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1) i) $\sum_{r=1}^{5}(3r+2)$

$= 5+8+11+14+17$

$= 55$

ii) AP $a = 4.2$ ①

6^{th} $a+5d = 1.8$ ②

② − ① $5d = -2.4$

$d = \frac{-2.4}{5}$

$d = -0.48$

2) i) $\int_1^5 4x\,dx = \left[\frac{4x^2}{2}\right]_1^5$

$= \left[2x^2\right]_1^5$

$= 2\times5^2 - 2\times1^2$

$= 50 - 2$

$= 48$

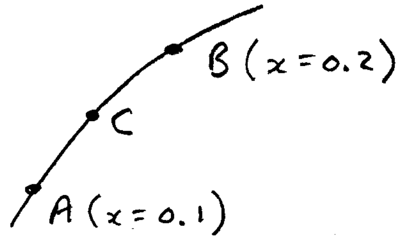
ii) $\int 6x^{\frac{1}{2}}dx = \frac{6x^{3/2}}{\frac{3}{2}} + c$

$= 6\times\frac{2}{3}x^{3/2} + c$

$= 4x^{3/2} + c$

3)

i)
ii)



$y = \log_{10}x$

gradient $= \frac{\log_{10}0.2 - \log_{10}0.1}{0.2 - 0.1}$

$= 3.01$ to 3 s.f.

4) $y = 2x^3$

$\frac{dy}{dx} = 6x^2$

When $x=2$, $\frac{dy}{dx} = 6\times2^2 = 24$

$\therefore$ gradient of normal $= -\frac{1}{24}$

$y - y_1 = m(x - x_1)$

(when $x=2$, $y = 2\times2^3 = 16$)

$y - 16 = -\frac{1}{24}(x-2)$

$24y - 384 = -x + 2$

$x + 24y = 386$

5) i) $y = x^2+3 \rightarrow y = 2x^3+6$

$y = 2(x^2+3)$

One way stretch parallel to y-axis by scale factor 2

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5ii) $y = 2x^2 \rightarrow y = 2(x-3)^2$

Translation by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

6) $\frac{dy}{dx} = 12x^3 - 7$

$\Rightarrow y = \frac{12x^4}{4} - 7x + c$

$y = 3x^4 - 7x + c$

through $(2, 10)$

$10 = 3(2)^4 - 7(2) + c$

$10 = 48 - 14 + c$

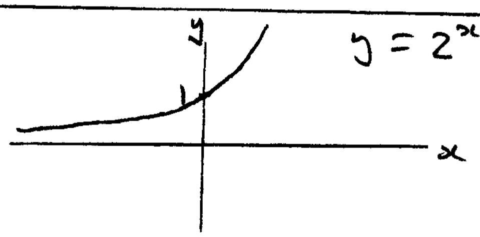
$10 + 14 - 48 = c$

$-24 = c$

Curve

$y = 3x^4 - 7x - 24$

7)i) $y = 2^x$



ii) $\log_a w = 3 + \log_a x^5 - \log_a 2x + \log_a 6$

$\log_a w = 3 + \log_a\left(\frac{6x^5}{2x}\right)$

$\log_a w = 3 + \log_a(3x^4)$

$\log_a w - \log_a(3x^4) = 3$

$\log_a\left(\frac{w}{3x^4}\right) = 3$

$\frac{w}{3x^4} = a^3$

$w = 3a^3x^4$

8) $6\cos^2 x = 5 - \sin x$

$6(1 - \sin^2 x) = 5 - \sin x$

$6 - 6\sin^2 x = 5 - \sin x$

$0 = 6\sin^2 x - \sin x + 5 - 6$

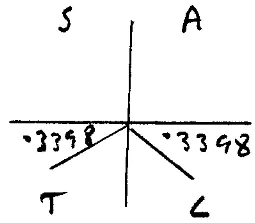
$0 = 6\sin^2 x - \sin x - 1$

$0 = (3\sin x + 1)(2\sin x - 1)$

$\Rightarrow \sin x = -\frac{1}{3}$ or $\sin x = \frac{1}{2}$

$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$\sin^{-1}\frac{1}{3} = 0.3398$ radians



$x = \pi + 0.3398 = 3.481$ rad

$x = 2\pi - 0.3398 = 5.943$ rad

Solution:

$x = \frac{\pi}{6}, \frac{5\pi}{6}, 3.481, 5.943$

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9)

i) $V = \pi r^2 h$, $A = 2\pi r^2 + 2\pi r h$

If $V = 400$,

$$h = \frac{400}{\pi r^2}$$

$$\Rightarrow A = 2\pi r^2 + 2\pi r \times \frac{400}{\pi r^2}$$

$$A = 2\pi r^2 + \frac{800}{r}$$

ii) $A = 2\pi r^2 + 800 r^{-1}$

$$\frac{dA}{dr} = 4\pi r - 800 r^{-2}$$

$$\frac{d^2A}{dr^2} = 4\pi + 1600 r^{-3}$$

iii) Min or max when $\frac{dA}{dr} = 0$

$$\Rightarrow 4\pi r - \frac{800}{r^2} = 0$$

$$\Rightarrow 4\pi r^3 - 800 = 0$$

$$\Rightarrow 4\pi r^3 = 800$$

$$r^3 = \frac{800}{4\pi}$$

$$r = \sqrt[3]{\frac{200}{\pi}}$$

$r = 3.99$ to 3 s.f.

Check this is a min

When $r = 3.99$

$$\frac{d^2A}{dr^2} = 4\pi + \frac{1600}{3.99^3} > 0$$

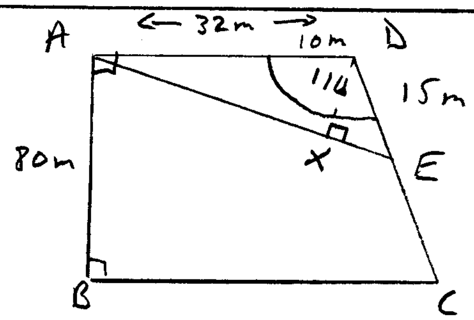
$\therefore$ a minimum

$$A = 2\pi \times 3.99^2 + \frac{800}{3.99}$$

$A = 300.53$

$A = 301$ to 3 s.f.

10)



i) Find AE

$AE^2 = 32^2 + 15^2 - 2 \times 32 \times 15 \cos 116$

$\Rightarrow AE = 40.8636$

$AE = 40.9$ m to 3 s.f.

ii) Area of $\triangle ADE$

$= \frac{1}{2} \times 32 \times 15 \times \sin 116^\circ$

$= 215.711$ m^2

From diagram, Area of $\triangle ADE$

$= \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} \times 40.8636 \times DX$

$\Rightarrow 20.4318\, DX = 215.711$

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10ii cont) $DX = \frac{215.711}{20.4318} = 10.5576$

$DX = 10.6$ m to 3 s.f.

Since radius of pond is 10m centred on D, it lies within $\triangle ADE$.

iii) Area of pond $= \pi \times 10^2 \times \frac{116}{360}$

$= 101.23$ m^2

Area of meadow

= Area of $\triangle ADE$ − Area of pond

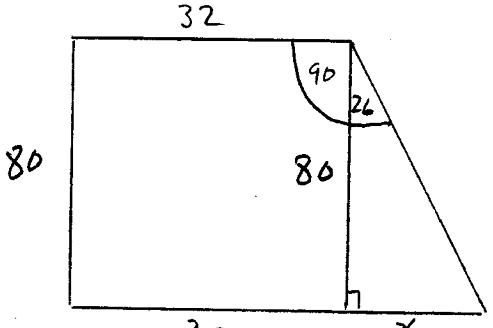
$= 215.71 - 101.23 = 114.48$ m^2

So

Meadow 114.5 m^2 to 4 s.f

Pond 101.2 m^2 to 4 s.f

iv)



$\sin 26^\circ = \frac{x}{80}$

$\Rightarrow x = 80 \sin 26^\circ = 35.1$ m to 3s.f

Area of trapezium

$= \frac{1}{2}\left(32 + (32 + 35.1)\right) \times 80$

$= 3964$ m^2

Area of car park

= Area of trapezium − Area of $\triangle ADE$

$= 3964 - 216 = 3748$ m^2

$\frac{3748}{3964} \times 100 = 94.55\%$

$\therefore$ car park uses more than 90%

11) i) AP $a = 30000$ $d = 1000$

10^{th} year $= 30000 + 9 \times 1000 = 39000$

11^{th} year $= 30000 + 10 \times 1000 = 40000$

GP

$a = 25000$, $r = 1.05$

10^{th} year 25000×1.05^9

$= 38783$

11^{th} year 25000×1.05^{10}

$= 40722$

YR10 Arif £39000 Bettina £38783
so Arif more

YR11 Arif £40000 Bettina £40722

So Bettina more

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11 ii) AP $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$S_{17} = \frac{17}{2}\left(60000 + 16 \times 1000\right)$

$S_{17} = £646,000$

GP

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_{17} = \frac{25000(1.05^{17} - 1)}{1.05 - 1}$

$S_{17} = £646,009$

To nearest £100 both these sums are £646,000

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iii) $\frac{a(r^n - 1)}{r-1} > M$

$a(r^n - 1) > M(r-1)$

$r^n - 1 > \frac{M(r-1)}{a}$

$r^n > \frac{M(r-1)}{a} + 1$

$n \log_{10} r > \log_{10}\left(\frac{M(r-1)}{a} + 1\right)$

$n > \frac{\log_{10}\left(\frac{M(r-1)}{a} + 1\right)}{\log_{10} r}$

$n > \frac{\log_{10}\left(\frac{M(0.05)}{25000} + 1\right)}{\log_{10} 1.05}$

$n > \frac{\log_{10}\left(\frac{M \times \frac{1}{20}}{25000} + 1\right)}{\log_{10} 1.05}$

$n > \frac{\log_{10}\left(\frac{M}{500000} + 1\right)}{\log_{10} 1.05}$

$n > \frac{\log_{10}\left(\frac{M + 500000}{500000}\right)}{\log_{10} 1.05}$

$n > \frac{\log_{10}(M + 500000) - \log_{10} 500000}{\log_{10} 1.05}$

$n > \frac{\log_{10}(1,700,000) - \log_{10} 500000}{\log_{10} 1.05}$

$n > 25.08$

$n = 26$ when total exceeds £1.2 Mill
