**Monday**

**23 MAY 2005**

**Morning**

**1 hour 30 minutes**

Additional materials:

Answer booklet

Graph paper

MEI Examination Formulae and Tables (MF2)

**TIME** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are **not** permitted to use a calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.



**WARNING**

**You are not allowed to use  
a calculator in this paper**

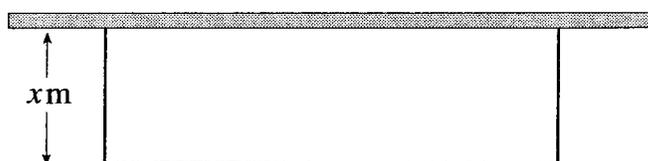
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**This question paper consists of 4 printed pages.**

## Section A (36 marks)

- 1 Find the remainder when  $x^3 + 2x^2 - 5$  is divided by  $x - 3$ . [2]
- 2 Make  $x$  the subject of
- $$3x - 5y = y - mx. \quad [3]$$
- 3 The smallest of three consecutive integers is  $n$ .  
Write down the other two integers.  
Prove that the sum of any three consecutive integers is divisible by 3. [3]
- 4 A line has equation  $3x + 5y = 12$ . Find its gradient and the coordinates of the points where it crosses the axes. [4]
- 5 Find the binomial expansion of  $(2 - x)^3$ . [4]
- 6 Simplify the following.
- (i)  $a^0$  [1]
- (ii)  $a^6 \div a^{-2}$  [1]
- (iii)  $(9a^6b^2)^{-\frac{1}{2}}$  [3]
- 7 (i) Simplify  $\sqrt{24} + \sqrt{6}$ . [2]
- (ii) Express  $\frac{36}{5 - \sqrt{7}}$  in the form  $a + b\sqrt{7}$ , where  $a$  and  $b$  are integers. [3]

- 8 Fig. 8 is a plan view of a rectangular enclosure. A wall forms one side of the enclosure. The other three sides are formed by fencing of total length 30 m. The width of the rectangle is  $x$  m and the area enclosed is  $112 \text{ m}^2$ .



Not to  
scale

Fig. 8

Show that  $x^2 - 15x + 56 = 0$ .

By factorising, solve this equation and find the possible dimensions of the rectangle. [5]

- 9 Find the  $x$ -coordinates of the points of intersection of the line  $y = 3x + 2$  and the curve  $y = 3x^2 - 7x + 1$ . Leave your answers in surd form. [5]

**Section B (36 marks)**

- 10 (i) Write  $x^2 - 8x + 25$  in the form  $(x - a)^2 + b$ . [3]
- (ii) State the coordinates of the minimum point on the graph of  $y = x^2 - 8x + 25$  and sketch this graph. [4]
- (iii) Solve the inequality  $x^2 - 8x + 25 > 18$ . [3]
- (iv) The graph of  $y = x^2 - 8x + 25$  is translated by  $\begin{pmatrix} 0 \\ -20 \end{pmatrix}$ . State an equation for the resulting graph. [1]

- 11 The points  $A(0, 2)$ ,  $B(7, 9)$  and  $C(6, 10)$  lie on the circumference of a circle, as shown in Fig.11.

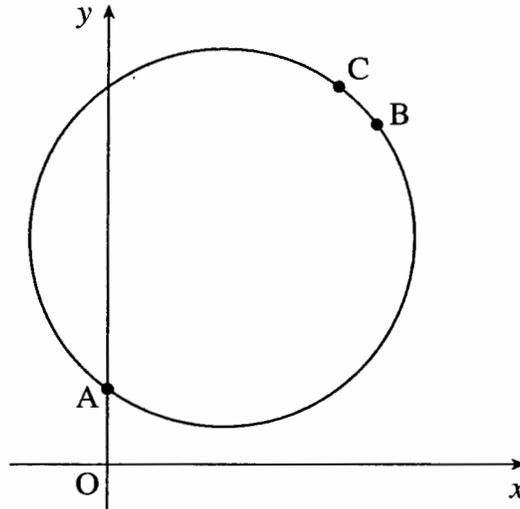


Fig. 11

- (i) Find the length of AC.

Prove that triangle ABC is right-angled at B.

[4]

- (ii) Hence show that the centre of the circle is  $(3, 6)$  and its radius is 5.

Find the equation of the circle.

[4]

- (iii) Find an equation for the tangent to the circle at C.

Find the coordinates of the points where this tangent crosses the axes.

[5]

- 12 In the cubic polynomial  $f(x)$ , the coefficient of  $x^3$  is 1. The roots of  $f(x) = 0$  are  $-1, 2$  and  $5$ .

- (i) Write  $f(x)$  in factorised form.

Show that  $f(x)$  may be written as

$$f(x) = x^3 - 6x^2 + 3x + 10. \quad [3]$$

- (ii) Sketch the graph of  $y = f(x)$ .

[3]

- (iii) Show that  $x = 4$  is one root of the equation  $f(x) + 10 = 0$ .

Hence find a quadratic equation which is satisfied by the other two roots of the equation  $f(x) + 10 = 0$ . [6]

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Mark Scheme

June 2005

**Mark Scheme 4751**  
**June 2005**

## Section A

1	40	2	M1 subst of 3 for $x$ or attempt at long divn with $x^3 - 3x^2$ seen in working; 0 for attempt at factors by inspection	2
2	$[x = ] \frac{6y}{3+m}$ as final answer	3	M1 for $3x + mx = y + 5y$ o.e. and M1 for $x(3 + m)$ or ft sign error	3
3	$n + 1$ and $n + 2$ both seen $3n + 3$  $= 3(n + 1)$ o.e.	1 M1  A1	condone e.g. $a$ instead of $n$ for last 2 marks or starting again with full method for middle number = $y$ etc or 3 a factor of both terms so divisible by 3	3
4	$-0.6$ o.e.  (4, 0) (0, 12/5) o.e.	2  1 1	M1 for 0.6 or $-0.6x$ o.e. or rearrangement to 'y =' form [need not be correct] condone values of $x$ and $y$ given	4
5	$8 - 12x + 6x^2 - x^3$ isw	4	B3 for 3 terms correct or all correct except for signs; B2 for two terms correct including at least one of $-12x$ and $6x^2$ ; B1 for 1 3 3 1 soi or for 8 and $-x^3$	4
6	(i) 1  (ii) $a^8$ cao  (iii) $\frac{1}{3a^3b}$ or $\frac{1}{3}a^{-3}b^{-1}$ isw	1  1  3	M2 for two 'terms' correct or M1 for $3a^3b$ or $\frac{1}{(9a^6b^2)^{\frac{1}{2}}}$ or $\frac{1}{\sqrt{9a^6b^2}}$ ; ignore $\pm$	5
7	(i) $3\sqrt{6}$ or $\sqrt{54}$ isw  (ii) $10 + 2\sqrt{7}$	2  3	M1 for $\sqrt{(4 \times 6)}$ or $2\sqrt{6}$ or $3\sqrt{2}\sqrt{3}$ seen  M1 for attempt to multiply num. and denom. by $5 + \sqrt{7}$ and M1 for 18 or $25 - 7$ seen	5
8	$x(30 - 2x) = 112$ $x(15 - x) = 56$ or $30x - 2x^2 = 112$  ( $x - 7$ )( $x - 8$ ) $x = 7$ or 8 7 by 16 or 8 by 14	M1 A1  1 1 1	allow M1 for length = $30 - 2x$ soi NB answer given  0 for formula or completing sq etc must be explicit; both values required allow for 16 and 14 found following 7 and 8; both required	5
9	$[y = ] 3x + 2 = 3x^2 - 7x + 1$  $[0 = ] 3x^2 - 10x - 1$ or $-3x^2 + 10x + 1$ $x = \frac{10 \pm \sqrt{100 + 12}}{6}$  $= \frac{10 \pm \sqrt{112}}{6}$ or $\frac{5 \pm \sqrt{28}}{3}$ o.e. isw	M1  M1 M1  A2	or rearrangement of linear and subst for $x$ in quadratic attempted condone one error; dep on first M1 attempt at formula [dep. on first M1 and quadratic = 0]; M2 for whole method for completing square or M1 to stage before taking roots A1 for two of three 'terms' correct [with correct fraction line] or for one root	5

## Section B

10	i	$(x - 4)^2 + 9$	3	B1 for 4, B2 for 9 or M1 for 25 - 16	3
	ii	(4, 9) or ft	1+1		
	iii	parabola right way up 25 at intersection on y-axis (mark intent)	G1 G1	condone stopping at y axis ignore posn of min: can ft theirs	4
		$x > 7$ or $x < 1$	3	M1 for $x^2 - 8x + 7$ [ $> 0$ ] and M1 for $(x - 7)(x - 1)$ [ $> 0$ ] or M1 for $(x - 4)^2$ [ $>$ ] 9 and M1 for $x - 4 > 3$ and $x - 4 < -3$ or B2 for 1 and 7	3
iv	$[y =] x^2 - 8x + 5$	1	or $[y =] (x - 4)^2 - 11$	1	
11	i	$(6 - 0)^2 + (10 - 2)^2$ AC = 10 AB = $\sqrt{98}$ and BC = $\sqrt{2}$ clear correct use of Pythagoras's theorem	M1 A1 1 1	or 1 for grad AB = 1 and grad BC = -1 and 1 for comment/ showing $m_1 m_2 = -1$ o.e.	4
		ii	[angle in a semicircle so ]AC diameter [so radius = 5] midpt of AC = (6/2, [10+2]/2)	1 1	$d$ or diameter needed; NB ans given method must be shown; NB ans given
	iii	$(x - 3)^2 + (y - 6)^2 = 5^2$ o.e. isw	2	B1 for one side correct	4
		[grad AC =] 8/6 or 4/3 grad tgt = -3/4 $y - 10 = [-3/4](x - 6)$ o.e. [e.g. $3x + 4y = 58$ ] or ft (58/3, 0) and (0, 58/4) o.e. isw	1 M1 M1 A2	for grad tgt = -1/their grad AC or M1 for $y =$ their $m x + c$ then subst (6, 10) to find $c$ 1 each cao; condone not as coords	5
12	i	$(x + 1)(x - 2)(x - 5)$ $(x + 1)(x^2 - 7x + 10)$ correct step shown towards completion [answer given]	M1 A1 A1	o.e. with two other factors; condone missing brackets if expanded correctly; A2 for $x^3 - 5x^2 - 2x^2 + x^2 + 10x - 5x - 2x + 10$	3
		ii	cubic the right way up -1, 2 and 5 indicated on x axis 10 indicated at intn on y axis	G1 G1 G1	must extend beyond $x = -1$ and 5 at intersections of curve and axis
	iii	f(4) attempted  $= 64 - 96 + 12 + 10$	M1  A1	or $f(4) + 10$ ; or '4 a root implies $(x - 4)$ a factor' or vv or $5 \times 2 \times -1$ etc or correct long division if first M1 earned	
		attempt at long division of $x^3 - 6x^2 + 3x + 20$ by $x - 4$ as far as $x^3 - 4x^2$ in working  $x^2 - 2x - 5 = 0$	M2  A2	or M2 for $(x - 4)(x^2 + \dots - 5)$ or $(x - 4)(x^2 - 2x + k)$ seen; M1 for realising long divn by $x - 4$ needed but not doing it A1 for $x^2 - 2x - 5$ SC2 for finding $f(x) \div (x - 4) = x^2 - 2x - 5$ rem - 10 without further explanation	6

**4751 - Introduction to Advanced Mathematics****General Comments**

This was the second time this paper has been sat. The candidature was smaller than in January, with fewer year 13 students transferring to the new specification.

There were many excellent scripts, but also a long tail of very weak candidates who appeared to have gained little from the course.

A calculator is not allowed in this paper and, as in January, some candidates found this a considerable disadvantage, making errors in basic arithmetic as well as when they used longer methods such as using the quadratic formula when an equation factorises. In question 8, for instance, some were unable to divide 112 by 7 or 8, in spite of the fact that they had already established  $7 \times 8 = 56$  and that short division and other options were available to them. In general, time was not a problem, but for candidates who used long methods, or who re-worked questions in an attempt to eliminate their errors, there was some evidence of 'rushing' on the last question.

As in January, examiners found that many candidates wasted time by plotting graphs on graph paper when a simple sketch was requested. Teachers are asked to note that in a sketch, relevant points such as the intersections with the axes should be labelled appropriately. It is quicker and better for both candidates and examiners if sketches are done in the examination booklet rather than on graph paper.

Presentation was generally good, but some were careless with algebraic notation and language. For instance, in question 12 many omitted brackets for the quadratic factor when writing expressions such as  $(x + 1)(x^2 - 7x + 10)$ , and many gave an expression not equation as requested for the answer to the last part.

Although the content for this paper does not include calculus, some students were able to use their knowledge of calculus to answer some questions using alternative methods, most frequently in question 10 part (i). However, some used their knowledge of the C2 content inappropriately, for instance in question 4 to find the sum of 3 consecutive integers using the formula for the sum of an arithmetic progression and making the question rather harder for themselves in the process.

**Comments on Individual Questions****Section A**

- 1) Most candidates attempted a long division, with many gaining a method mark for the start of the long division but being unable to cope with the subtraction or with the lack of an  $x$  term. Nearly all those who used the simpler remainder theorem method were successful and gained the simple two marks intended for the start of the paper.
- 2) Many gained a method mark for collecting  $x$  and  $y$  terms on different sides of the equation. However, many errors were made after this, with weaker candidates often trying to divide by  $m$  and making further errors. Many failed to factorise as required, or did not realise that  $y + 5y$  could and should be simplified at this level.
- 3) Some omitted this question, not knowing what consecutive integers were. Most candidates who attempted it managed to write down  $n + 1$  and  $n + 2$ , although some gave  $2n$  and  $3n$ . However, many then used numerical examples as ‘proof’ rather than following the hint of the algebra. Poor algebra abounded from weaker candidates, such as  $\frac{3n+3}{3} = n$ .
- 4) Many candidates did this well, although there were errors in rearranging, especially by weak candidates, with some failing to divide by 5 and giving the gradient as 3 and the  $y$ -intercept as 12. Some sensibly and elegantly used the given form to find the intercepts and some then used to find the gradient. It was fairly common for candidates to miss out one or more of the three things they were asked to find.
- 5) The majority used the binomial theorem and substituted in, although carelessness with brackets and signs led to errors. Some completely ignored the minus sign. Few of those who multiplied out the brackets did so successfully.
- 6) The first two parts were often correct, although some gave the wrong answers of 0 and  $a^4$  or  $a^{-3}$ . The last part was found to be much more challenging, with the 9 causing most problems although weaker candidates also made errors such as  $a^6b^2 = ab^8$ .
- 7) The second part was often attempted with more success than the first, with candidates recognising more easily what was required in rationalising the denominator.  $\sqrt{30}$  rather than the correct answer was probably the most common answer for the first part, whilst  $\sqrt{24} = 4\sqrt{6}$  was another common error. Some who correctly obtained  $2\sqrt{6} + \sqrt{6}$  then went on to equate it to  $2 + 2\sqrt{6}$ . In the second part, many found the answer competently but others realised they should be multiplying numerator and denominator by something but could not remember what that something was, or made errors in their multiplying or in dividing only one term by 18.

- 8) Many candidates omitted the first part completely and went straight into the factorisation. Those who did attempt it sometimes wrote down two equations in two variables and did not know how to proceed. Those who realised that the length was  $30 - 2x$  or  $\frac{112}{x}$  usually then went on to successful completion. A number of candidates found the factorisation difficult, not recognising that  $56 = 7 \times 8$ . A few used the quadratic formula to find the roots and then found the factors. Relating the values of  $x$  back to finding the dimensions of the rectangle was found to be more difficult than expected.
- 9) As in the January paper, many candidates knew what to do here and many did so successfully, although some attempted to factorise or did not remember the quadratic formula accurately. A few attempted completing the square, with most being unsuccessful. Mercifully few attempted to substitute for  $x$  – only one or two candidates did so successfully. Some wasted time by going on to find the  $y$ -coordinates.

**Section B**

- 10 (i) Many candidates were successful in this simple example of completing the square, although there were the usual errors in the value of  $b$ , with 41 being a common wrong answer.
- (ii) Many candidates started again, with calculus being a popular method, to find the coordinates of the minimum point. With only two marks available for the sketch graph, candidates who did not mark the minimum point were not penalised, but those who did not mark the  $y$ -intercept were.
- (iii) Most candidates attempted to solve the inequality, with a variety of standard methods being employed. A number of candidates simply solved the equation and stopped there. Only a minority sorted out the direction of the inequalities satisfactorily. Many attempted to combine them to get  $1 > x > 7$ . A few candidates drew a sketch graph or referred to the one they had already sketched, to help them to draw a correct conclusion.
- (iv) Many candidates gained this mark but some were unsure as to whether they should add or subtract 20. Some gave the answer without the ' $y =$ '. A few candidates attempted to multiply some terms by 20.
- 11 (i) Candidates were usually successful in calculating the length of AC, but were less rigorous in showing the right angle. The gradient and the Pythagoras methods were equally common, and equally successful, though some candidates did not draw a clear conclusion from their calculations.
- (ii) Most candidates knew how to write down the equation of a circle, but were less sure about showing that they understood why the centre was at (3, 6) and the radius 5. Better candidates did refer to the diameter, but hardly any referred to the reason why AC was the diameter.
- (iii) Many candidates attempted this well though they often got lost in the arithmetic at the end, thus gaining method marks but not accuracy marks. Some candidates only found one of the intercepts. A few candidates tried to use calculus methods to determine the gradient of the tangent, but these were generally unsuccessful. A small minority thought that BC would be the tangent and found the equation of that line.
- 12 (i) Those who knew how to write the equation in factorised form then usually were successful in multiplying the factors out, with the given answer enabling them to spot and correct their errors. A few worked backwards by attempting to find factors for the cubic.
- (ii) Attempts at the sketch of the graph were reasonable. Candidates usually gave the values of the intersections with the  $x$ -axis, but some omitted that for the  $y$ -axis.

- (iii) Candidates who determined the value of  $f(4)$  were generally successful, though a few did make an arithmetic slip. Many found the quadratic factor correctly, but very few candidates wrote down the required quadratic equation, most choosing to leave it as a quadratic expression. Quite a number of candidates thought that they were required to solve this equation. A number of candidates only did the long division in this part of the question and some did not make it clear that they could conclude from their results that they had shown  $x = 4$  to be one root of the equation. A few divided  $(x - 4)$  into  $f(x)$  rather than  $f(x) + 10$  but were rarely able to interpret the implications of the result.