1) $\frac{d}{dx}(10x^4 + 12)$

$= 40x^3$

2) 1 2 3 4 5 1 2 3 4 5 1 ..

i) Repeats every 5 terms

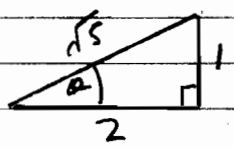
48^{th} term $= 3$

ii) $1 + 2 + 3 + 4 + 5 = 15$

$S_{48} = 9 \times 15 + 1 + 2 + 3$

$= 141$

3) $\tan\theta = \frac{1}{2}$

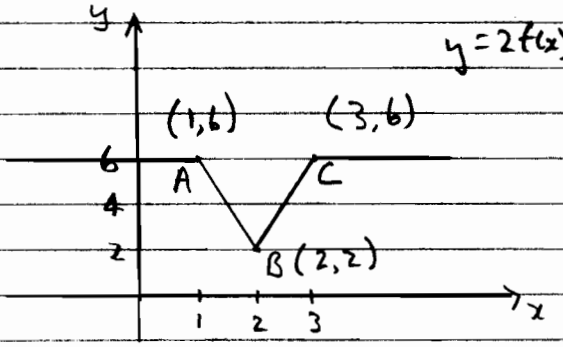


By Pythagoras hypotenuse $= \sqrt{5}$

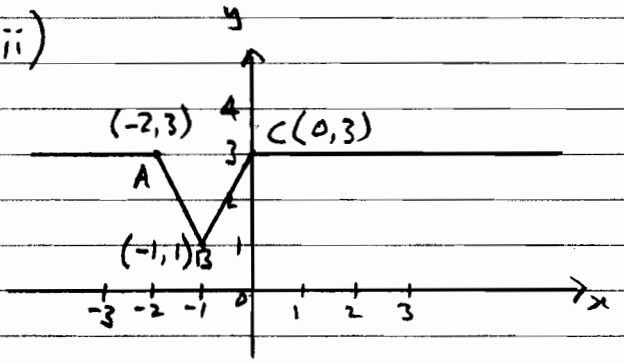
$\cos\theta = \frac{2}{\sqrt{5}}$

$\therefore \cos^2\theta = \frac{4}{5}$

4) i)



4ii)



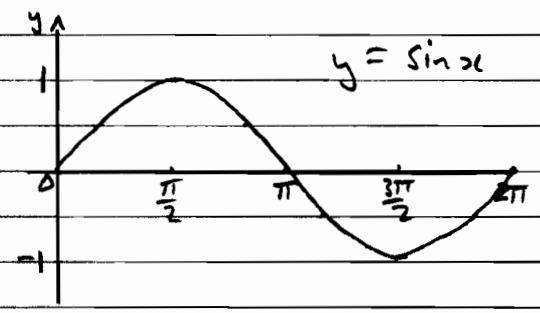
5) $\int \left(12x^5 + \sqrt[3]{x} + 7\right) dx$

$= \int \left(12x^5 + x^{\frac{1}{3}} + 7\right) dx$

$= \frac{12x^6}{6} + \frac{x^{4/3}}{4/3} + 7x + C$

$= 2x^6 + \frac{3x^{4/3}}{4} + 7x + C$

6) i)



6ii) $2\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$

$\Rightarrow \theta = \frac{7\pi}{6}$ or $\theta = \frac{11\pi}{6}$

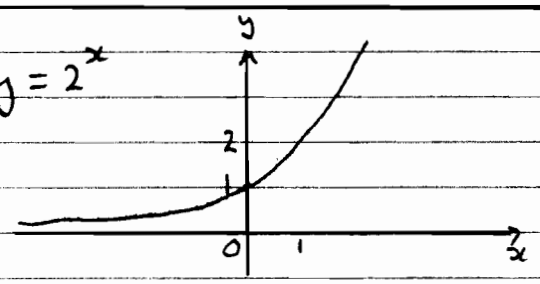
for $0 \leq \theta \leq 2\pi$

7) i) $\sum_{r=2}^{5} 2^r = 2^2 + 2^3 + 2^4 + 2^5$

$= 4 + 8 + 16 + 32 = 60$

7ii) $2^n = \frac{1}{64} \Rightarrow n = -6$

7iii)



8) GP $ar = 18$ ①

$ar^3 = 2$ ②

② ÷ ① $r^2 = \frac{2}{18} = \frac{1}{9}$

$\Rightarrow r = \frac{1}{3}$ since $r > 0$

Subst for r in ①

$\frac{1}{3}a = 18$

$\Rightarrow a = 54$

$S_\infty = \frac{a}{1-r} = \frac{54}{1-\frac{1}{3}}$

$S_\infty = \frac{54}{\frac{2}{3}} = 81$

9) $\log_{10} y = 3x + 2$

i) $\log_{10} 500 = 3x + 2$

$\log_{10} 500 - 2 = 3x$

$x = \frac{\log_{10} 500 - 2}{3}$

$x = 0.23$ to 2 d.p.

9ii) $\log_{10} y = 3 \times (-1) + 2$

$\log_{10} y = -1$

$y = 10^{-1}$

$y = \frac{1}{10}$

9iii) $\log_{10}(y^4) = 4 \log_{10} y$

$= 4(3x+2)$

$= 12x + 8$

9iv) $\log_{10} y = 3x + 2$

$\Rightarrow y = 10^{3x+2}$

10) $V = x^2 h = 120$ cm^3

i) $\Rightarrow h = \frac{120}{x^2}$ cm

Surface area = 6 faces

$= 2x^2 + 4xh$

$= 2x^2 + \frac{4x \times 120}{x^2}$

$= 2x^2 + \frac{480}{x}$

10 ii) $A = 2x^2 + \frac{480}{x}$

$$\frac{dA}{dx} = 4x - \frac{480}{x^2}$$

$$\frac{d^2A}{dx^2} = 4 + \frac{960}{x^3}$$

iii) Min or Max when $\frac{dA}{dx} = 0$

$$\Rightarrow 4x - \frac{480}{x^2} = 0$$

$$\Rightarrow 4x^3 - 480 = 0$$

$$\Rightarrow x^3 = 120$$

$$\Rightarrow x = \sqrt[3]{120} = 4.93 \text{ cm to 2 dp}$$

when $x = 120^{\frac{1}{3}}$

$$\frac{d^2A}{dx^2} = 4 + \frac{960}{120}$$

$$= 12 > 0$$

$\therefore$ a minimum

Min surface area

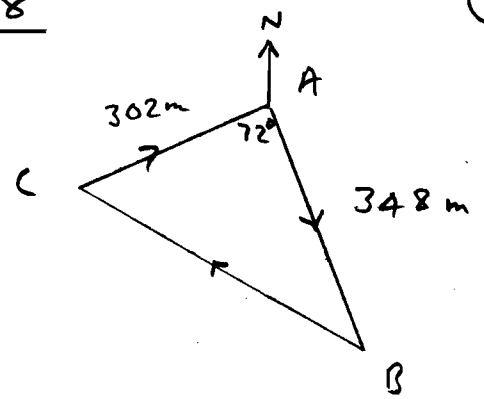
$$= 2 \times 120^{2/3} + \frac{480}{120^{1/3}}$$

$$= 145.97 \text{ cm}^2 \text{ to 2 d.p}$$

11 i)

Cosine Rule

A) $BC^2 = 302^2 + 348^2 - 2 \times 302 \times 348 \cos 72^\circ$



$BC^2 = 147355$

$BC = 383.87$ m

$BC = 384$ m to nearest m

Course $= 302 + 348 + 384$

$= 1034$ m

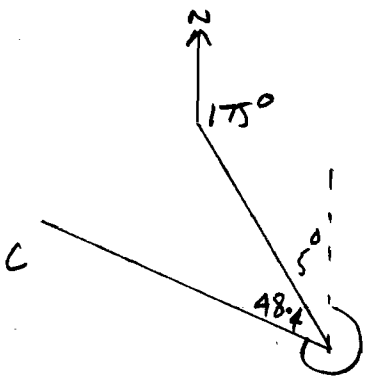
B) Sine Rule

$$\frac{302}{\sin B} = \frac{383.87}{\sin 72^\circ}$$

$$\sin B = \frac{\sin 72^\circ \times 302}{383.87}$$

$\sin B = 0.7482$

$B = 48.4^\circ$



Bearing of 2nd stage

$= 360 - (48.4 + 5) = 306.6^\circ$

11 ii) Arc length $PQ = r\theta$

$= 120 \times \frac{224}{360} \times 2\pi$

$= 469.1$ m

Find straight line PQ by cosine rule

$PQ^2 = 120^2 + 120^2 - 2 \times 120 \times 120 \cos 136°$

$PQ^2 = 49516.986$

$PQ = 222.5$ m

Course length $= 469.1 + 222.5$

$= 691.6$ m

12) i)

A) $y = x^4$

$y = 8x$

At P $8x = x^4$

$\Rightarrow x^4 - 8x = 0$

$\Rightarrow x(x^3 - 8) = 0$

$\Rightarrow x = 0$ or $x = 2$

P is point $(2, 16)$

Area of $\Delta = \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} \times 2 \times 16 = 16$ units^2

B) Shaded area $= \int_0^2 (8x - x^4)\, dx$

$= \left[4x^2 - \frac{x^5}{5} \right]_0^2$

$= \left(4 \times 2^2 - \frac{2^5}{5}\right) - (0 - 0)$

$= 9.6$ units^2

12 ii)

A) $f(x) = x^4$

$f(x+h) = (x+h)^4$

$= x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

B) $\frac{f(x+h) - f(x)}{h}$

$= 4x^3 + 6x^2h + 4xh^2 + h^3$

C) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= 4x^3$

D) $4x^3 = \frac{d}{dx} f(x)$