OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4751

Introduction to Advanced Mathematics (C1)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 20 May 2009
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

No calculator can be used for this paper

Section A (36 marks)

1 A line has gradient -4 and passes through the point (2, 6). Find the coordinates of its points of intersection with the axes. **[4]**

2 Make a the subject of the formula $s = ut + \frac{1}{2}at^2$. **[3]**

3 When $x^3 - kx + 4$ is divided by $x - 3$, the remainder is 1. Use the remainder theorem to find the value of k. **[3]**

4 Solve the inequality $x(x - 6) > 0$. **[2]**

5 **(i)** Calculate ${}^5C_3$. **[2]**

(ii) Find the coefficient of x^3 in the expansion of $(1 + 2x)^5$. **[2]**

6 Prove that, when n is an integer, $n^3 - n$ is always even. **[3]**

7 Find the value of each of the following.

(i) $5^2 \times 5^{-2}$ **[2]**

(ii) $100^{\frac{3}{2}}$ **[1]**

8 **(i)** Simplify $\dfrac{\sqrt{48}}{2\sqrt{27}}$. **[2]**

(ii) Expand and simplify $\left(5 - 3\sqrt{2}\right)^2$. **[3]**

9 **(i)** Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$. **[3]**

(ii) Write down the coordinates of the minimum point on the graph of $y = x^2 + 6x + 5$. **[2]**

10 Find the real roots of the equation $x^4 - 5x^2 - 36 = 0$ by considering it as a quadratic equation in x^2. **[4]**

Section B (36 marks)

11

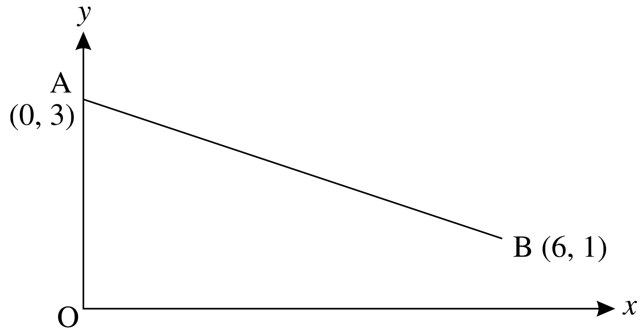

Fig. 11

Fig. 11 shows the line joining the points A $(0, 3)$ and B $(6, 1)$.

(i) Find the equation of the line perpendicular to AB that passes through the origin, O. **[2]**

(ii) Find the coordinates of the point where this perpendicular meets AB. **[4]**

(iii) Show that the perpendicular distance of AB from the origin is $\frac{9\sqrt{10}}{10}$. **[2]**

(iv) Find the length of AB, expressing your answer in the form $a\sqrt{10}$. **[2]**

(v) Find the area of triangle OAB. **[2]**

12 **(i)** You are given that $f(x) = (x + 1)(x - 2)(x - 4)$.

(A) Show that $f(x) = x^3 - 5x^2 + 2x + 8$. **[2]**

(B) Sketch the graph of $y = f(x)$. **[3]**

(C) The graph of $y = f(x)$ is translated by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

State an equation for the resulting graph. You need not simplify your answer.

Find the coordinates of the point at which the resulting graph crosses the y-axis. **[3]**

(ii) Show that 3 is a root of $x^3 - 5x^2 + 2x + 8 = -4$. Hence solve this equation completely, giving the other roots in surd form. **[5]**

13 A circle has equation $(x - 5)^2 + (y - 2)^2 = 20$.

(i) State the coordinates of the centre and the radius of this circle. **[2]**

(ii) State, with a reason, whether or not this circle intersects the y-axis. **[2]**

(iii) Find the equation of the line parallel to the line $y = 2x$ that passes through the centre of the circle. **[2]**

(iv) Show that the line $y = 2x + 2$ is a tangent to the circle. State the coordinates of the point of contact. **[5]**

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Section A

1	(0, 14) and (14/4, 0) o.e. isw	4	M2 for evidence of correct use of gradient with (2, 6) eg sketch with 'stepping' or $y - 6 = -4(x - 2)$ seen or $y = -4x + 14$ o.e. or M1 for $y = -4x + c$ [accept any letter or number] and M1 for $6 = -4 \times 2 + c$; A1 for (0, 14) [$c = 14$ is not sufficient for A1] and A1 for (14/4, 0) o.e.; allow when $x = 0$, $y = 14$ etc isw	4
2	$[a =]\dfrac{2(s-ut)}{t^2}$ o.e. as final answer [condone $[a =]\dfrac{(s-ut)}{0.5t^2}$]	3	M1 for each of 3 complete correct steps, ft from previous error if equivalent difficulty [eg dividing by t does not count as step – needs to be by t^2] $[a =]\dfrac{(s-ut)}{\frac{1}{2}t^2}$ gets M2 only (similarly other triple-deckers)	3
3	10 www	3	M1 for f(3) = 1 soi and A1 for $31 - 3k = 1$ or $27 - 3k = -3$ o.e. [a correct 3-term or 2-term equation] long division used: M1 for reaching $(9 - k)x + 4$ in working and A1 for $4 + 3(9 - k) = 1$ o.e. equating coeffts method: M2 for $(x - 3)(x^2 + 3x - 1)$ [+ 1] o.e. (from inspection or division)	3
4	$x < 0$ or $x > 6$ (both required)	2	B1 each; if B0 then M1 for 0 and 6 identified;	2
5	(i) 10 www (ii) 80 www or ft 8 × their (i)	2 2	M1 for $\dfrac{5\times4\times3}{3\times2(\times1)}$ or $\dfrac{5\times4}{2(\times1)}$ or for 1 5 10 10 5 1 seen B2 for $80x^3$; M1 for 2^3 or $(2x)^3$ seen	4
				16

6	any general attempt at n being odd <u>and</u> n being even even n odd implies n^3 odd and odd − odd = even n even implies n^3 even and even − even = even	M1 A1 A1	M0 for just trying numbers, even if some odd, some even or $n(n^2 - 1)$ used with n odd implies $n^2 - 1$ even and odd × even = even etc [allow even × odd = even] or A2 for $n(n - 1)(n + 1)$ = product of 3 consecutive integers; at least one even so product even; odd^3 − odd = odd etc is not sufft for A1 SC1 for complete general method for only one of odd or even eg $n = 2m$ leading to $2(4m^3 - m)$	3
7	(i) 1 (ii) 1000	2 1	B1 for 5^0 or for 25 × 1/25 o.e.	3
8	(i) 2/3 www (ii) $43 - 30\sqrt{2}$ www as final answer	2 3	M1 for 4/6 or for $\sqrt{48} = 2\sqrt{12}$ or $4\sqrt{3}$ or $\sqrt{27} = 3\sqrt{3}$ or $\sqrt{108} = 3\sqrt{12}$ or for $\sqrt{\frac{4}{9}}$ M2 for 3 terms correct of $25 - 15\sqrt{2} - 15\sqrt{2} + 18$ soi, M1 for 2 terms correct	5
9	(i) $(x + 3)^2 - 4$ (ii) ft their $(-a, b)$; if error in (i), accept (−3, −4) if evidence of being independently obtained	3 2	B1 for $a = 3$, B2 for $b = -4$ or M1 for $5 - 3^2$ soi B1 each coord.; allow $x = -3$, $y = -4$; or M1 for $\begin{bmatrix} -3 \\ -4 \end{bmatrix}$ o.e. oe for sketch with −3 and −4 marked on axes but no coords given	5
10	$(x^2 - 9)(x^2 + 4)$ $x^2 = 9$ [or −4] or ft for integers /fractions if first M1 earned $x = \pm 3$ cao	M2 M1 A1	or correct use of quad formula or comp sq reaching 9 and −4; allow M1 for attempt with correct eqn at factorising with factors giving two terms correct, or sign error, or attempt at formula or comp sq [no more than two errors in formula/substn]; for this first M2 or M1 allow use of y etc or of x instead of x^2 must have x^2; or M1 for $(x + 3)(x - 3)$; this M1 may be implied by $x = \pm 3$ A0 if extra roots if M0 then allow SC1 for use of factor theorem to obtain both 3 and −3 as roots or $(x + 3)$ and $(x - 3)$ found as factors and SC2 for $x^2 + 4$ found as other factor using factor theorem [ie max SC3]	4
				20

Section B

11	**i**	$y = 3x$	2	M1 for grad AB = $\frac{1-3}{6}$ or − 1/3 o.e.	2
	ii	eqn AB is $y = -1/3\, x + 3$ o.e. or ft	M1	need not be simplified; no ft from midpt used in (i); may be seen in (i) but do not give mark unless used in (ii)	
		$3x = -1/3x + 3$ or ft $x = 9/10$ or 0.9 o.e. cao	M1 A1	eliminating *x* or *y* , ft their eqns if find *y* first, cao for *y* then ft for *x*	
		$y = 27/10$ oe ft their 3 × their x	A1	ft dep on both Ms earned	4
	iii	$\left(\frac{9}{10}\right)^2(1+3^2)$ o.e and completion to given answer	2	or square root of this; M1 for $\left(\frac{9}{10}\right)^2+\left(\frac{27}{10}\right)^2$ or 0.81 + 7.29 soi or ft their coords (inc midpt) or M1 for distance = 3 cos θ and tan θ = 3 and M1 for showing $\sin\theta = \frac{3}{\sqrt{10}}$ and completion	2
	iv	$2\sqrt{10}$	2	M1 for $6^2 + 2^2$ or 40 or square roots of these	2
	v	9 www or ft their $a\sqrt{10}$	2	M1 for ½ × 3 × 6 or $\frac{1}{2}\times \text{their}\, 2\sqrt{10}\times\frac{9}{10}\sqrt{10}$	2
					12

12	**iA**	expansion of one pair of brackets correct 6 term expansion	M1 M1	eg [$(x + 1)$]$(x^2 - 6x + 8)$; need not be simplified eg $x^3 - 6x^2 + 8x + x^2 - 6x + 8$; or M2 for correct 8 term expansion: $x^3 - 4x^2 + x^2 - 2x^2 + 8x - 4x - 2x + 8$, M1 if one error allow equivalent marks working backwards to factorisation, by long division or factor theorem etc or M1 for all three roots checked by factor theorem and M1 for comparing coeffts of x^3	2
	iB	cubic the correct way up x-axis: −1, 2, 4 shown y-axis 8 shown	G1 G1 G1	with two tps and extending beyond the axes at 'ends' ignore a second graph which is a translation of the correct graph	3
	iC	[$y=$]$(x - 2)(x - 5)(x - 7)$ isw or $(x - 3)^3 - 5(x - 3)^2 + 2(x - 3) + 8$ isw or $x^3 - 14x^2 + 59x - 70$ $(0, -70)$ or $y = -70$	2 1	M1 if one slip or for [$y =$] $f(x - 3)$ or for roots identified at 2, 5, 7 or for translation 3 to the left allow M1 for complete attempt: $(x + 4)(x + 1)(x - 1)$ isw or $(x + 3)^3 - 5(x + 3)^2 + 2(x + 3) + 8$ isw allow 1 for $(0, -4)$ or $y = -4$ after $f(x + 3)$ used	3
	ii	$27 - 45 + 6 + 8 = -4$ or $27 - 45 + 6 + 12 = 0$ long division of $f(x)$ or their $f(x) + 4$ by $(x - 3)$ attempted as far as $x^3 - 3x^2$ in working $x^2 - 2x - 4$ obtained $[x =]\dfrac{2 \pm \sqrt{(-2)^2 - 4\times(-4)}}{2}$ or $(x - 1)^2 = 5$ $\dfrac{2 \pm \sqrt{20}}{2}$ o.e. isw or $1 \pm \sqrt{5}$	B1 M1 A1 M1 A1	or correct long division of $x^3 - 5x^2 + 2x + 12$ by $(x - 3)$ with no remainder or of $x^3 - 5x^2 + 2x + 8$ with rem −4 or inspection with two terms correct eg $(x - 3)(x^2 \ldots\ldots\ldots - 4)$ dep on previous M1 earned; for attempt at formula or comp square on their other 'factor'	5 13

13	i	(5, 2) $\sqrt{20}$ or $2\sqrt{5}$	1 1	0 for $\pm\sqrt{20}$ etc	2
	ii	no, since $\sqrt{20}$ < 5 or showing roots of $y^2 - 4y + 9 = 0$ o.e. are not real	2	or ft from their centre and radius M1 for attempt (no and mentioning $\sqrt{20}$ or 5) or sketch or solving by formula or comp sq $(-5)^2 + (y-2)^2 = 20$ [condone one error] or SC1 for fully comparing distance from x axis with radius and saying yes	2
	iii	$y = 2x - 8$ or simplified alternative	2	M1 for $y - 2 = 2(x - 5)$ or ft from **(i)** or M1 for $y = 2x + c$ and subst their **(i)** or M1 for ans $y = 2x + k$, $k \neq 0$ or -8	2
	iv	$(x-5)^2 + (2x)^2 = 20$ o.e. $5x^2 - 10x + 5$[= 0] or better equiv. obtaining $x = 1$ (with no other roots) or showing roots equal one intersection [so tangent] (1, 4) cao alt method $y - 2 = -½ (x - 5)$ o.e. $2x + 2 - 2 = -½ (x - 5)$ o.e. $x = 1$ $y = 4$ cao showing (1, 4) is on circle alt method perp dist between $y = 2x - 8$ and $y = 2x + 2 = 10 \cos \theta$ where $\tan \theta = 2$ showing this is $\sqrt{20}$ so tgt $x = 5 - \sqrt{20} \sin \theta$ $x = 1$ (1, 4) cao	M1 M1 M1 A1 A1 M1 M1 A1 A1 B1 M1 M1 M1 A1 A1	subst $2x + 2$ for y [oe for x] expanding brackets and rearranging to 0; condone one error; dep on first M1 o.e.; must be explicit; or showing line joining (1,4) to centre is perp to $y = 2x + 2$ allow $y = 4$ line through centre perp to $y = 2x + 2$ dep; subst to find intn with $y = 2x + 2$ by subst in circle eqn or finding dist from centre = $\sqrt{20}$ [a similar method earns first M1 for eqn of diameter, 2nd M1 for intn of diameter and circle A1 each for x and y coords and last B1 for showing (1, 4) on line – award onlyA1 if (1, 4) and (9, 0) found without (1, 4) being identified as the soln] or other valid method for obtaining x allow $y = 4$	5 11

4751 Introduction to Advanced Mathematics (C1)

General Comments

As usual, the full range of achievement has been seen on this paper. There was plenty of good, positive achievement seen and there were no parts of questions which proved inaccessible.

A few centres continue to enter large numbers of very weak candidates.

Time was not an issue in general, with very few candidates not having the opportunity to attempt all parts of questions, and some clearly having a second go at a few parts where they were dissatisfied with their answers. However, some candidates used long methods in some parts, and will not have helped themselves timewise in doing so.

Lack of facility with fractions continues to disadvantage some candidates. On this paper, question 11 parts (i) to (iii) were the places examiners experienced this most often. In questions 12(i)(*C*) and 13(ii) a surprising number confused the x and y axes.

Centres should note that this paper will be marked on-line as from January 2010. Candidates will be issued with a lined Printed Answer Book that is specific to the paper. Samples based on previous papers will be available to centres on the OCR website, before next January's examination, so that candidates have an opportunity to gain familiarity with the style of booklet and the space provided for answering each question.

Comments on Individual Questions

Section A

1) Many candidates gained 4 marks, but some stopped after finding the equation of the line and did not go on to find the intersections with the axes. Some made errors in the equation of the line because of negative signs and brackets; some did not notice that they were asked to find two intersections.

2) Most candidates gained at least one mark in the rearrangement. Some made errors in coping with the ½ , whilst a surprising number attempted a square root – perhaps they had practised the same formula making t the subject?

3) The majority understood they should substitute 3 and use f(3) = 1. The commonest error after that was $3^3 = 9$. Those who attempted long division rarely succeeded completely. Some achieved success from equating coefficients or using a mixture of that and long division, working backwards and finding k had to be 10 to make the working correct. Some candidates omitted this question or used f(3) = 0 or used f(−3) instead of f(3).

4) Many candidates made life more difficult for themselves here by multiplying out the brackets and working with $x^2 > 6x$. In so doing, they often reached $x > 6$ but usually lost the $x < 0$ part of the solution. Others correctly identified 0 and 6 as the endpoints, but had wrong inequalities. Gaining both marks here was relatively rare. Few candidates drew sketches of $y = x(x - 6)$ to help themselves, but those who did usually had the correct answer.

5) Many gained full marks on this question but relatively few saw the link between the parts, with most starting again with Pascal's triangle in part (ii). As expected, some candidates often failed to use factorials correctly in the first part or to cube the 2 in the second part.

6) Many candidates tried only examples of odd and even numbers and did not attempt a general argument. Of those that did, the majority used the approach 'odd^3 = odd, odd − odd = even (and similarly with even numbers). Some factorised the expression as $n(n^2 - 1)$ and argued successfully from there. Very few factorised it further. Some of the better candidates used $n = 2m$ and $n = 2m + 1$, but some of these attempts were spoiled by errors in their algebra, particularly in expanding and simplifying $(2m + 1)^3 - (2m + 1)$.

7) This was usually correct, but errors in the first part included $5^{2-2} = 5$ and $5^{\frac{1}{2}} = \sqrt{5}$. In the second part, those who cubed first often had problems and some candidates who did obtain $\sqrt{100} = 10$ occasionally gave the answer to cubing it as 30.

8) A surprising number struggled with the first part. Some tried to multiply top and bottom by $2\sqrt{27}$, which tended to lead to problems with arithmetic. Others got part of the way, but failed to simplify, including quite a few who reached $\frac{2\sqrt{3}}{3\sqrt{3}}$ or even $\frac{4}{6}$ and then failed to complete the simplification. Nevertheless many candidates did get the method mark. One common error was go from $2 \times 3\sqrt{3}$ to $5\sqrt{3}$ in the denominator.

Most managed the first three terms in the expansion, but many could not deal with $(3\sqrt{2})^2$. Some got 18 but thought it was negative, and a few got 18 but could not add 25 and 18 correctly. $(3\sqrt{2})^2 = 9\sqrt{2}$ or 36 were other common errors.

9) With no fractions to cope with, and the coefficient of x^2 being 1, this completing the square question was done well. Two common errors were: $(x + 3)^2 + 14$ and $(x - 3)^2 - 4$.

Many candidates were able to use their answer to the first part to write down the coordinates of the minimum, though a few failed to see the connection and started again using calculus. Others attempted to solve the quadratic equation and could have arrived at a correct conclusion using symmetry but failed to do so. The most common error was one of sign.

10) Few gained full marks for this question. Some tried to square root the whole expression on the LHS, to get equations such as $x^2 - 5x - 6 = 0$. Others tried taking out a factor of x^2 and worked from $x^2(x^2 - 5) = 36$. It was evident that many candidates did not understand the concept of a quadratic equation in x^2. Of those who did, some solved the quadratic equation correctly but then left the answer as $x = 9$ or -4, others gave answers of ±2 as well as ±3, or just 2 and 3, and a few gave 16 and 9. The word "real" in the question led a number of candidates to consider the value of the discriminant without taking further steps towards a solution.

Section B

11) (i) Most knew what to do and many found the equation of the perpendicular line correctly. However, $2 \div -6 = -3$ was a common error after a correct expression for the gradient of AB.

(ii) Many candidates successfully equated the correct equations. However, there were frequent errors in coping with the fact that the gradient of AB was −1/3. Those who multiplied up frequently failed to multiply all terms. Those who kept the fraction and reached $3x = -\frac{1}{3}x + 3$ or $3\frac{1}{3}x = 3$ frequently made errors in finding *x*. Follow-through marks limited the damage for such candidates.

(iii) Some did not realise that the coordinates they had just found were needed here, but those who used them correctly were able to gain a mark even if their answers were wrong. From the correct coordinates, following through to achieve our given answer required some squaring of fractions and coping with surds where errors were frequent.

(iv) This part was independent of the previous ones and was done quite well, with the surd in the required form of answer giving a helpful hint. In spite of this, some who had the correct method left their answers as $\sqrt{40}$, whilst the error of thinking this was $4\sqrt{10}$ was also quite common.

(v) Many made this more difficult than they needed to, with some very complicated routes to find the area of OAB, frequently involving two triangles to be added. There was frequent involvement of length OB and Pythagoras being invoked to find length OA.

12) (i)(*A*) Having the answer given to the expansion helped to concentrate candidates' minds on finding any errors in their work, and most gained 2 marks here. Expanding one pair of brackets first was the usual method.

(i)(*B*) The sketch of the cubic graph was often done well – the mark lost most often was for failure to find and show the intersection with the *y* axis. A few stopped their graph at the intersections with the *x* axis at −1 and 4.

(i)(*C*) Some simply put $f(x - 3)$ and did not produce an explicit equation for *y* in terms of *x*. Using $f(x + 3)$ or $f(x) + 3$ were the common errors. Several candidates omitted this part. Of those who gave an equation, few realised they could substitute $x = 0$ into their unsimplified form to obtain the intersection with the *y* axis – mistakes in the algebra then sometimes led to a wrong answer from those who had a correct equation for *y*. Some gave the intersections with the *x* axis rather than the required *y* axis.

(ii) Most used f(3) to obtain the required result although some candidates went straight into long division. Some candidates gained the first mark but did not have much idea about the division. Some used inspection to find the other factor. There were some errors in the formula, but many candidates gained full marks on this part.

13) Many candidates answered this question well.

(i) Most candidates picked up both marks in this first part. A few candidates got the signs of the coordinates wrong and a few gave the radius as 20 rather than $\sqrt{20}$.

(ii) Some who appeared to understand the question lost marks by not being sufficiently clear (e.g. "the radius is not big enough" without comparing 5 and $\sqrt{20}$). Others compared the radius with the distance from the x-axis and said that it did cross. One or two found the distance from the origin to the centre and tried to base an argument on that. Some put $x = 0$ into the equation of the circle in order to find the intersection and gained full credit by showing that the discriminant of the resulting quadratic equation was negative, thus showing that there were no real roots and so no intersection. Some showed this from the equation $(y - 2)^2 = -5$.

(iii) This part was done very well, with just a few using the perpendicular gradient.

(iv) A good number of candidates produced efficient solutions to find the point (1, 4), although many failed to gain the mark for explaining that the repeated root meant that the line was a tangent. Many had the right idea but made algebraic errors, especially in squaring $2x$. Those who multiplied out before substituting $y = 2x + 2$ were more error-prone. A substantial minority made no attempt at this part.