1) $\frac{d}{dx}\left(6x^{5/2}+4\right) = \frac{5}{2} \times 6x^{3/2}$

$= 15x^{3/2}$

2) GP $a = 6$, $S_\infty = 5$

$S_\infty = \frac{a}{1-r}$ for $|r|<1$

$5 = \frac{6}{1-r}$

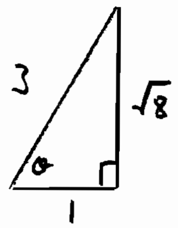
$5(1-r) = 6$

$5 - 5r = 6$

$5 - 6 = 5r$

$\Rightarrow r = -0.2$

3) $\cos\theta = \frac{1}{3}$




$\tan\theta = \frac{\sqrt{8}}{1}$

$\tan\theta = \sqrt{8}$

4)

i) C is periodic

ii) B is convergent

iii)

1, 2, 4, 8, 16, 32

$2^0 \quad 2^1 \quad 2^2 \quad 2^3 \quad 2^4 \quad 2^5$

$n^{th}$ term $= 2^{n-1}$

5) i) $y = \frac{4}{x^2}$

$A(2,1)$ $\qquad B\left(2.1, \frac{4}{2.1^2}\right)$

$B(2.1, 0.90702948)$

Gradient of chord AB $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0.90702948 - 1}{2.1 - 2}$

$= -0.9297$ to 4 d.p

ii) Any point between 2 and 2.1

say $C = 2.05$

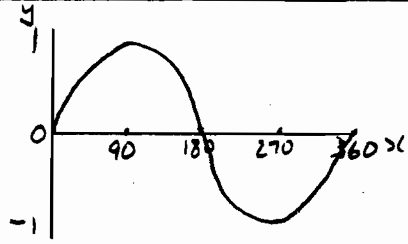
iii) $y = \frac{4}{x^2} = 4x^{-2}$

$\frac{dy}{dx} = -8x^{-3} = -\frac{8}{x^3}$

At $(2,1)$ $\frac{dy}{dx} = \frac{-8}{2^3} = -\frac{8}{8} = -1$

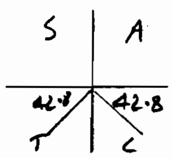
Gradient at A $= -1$

6)




$\sin x = -0.68$

$\sin^{-1}(0.68) = 42.8°$



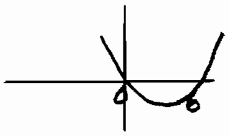

$x = 222.8°, 317.2°$

7) $\frac{dy}{dx} = x^2 - 6x$

y is an increasing function when $\frac{dy}{dx} > 0$

$\Rightarrow \quad x^2 - 6x > 0$

$x(x-6) > 0$




Increasing function when

$x < 0 \quad$ or $\quad x > 6$

8) $7^{th}$ term of AP $\quad a + 6d = 6$ 1

$S_{10} = 30 \qquad S_n = \frac{n}{2}[2a + (n-1)d]$

$30 = \frac{10}{2}[2a + 9d]$

$\Rightarrow \quad 6 = 2a + 9d \qquad$ 2

Also $\quad 6 = a + 6d \qquad$ 1

2 $- 2 \times$ 1

$-6 = -3d$

$\Rightarrow \quad d = 2$

Subst in 1 $\qquad a + 12 = 6$

$\Rightarrow \quad a = -6$

$5^{th}$ term $= a + 4d$

$= -6 + 4 \times 2$

$= 2$

9) $\frac{dy}{dx} = 6x^2 + 8x$

$\Rightarrow \quad y = \frac{6x^3}{3} + \frac{8x^2}{2} + c$

$y = 2x^3 + 4x^2 + c$

$(1,5)$ on curve so

$5 = 2(1)^3 + 4(1)^2 + c$

$5 = 2 + 4 + c$

$\Rightarrow \quad c = -1$

$\therefore \quad y = 2x^3 + 4x^2 - 1$

10) i) $\log_a x^4 + \log_a\left(\frac{1}{x}\right)$

$= 4\log_a x + \log_a x^{-1}$

$= 4\log_a x - \log_a x$

$= 3\log_a x$

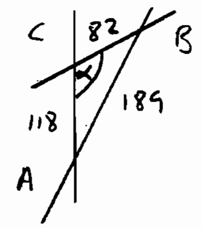
10 ii) $\log_{10} b + \log_{10} c = 3$

$\Rightarrow \quad \log_{10} bc = 3$

$\Rightarrow \quad bc = 10^3$

$\Rightarrow \quad b = \frac{1000}{c}$

**11) i)**




Bearing of B from C $= 180° - \alpha$

$$\cos\alpha = \frac{82^2 + 118^2 - 189^2}{2 \times 118 \times 82}$$

$\Rightarrow \alpha = 141.2°$

$\therefore$ Bearing of B from C $= 038.8°$

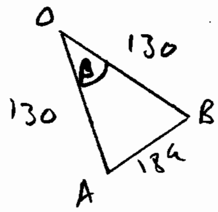
**11 ii)**

Area $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 118 \times 82 \times \sin 141.2°$

$= 3031.5 \text{ m}^2$

**11 iii) A)**




$$\cos\beta = \frac{130^2 + 130^2 - 189^2}{2 \times 130 \times 130}$$

$\angle AOB = \beta = 93.258°$

$= 93.258 \times \frac{\pi}{180}$ radians

$= 1.62766$ radians

$= 1.63$ radians to 3 s.f.

**B)** Area added

$=$ Area of Sector AOB $-$ Area of $\triangle AOB$

$= \frac{1}{2} r^2\theta - \frac{1}{2} r^2 \sin\theta$

$= \frac{1}{2} \times 130^2 (1.62766 - \sin 1.62766)$

$= 5317 \text{ m}^2 = 5300 \text{ m}^2$ to 2 s.f.

**12) i)**

$y = 2x^2 - 11x + 12$

$y = (2x - 3)(x - 4)$

Cuts $x$ axis when $y = 0$

$\Rightarrow x = 4$ or $x = \frac{3}{2}$

$\therefore$ cuts $x$ axis at $(4, 0)$ and $(\frac{3}{2}, 0)$

**ii)**

$\frac{dy}{dx} = 4x - 11$

At $(4, 0)$ $\frac{dy}{dx} = 16 - 11 = 5$

$\therefore$ gradient of normal $= -\frac{1}{5}$

Using $y - y_1 = m(x - x_1)$

$y - 0 = -\frac{1}{5}(x - 4)$

Normal is $y = -\frac{1}{5}x + \frac{4}{5}$

For normal when $x = 0$, $y = \frac{4}{5}$

when $y = 0$, $-\frac{1}{5}x + \frac{4}{5} = 0$

$\Rightarrow x = 4$

Normal cuts axes at $(4, 0)$ and $(0, \frac{4}{5})$

Area of $\triangle = \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} \times 4 \times \frac{4}{5} = 1.6$ units$^2$

12iii) Area $= \int_{\frac{3}{2}}^{4} y\,dx$

$$= \int_{\frac{3}{2}}^{4} (2x^2 - 11x + 12)\,dx$$

$$= \left[\frac{2x^3}{3} - \frac{11x^2}{2} + 12x\right]_{\frac{3}{2}}^{4}$$

$$= \left(\frac{2(4)^3}{3} - \frac{11(4)^2}{2} + 12(4)\right) - \left(\frac{2(1.5)^3}{3} - \frac{11(1.5)^2}{2} + 12(1.5)\right)$$

$$= 2\tfrac{2}{3} - 7\tfrac{7}{8}$$

$$= -5\tfrac{5}{24} \text{ or } -5.208$$

Area $= 5\frac{5}{24}$ units$^2$

(– sign indicates area is below $x$ axis)

13) i)

$$y = k \times 10^{ax}$$

$$\Rightarrow \log_{10} y = \log_{10}(k \times 10^{ax})$$

$$\Rightarrow \log_{10} y = \log_{10} k + \log_{10} 10^{ax}$$

$$\Rightarrow \log_{10} y = \log_{10} k + ax \log_{10} 10$$

$$\Rightarrow \log_{10} y = \log_{10} k + ax$$

compares with

$$y = c + mx$$

$a$ = gradient

$\log_{10} k$ = vertical intercept

13ii) See insert

13iii) From graph $\log_{10} k \approx 2.5$

$$k = 316$$

$$a = \text{gradient} = \frac{3.85 - 2.5}{7 - 0}$$

$$a = 0.193$$

$$\Rightarrow \quad y = 316 \times 10^{0.193x}$$

13iv)

$$75000 \approx 316 \times 10^{0.193x}$$

$$\frac{75000}{316} = 10^{0.193x}$$

$$\log_{10}\left(\frac{75000}{316}\right) = 0.193x$$

$$x \approx \frac{\log_{10}\left(\frac{75000}{316}\right)}{0.193} \approx 12.3$$

When $x = 12$ $\quad y = 316 \times 10^{0.193 \times 12} = 65416$

when $x = 13$ $\quad y = 316 \times 10^{0.193 \times 13} = 102020$

Month 12 closest to £75000

13v) Exponential growth – it grows to fast to soon and is unbounded.

**13 (ii)**

| Number of months after start-up ($x$) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Profit for this month (£$y$) | 500 | 800 | 1200 | 1900 | 3000 | 4800 |
| $\log_{10} y$ | 2.70 | 2.90 | 3.08 | 3.28 | 3.48 | 3.68 |