MEI CORE 2 JAN 2005

1) $y = x^6 + \sqrt{x}$

$y = x^6 + x^{\frac{1}{2}}$

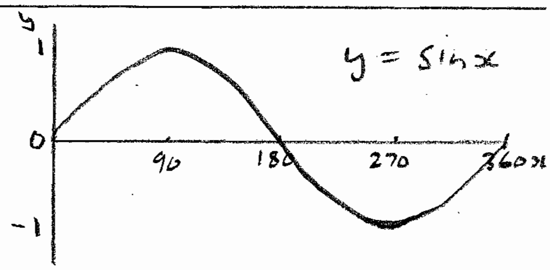
$\frac{dy}{dx} = 6x^5 + \frac{1}{2}x^{-\frac{1}{2}}$

$\frac{dy}{dx} = 6x^5 + \frac{1}{2\sqrt{x}}$

2) $\int \left(x^3 + \frac{1}{x^3} \right) dx$

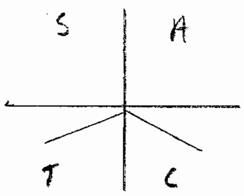
$= \frac{x^4}{4} - \frac{1}{2x^2} + c$

3)



$\sin x = -0.2$

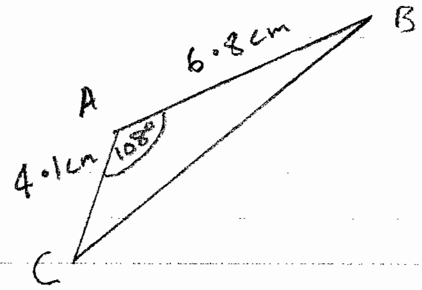
$\sin^{-1}(0.2) = 11.5^\circ$



$x = 191.5^\circ, 348.5^\circ$

for $0^\circ \le x \le 360^\circ$

4)



i) Find BC

Cosine Rule

$|BC|^2 = 4.1^2 + 6.8^2 - 2 \times 4.1 \times 6.8 \cos 108^\circ$

$\Rightarrow |BC| = 8.96$ cm to 2 d.p.

ii) Area of $\Delta = \frac{1}{2} bc \sin A$

$= \frac{1}{2} \times 4.1 \times 6.8 \times \sin 108^\circ$

$= 13.26$ cm^2 to 2 dp

5) GP $a = 4$
$ar = 2$
$ar^2 = 1$

$a = 4, \quad r = \frac{1}{2}$

20^{th} term $= ar^{19}$

$= 4 \times \left(\frac{1}{2}\right)^{19}$

$= 2^2 \times 2^{-19}$

$= 2^{-17}$

$S_\infty = \frac{a}{1-r} = \frac{4}{1-\frac{1}{2}}$

$= \frac{4}{\frac{1}{2}}$

$S_\infty = 8$

6) $a_1 = 4, \; a_{r+1} = a_r + 3$

$a_1 = 4$
$a_2 = 7$
$a_3 = 10$
$a_4 = 13$

AP $a = 4, \; d = 3$

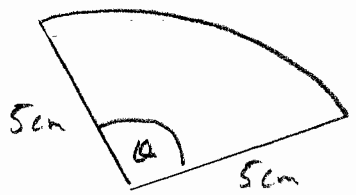
$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$S_{100} = \frac{100}{2}\left(8 + 99 \times 3\right)$

$= 50 \times 305$

$= 15{,}250$

7)



Area of sector $= \frac{1}{2}r^2\theta$

i) $30 = \frac{1}{2} \times 5^2 \times \theta$

$60 = 25\theta$

$\theta = \frac{60}{25} = 2.4$ radians

ii

Arc length $= r\theta$

$= 5 \times 2.4$

$= 12$ cm

Perimeter $12 + 5 + 5 = 22$ cm

8) $10^x = 316$

i) $\log_{10} 10^x = \log_{10} 316$

$x \log_{10} 10 = \log_{10} 316$

$x = \log_{10} 316$

$x = 2.50$ to 3 sig fig

ii) $\log_a(a^2) - 4\log_a\left(\frac{1}{a}\right)$

$= 2\log_a a - 4\log a^{-1}$

$= 2 + 4\log_a a$

$= 2 + 4 = 6$

Section B

9) $y = \frac{1}{4}\left(10x - x^2\right)$

i) A) $y = \frac{5x}{2} - \frac{x^2}{4}$

$\frac{dy}{dx} = \frac{5}{2} - \frac{x}{2}$

At t.p $\frac{dy}{dx} = 0$

$\frac{5}{2} - \frac{x}{2} = 0$

$\Rightarrow x = 5$

Max height when $x = 5$

$y = \frac{1}{4}\left(10(5) - (5)^2\right)$

9 i) A) cont

$y = \frac{25}{4}$

Max height $= \frac{25}{4}$ metres

9 i) B)

Volume = Area of Cross-Section × Length

$\int_0^{10} y\,dx$ = Area of cross-section

100 m = length

$\therefore$ Volume $= 100\int_0^{10} y\,dx$

$100\int_0^{10} \frac{1}{4}(10x - x^2)\,dx$

$= 25\int_0^{10} (10x - x^2)\,dx$

$= 25\left[5x^2 - \frac{x^3}{3}\right]_0^{10}$

$= 25\left[\left(500 - \frac{1000}{3}\right) - (0 - 0)\right]$

$= 25\left[500 - 333\frac{1}{3}\right]$

$= 4166\frac{2}{3}$ metres^3

9 ii)

$T_5 = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4) + y_5\right]$

$= \frac{2}{2}\left[2.15 + 2(5.64 + 6.44 + 6.44 + 5.64) + 2.15\right]$

$T_5 = 52.62$ m^2

New total volume removed

≈ 5262 m^3

If question requires extra volume removed then estimate would be

$5262 - 4167$

$= 1095$ m^3

10) i) $y = x^3 - 6x^2 + 12$

$\frac{dy}{dx} = 3x^2 - 12x$

At t.p $\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 12x = 0$

$3x(x - 4) = 0$

$\Rightarrow x = 0$ or $x = 4$

When $x = 0$, $y = 12$

when $x = 4$, $y = (4)^3 - 6(4)^2 + 12$

$= 64 - 96 + 12$

$= -20$

t.ps at $(0, 12)$ and $(4, -20)$

$\frac{d^2y}{dx^2} = 6x - 12$

When $x = 0$ $\frac{d^2y}{dx^2} = -12 < 0$

10 i) cont

$\therefore (0, 12)$ is a maximum

When $x = 4$

$$\frac{d^2y}{dx^2} = 6\times4 - 12 = +12 > 0$$

$\therefore (4, -20)$ is a minimum

ii)

When $x = 2$

$$\frac{dy}{dx} = 3(2)^2 - 12(2)$$

$$= 12 - 24 = -12$$

$\therefore$ gradient of normal $= +\frac{1}{12}$

Line through $(2, -4)$ with gradient $= \frac{1}{12}$

Using $y - y_1 = m(x - x_1)$

$$y - -4 = \frac{1}{12}(x - 2)$$

$$y + 4 = \frac{1}{12}x - \frac{1}{6}$$

$$y = \frac{1}{12}x - \frac{25}{6}$$

11)

$$z = z_0 10^{-kt}$$

i) When $t = 0$, $z = z_0$

$\therefore z_0$ is the initial difference between the temperature of the drink and room temperature.

ii)

$$z = z_0 10^{-kt}$$

$$\log_{10} z = \log_{10} z_0 10^{-kt}$$

$$\log_{10} z = \log_{10} z_0 + \log_{10} 10^{-kt}$$

$$\log_{10} z = \log_{10} z_0 - kt \log_{10} 10$$

$$\log_{10} z = \log_{10} z_0 - kt$$

iii) See graph on (5)

t	10	20	30	40	50
Temp	68	53	42	36	31
z	46	31	20	14	9
$\log_{10} z$	1.66	1.49	1.30	1.15	0.95

Gradient $= -k$

$$-k = \frac{0.95 - 1.66}{50 - 10}$$

$$-k \approx -0.01775$$

$$k \approx 0.01775$$

From graph $\log_{10} z_0 \approx 1.85$

$$z_0 \approx 10^{1.85}$$

$$z_0 \approx 70.8^\circ$$

$$z = (70.8)\,10^{-0.01775t}$$

When $t = 70$, $z = (70.8)10^{-0.01775\times70}$

$$z = 4.05$$

Temp $= z + z_0 \approx 22 + 4 \approx 26^\circ C$

INSERT for C2 Jan 2005

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11 (iii)

t	10	20	30	40	50
z	46	31	20	14	9
$\log_{10} z$	1.66	1.49	1.30	1.15	0.95