OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4751:** Introduction to Advanced Mathematics

Mark Scheme for January 2011

Oxford Cambridge and RSA Examinations

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Marking instructions for GCE Mathematics (MEI): Pure strand

1. You are advised to work through the paper yourself first. Ensure you familiarise yourself with the mark scheme before you tackle the practice scripts.

2. You will be required to mark ten practice scripts. This will help you to understand the mark scheme and will not be used to assess the quality of your marking. Mark the scripts yourself first, using the annotations. Turn on the comments box and make sure you understand the comments. You must also look at the definitive marks to check your marking. If you are unsure why the marks for the practice scripts have been awarded in the way they have, please contact your Team Leader.

3. When you are confident with the mark scheme, mark the ten standardisation scripts. Your Team Leader will give you feedback on these scripts and approve you for marking. (If your marking is not of an acceptable standard your Team Leader will give you advice and you will be required to do further work. You will only be approved for marking if your Team Leader is confident that you will be able to mark candidate scripts to an acceptable standard.)

4. Mark strictly to the mark scheme. If in doubt, consult your Team Leader using the messaging system within *scoris*, by email or by telephone. Your Team Leader will be monitoring your marking and giving you feedback throughout the marking period.

   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

   Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

5. The following types of marks are available.

   M
   A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

   A
   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

   B
   Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

6. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

7. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

   Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

8. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

9. **Rules for crossed out and/or replaced work**

   If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

   If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

   If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

   If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

10. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

    Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

11. Annotations should be used whenever appropriate during your marking.

    The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

    For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

12. For answers scoring no marks, you must either award NR (no response) or 0, as follows:

    Award NR (no response) if:
    - Nothing is written at all in the answer space
    - There is a comment which does not in any way relate to the question being asked ("can't do", "don't know", etc.)
    - There is any sort of mark that is not an attempt at the question (a dash, a question mark, etc.)

    The hash key [#] on your keyboard will enter NR.

    Award 0 if:
    - There is an attempt that earns no credit. This could, for example, include the candidate copying all or some of the question, or any working that does not earn any marks, whether crossed out or not.

13. The following abbreviations may be used in this mark scheme.

M1	method mark (M2, etc, is also used)
A1	accuracy mark
B1	independent mark
E1	mark for explaining
U1	mark for correct units
G1	mark for a correct feature on a graph
M1 dep*	method mark dependent on a previous mark, indicated by *
cao	correct answer only
ft	follow through
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
sc	special case
soi	seen or implied
www	without wrong working

14. Annotating scripts. The following annotations are available:

✓and ✗
BOD Benefit of doubt
FT Follow through
ISW Ignore subsequent working (after correct answer obtained)
M0, M1 Method mark awarded 0, 1
A0, A1 Accuracy mark awarded 0, 1
B0, B1 Independent mark awarded 0,1
SC Special case
^ Omission sign
MR Misread
Highlighting is also available to highlight any particular points on a script.

15. The comments box will be used by the Principal Examiner to explain his or her marking of the practice scripts for your information. Please refer to these comments when checking your practice scripts.

    Please do not type in the comments box yourself. Any questions or comments you have for your Team Leader should be communicated by the *scoris* messaging system, e-mail or by telephone.

16. Write a brief report on the performance of the candidates. Your Team Leader will tell you when this is required. The Assistant Examiner's Report Form (AERF) can be found on the Cambridge Assessment Support Portal. This should contain notes on particular strengths displayed, as well as common errors or weaknesses. Constructive criticisms of the question paper/mark scheme are also appreciated.

17. Link Additional Objects with work relating to a question to those questions (a chain link appears by the relevant question number) – see scoris assessor Quick Reference Guide page 19-20 for instructions as to how to do this – this guide is on the Cambridge Assessment Support Portal and new users may like to download it with a shortcut on your desktop so you can open it easily! For AOs containing just formulae or rough working not attributed to a question, tick at the top to indicate seen but not linked. When you submit the script, *scoris* asks you to confirm that you have looked at all the additional objects. Please ensure that you have checked all Additional Objects thoroughly.

18. The schedule of dates for the marking of this paper is displayed under 'OCR Subject Specific Details' on the Cambridge Assessment Support Portal. It is vitally important that you meet these requirements. If you experience problems that mean you may not be able to meet the deadline then you must contact your Team Leader without delay.

SECTION A

1	$y = 5x + 3$	**3**	**M2** for $y - 13 = 5(x - 2)$ oe or **M1** for $y = 5x$ [+ k] [k = letter or number other than −4] and **M1** for $13 = \text{their } m \times 2 + k$	or **M1** for $y - b = 5(x - a)$ with wrong a, b or for $y - 13 = \text{their } 5(x - 2)$ oe **M0** for first M if −1/5 used as gradient even if 5 seen first; second M still available if earned
2	(i)(A) 1/16	**1**	isw attempted conversion of 1/16 to decimals	accept 0.0625
2	(i)(B) 1	**1**		set image 'fit to height' so that in marking this question you also check that there is no working on the back page attached to the image
2	(ii) 256/625	**2**	**M1** for num or denom correct or for 4/5 or 0.8	accept 0.4096
3	$\frac{9y^{10}}{2x^2}$ oe as final answer	**3**	**1** for each 'term'; 27/6 gets 0 for first term if **0**, allow **B1** for $(3xy^4)^3 = 27x^3y^{12}$	allow eg $4.5x^{-2}y^{10}$
4	$x > 5/2$ oe (−5/−2 oe not sufft)	**2**	**M1** for $5 < 2x$ or for 5/2 oe obtained with equation or wrong inequality	**M0** for just $-2x < -5$ (not sufft) ; **M1** for $x > -5/-2$

5	$\frac{3V}{\pi r^2} = \sqrt{l^2 - r^2}$	**M1**	for correctly getting non- '$l^2 - r^2$' terms on other side[**M0** for 'triple decker' fraction]	may be done in several steps, if so, condone omission of brackets in eg $9V^2 = \pi^2 r^4\ l^2 - r^2$ if they recover – if not, do not give 1st **M1** [but can earn the 2nd **M1**]
	$\left(\frac{3V}{\pi r^2}\right)^2 = l^2 - r^2$	**M1**	oe or ft; for squaring correctly	for combined steps, allow credit for correct process where possible;
	$l^2 = \left(\frac{3V}{\pi r^2}\right)^2 + r^2$	**M1**	oe or ft; for getting l term as subject	eg $\pi^2 r^4 l^2$ as the term on one side
	$[l =]\sqrt{\left(\frac{3V}{\pi r^2}\right)^2 + r^2}$	**M1**	oe. or ft; mark final answer; for finding square root (and dealing correctly with coefficient of l term if needed at this stage); condone $\pm\sqrt{}$etc	For **M4**, the final expression must be totally correct, [condoning omission of l and insertion of $\pm$] eg **M4** for $\frac{\sqrt{9V^2 + \pi^2 r^6}}{\pi r^2}$
6	$32 - 240x + 720x^2$ isw	**4**	**B3** for all correct except for sign error(s) **B2** for 2 terms correct numerically, ignoring any sign error or for 32, –240 and 720 found or B2 for all correct, including signs, but unsimplified **B1** for binomial coeffts 1, 5, 10 used or 1 5 10 10 5 1 seen **SC3** for $-240x + 720x^2 - 1080x^3$ isw or for $-243x^5 + 810x^4 - 1080x^3$ or **SC2** for these terms with sign error(s)	accept terms listed separately; condone $-240x^1$ expressions left in ${}^nC_r$ form or with factorials not sufft

7	(i) $3^{7/2}$ oe or $k = 7/2$ oe	**2**	**M1** for $\frac{3^4}{\sqrt{3}}$ or $\frac{81}{3^{1/2}}$ or $81\times3^{-1/2}$ or $3^3\sqrt{3}$ or $27 \times 3^{1/2}$ or better or for $81 = 3^4$ or $\sqrt{3} = 3^{1/2}$ or $\frac{1}{\sqrt{3}} = 3^{-1/2}$ or (following correct rationalisation of denominator) for $27 = 3^3$ isw conversion of 7/2 oe	**M0** for just $81 = 3 \times 3 \times 3 \times 3$ oe – indices needed allow an M mark for partially correct work still seen in fraction form eg $\frac{3^4}{3^{-1/2}}$ gets mark for $81 = 3^4$
7	(ii) $\frac{14+5\sqrt{3}}{11}$ or $\frac{28+10\sqrt{3}}{22}$ www isw	**3**	**M1** for multiplying num and denom by $5 + \sqrt{3}$ <u>and</u> **M1** for num or denom correct in final answer (M0 if wrongly obtained)	2[nd] **M1** is not dependent on 1[st] **M1**
8	(7/11, 24/11) oe www	**3**	**B2** for one coord correct; condone not expressed as coords, isw or **M1** for subst or elimination; eg $x + 2(5x - 1) = 5$ oe; condone one error **SC2** for mixed fractions and decimals eg (3.5/5.5, 12/5.5)	
9	(i) $\frac{1}{2} \times 2x \times (x + 2 + 3x + 6)$ oe $x(4x + 8) = 140$ oe and given ans $x^2 + 2x - 35 = 0$ obtained correctly with at least one further interim step	**M1** **A1**	correct statement of area of trap; may be rectangle ± triangle, or two triangles	eg $2x(x + 2) + \frac{1}{2} \times 2x \times (2x + 4)$ condone missing brackets for **M1;** condone also for **A1** if expansion is treated as if they were there

	(ii) [AB =] 21 www	**3**	or **B2** for x = [−7 or] 5 cao www or for AB = 21 or −15 or **M1** for $(x + 7)(x - 5)$ [= 0]or formula or completing square used eg $(x + 1)^2 - 36$ [= 0]; condone one error eg factors with sign wrong or which give two terms correct when expanded or **M1** for showing f(5) = 0 without stating $x = 5$	may be done in (i) if not here – allow the marks if seen in either part of the image – some candidates are omitting the request in (i) and going straight to solving the equation (in which case give 0 [not NR] for (i), but annotate when the image appears again in (ii)) 5 on its own or AB = 5 with no working scores 0; we need to see $x = 5$
10	(i) P $\Leftarrow$ Q (ii) none [of the above] (iii) P $\Rightarrow$ Q	**1** **1** **1**	or $\Leftarrow$ or 'Q $\Rightarrow$ P' or $\Rightarrow$	Condone single arrows

Section A Total: 36

SECTION B

11	(i) grad AB = $\frac{0-6}{1-(-1)}$ oe [= −3] isw grad BC = $\frac{0-4}{1-13}$ oe [= 1/3] isw product of grads = −1 [so lines perp] stated or shown numerically	**M1** **M1** **M1**	for full marks, it should be clear that grads are independently obtained or 'one grad is neg. reciprocal of other' **or** **M1** for length of one side (or square of it) **M1** for length of other two sides (or their squares) found independently **M1** for showing or stating that Pythag holds [so triangle rt angled]	eg grads of −3 and 1/3 without earlier working earn **M1M0** for **M3**, must be fully correct, with gradients evaluated at least to −6/2 and −4/−12 stage $AB^2 = 6^2 + 2^2 = 40$, $BC^2 = 4^2 + 12^2 = 160$, $AC^2 = 14^2 + 2^2 = 200$
11	(ii) AB = √40 or BC = √160 ½ × √40 × √160 oe or ft their AB, BC 40	**M1** **M1** **A1**	**or M1** for one of area under AC (=70), under AB (=6) under BC (=24) (accept unsimplified) and **M1** for their trap. − two triangles	allow **M1** for $\sqrt{(1-(-1))^2+(6-0)^2}$ or for $\sqrt{(13-1)^2+(4-0)^2}$ **or** for rectangle – 3 triangles method, [$6\times14-\frac{1}{2}(2)(6)-\frac{1}{2}(4)(12)-\frac{1}{2}(2)(14)$ =84 – 6 – 24 – 14] **M1** for two of the 4 areas correct and **M1** for the subtraction

11	(iii) angle subtended by diameter = 90° soi mid point M of AC = (6, 5) rad of circle = $\frac{1}{2}\sqrt{14^2+2^2}\,[=]\,\frac{1}{2}\sqrt{200}$ oe or equiv using r^2 $(x-a)^2+(y-b)^2=r^2$ seen or $(x - \text{their } 6)^2 + (y - \text{their } 5)^2 = k$ used, with $k > 0$ $(x-6)^2+(y-5)^2=50$ cao	**B1** **B2** **M1** **M1** **A1**	or angle at centre = twice angle at circumf = 2 × 90 = 180 soi or showing BM = AM or CM, where M is midpt of AC; or showing that BM = ½ AC allow if seen in circle equation ; **M1** for correct working seen for **both** coords accept unsimplified; or eg $r^2 = 7^2 + 1^2$ or $5^2 + 5^2$; may be implied by correct equation for circle or by correct method for AM, BM or CM ft their M or $x^2+y^2-12x-10y+11=0$	condone 'AB and BC are perpendicular' or 'ABC is right angled triangle' provided no spurious extra reasoning allow **M1** bod intent for AC = $\sqrt{200}$ followed by $r = \sqrt{100}$ must be simplified (no surds)
11	(iv) (11, 10) cao	**1**		
12	(i)(A) sketch of cubic correct way up and with two tps, crossing x-axis in 3 distinct points crossing x axis at 1, 2.5 and 4 crossing y axis at −20	**B1** **B1** **B1**	**0** if stops at x-axis; condone not crossing y-axis intersections labelled on graph or shown nearby in this part; mark intent for intersections with both axes (eg condone graphs stopping at axes) or $x = 0$, $y = -20$ seen in this part if consistent with graph drawn	No section to be ruled; no curving back; condone slight 'flicking out' at ends; condone some doubling (eg erased curves may continue to show) allow 2.5 indicated by graph crossing halfway between their marked 2 and 3 on scale; allow if no graph but 0 if graph inconsistent with values allow if no graph, but eg **B0** for graph with intn on +ve y-axis or nowhere near their indicated −20

12	(i)(B) correct expansion of two brackets correct interim step(s) multiplying out linear and quadratic factors before given answer **or** showing that 1, 2.5 and 4 all satisfy $f(x) = 0$ for cubic in $2x^3$ ... form comparing coeffts of eg x^3 in the two forms	**M1** **M1** **or** **M1** **M1**	or **M2** for all 3 brackets multiplied at once, showing all 8 terms (**M1** if error in one term): $2x^3 - 8x^2 - 2x^2 - 5x^2 + 8x + 5x + 20x - 20$ **or** **M1** for dividing $2x^3$ ... form by one of the linear factors and **M1** for factorising the resultant quadratic factor	eg **M1** for $(2x - 5)(x^2 - 5x + 4)$ condone missing brackets if intent clear /used correctly
12	(ii)(A) $250 - 375 + 165 - 40$ isw	**B1**	**or** showing that $x - 5$ is a factor by eg division and then stating that $x = 5$ is root or that $g(5) = 0$	'$2 \times 125 + 15 \times 25 + 33 \times 5 - 40$' is not sufft **or** $[g(5) =]\ f(5) - 20 = 5 \times 4 \times 1 - 20\ [= 0]$
12	(ii) (B) $(x - 5)$ seen or used as linear factor division by $(x - 5)$ as far as $2x^3 - 10x^2$ seen in working $2x^2 - 5x + 8$ obtained isw	**M1** **M1** **A1**	may be in attempt at division or inspection/equating coefficients with two terms correct eg ($2x^2$ ….. + 8) eg may be seen in grid; condone $g(x)$ not expressed as product	allow if seen in (ii)(A) for division: condone signs of $2x^3 - 10x^2$ changed for subtraction, or subtraction sign in front of first term

12	(ii)(C) $b^2 - 4ac$ used on their quadratic factor $(-5)^2 - 4 \times 2 \times 8$ oe and negative [or -39] so no [real] root [may say only one [real] root, thinking of $x = 5$]	**M1** **A1**	may be in formula [or allow **2** marks for complete correct attempt at completing square and conclusion, or using calculus to show min value is above x-axis and comment re curve all above x-axis]	no ft for A mark from wrong quadratic factor condone error in working out -39 if correct unsimplified expression seen and neg result obtained $-5^2 - 4 \times 2 \times 8$ evaluated correctly with comment is eligible for **A1**, otherwise bod for the **M1** only
12	(iii) translation $\begin{pmatrix} 0 \\ -20 \end{pmatrix}$	**B1** **B1**	NB 'Moves' not sufficient for this first mark or 20 down;	**B0** for second mark if choice of one wrong, one right description
13	(i) $(0, -2)$ or 'crosses y-axis at -2' oe isw $(\pm 2^{\frac{1}{4}}, 0)$ oe isw	**B1** **B2**	or [when $y = 0$], $[x =] \pm 2^{\frac{1}{4}}$ or $\pm\sqrt{\sqrt{2}}$ or $\pm\sqrt[4]{2}$ isw **B1** for one root correct	condone $y = -2$

13	(ii) $[y =]\ x^2 = x^4 - 2$ oe and rearrangement to $x^4 - x^2 - 2\ [= 0]$ or $y^2 - y - 2\ [=0]$	**M1**		
	$(x^2 - 2)(x^2 + 1) = 0$ oe in y	**M1**	or formula or completing square; condone one error; condone replacement of x^2 by another letter or by x for 2nd M1 (but not the 3rd M1)	if completing square, and haven't arranged to zero, can earn first **M1** as well for an attempt such as $(x^2 - 0.5)^2 = 2.25$
	$x^2 = 2$ [or −1] or $y = 2$ or −1 or ft or $x = \sqrt{2}$ or $x = -\sqrt{2}$ or ft	**M1**	dep on 2nd **M1;** allow inclusion of correct complex roots; **M0** if any incorrect roots are included for x^2 or x	NB for second and third M: **M0** for $x^2 - 2 = 0$ or $x^2 = 2$ oe straight from quartic eqn – some candidates probably thinking $x^4 - x^2$ simplifies to x^2; last two marks for roots are available as B marks
	$(\sqrt{2}, 2)$ and $(-\sqrt{2}, 2)$; with no other intersections given	**B2**	or **B1** for one of these two intersections (even if extra intersections given) or for $x = \pm\sqrt{2}$ (and no other roots) or for $y = 2$ (and no other roots), marking to candidates' advantage	some candidates having several attempts at solving this equation – mark the best in this particular case

13	(iii) from $x^4 - kx^2 - 2$ [= 0]: $k^2 + 8 > 0$ oe $k + \sqrt{k^2+8} \geq 0$ for all k [so there is a positive root for x^2 and hence real root for x and so intersection]	 **B1** **B1**	Allow x^2 replaced by other letters or x or from $y^2 - k^2y - 2k^2$ [= 0] $k^4 + 8k^2 > 0$ oe $k^2 + \sqrt{k^4+8k^2} > 0$ oe for all k [so there is a positive root for y and hence real root for x and so intersection] if **B0B0**, allow **SC1** for $\frac{k \pm \sqrt{k^2+8}}{2}$ or $\frac{k^2 \pm \sqrt{k^4+8k^2}}{2}$ obtained [need not be simplified]	[alt methods: may use completing square to show similarly, or comment that at $x = 0$ the quadratic is above the quartic and that as $x \to \infty$, $x^4 - 2 > kx^2$ for all k] condone lack of brackets in $(-k)^2$

Section B Total: 36

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

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OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553