1)

$$\log_a a = 1$$

$$\log_a(a^3) = 3\log_a a = 3$$

2)

GP $a = 8$, $S_\infty = 10$

$$S_\infty = \frac{a}{1-r}$$

$$\Rightarrow 10 = \frac{8}{1-r}$$

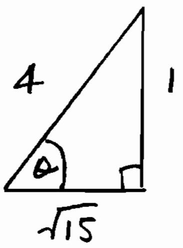
$$\Rightarrow 10(1-r) = 8$$

$$10 - 10r = 8$$

$$10 - 8 = 10r$$

$$r = 0.2$$

3)



$$\sin\theta = \frac{1}{4}$$

$$\tan\theta = \frac{1}{\sqrt{15}}$$

4)

$$\int_1^2 \left(x^4 - \frac{3}{x^2} + 1\right) dx$$

$$= \left[\frac{x^5}{5} + \frac{3}{x} + x\right]_1^2$$

$$= \left(\frac{2^5}{5} + \frac{3}{2} + 2\right) - \left(\frac{1^5}{5} + \frac{3}{1} + 1\right)$$

$$= \left(\frac{32}{5} + \frac{7}{2}\right) - \left(\frac{1}{5} + 4\right)$$

$$= \frac{31}{5} - \frac{1}{2} = \frac{62}{10} - \frac{5}{10}$$

$$= \frac{57}{10} = 5.7$$

5)

$$\frac{dy}{dx} = 3 - x^2$$

$$\Rightarrow y = 3x - \frac{x^3}{3} + c$$

Passes through $(6, 1)$

$$\Rightarrow 1 = 3 \times 6 - \frac{6^3}{3} + c$$

$$1 = 18 - \frac{216}{3} + c$$

$$1 - 18 + 72 = c$$

$$55 = c$$

$$\Rightarrow y = 3x - \frac{x^3}{3} + 55$$

6)

$$u_1 = 3$$

$$u_{n+1} = u_n + 5$$

i)

$$u_1 = 3$$

$$u_2 = 8$$

$$u_3 = 13$$

$$u_4 = 18$$

6ii) AP $a = 3,\ d = 5$

Find $S_{100} - S_{50}$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$S_{100} = \frac{100}{2}\left(6 + 99\times 5\right)$$

$$= 25{,}050$$

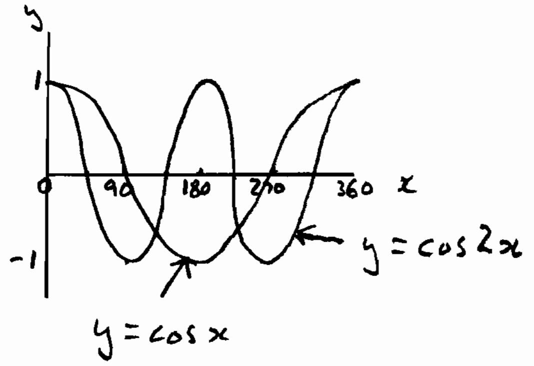
$$S_{50} = \frac{50}{2}\left(6 + 49\times 5\right)$$

$$= 6{,}275$$

$$S_{100} - S_{50} = 18{,}775$$

7)

i)



ii) $\cos 2x = 0.5$

$\Rightarrow 2x = 60^\circ, 300^\circ, 420^\circ, 660^\circ$

$\Rightarrow x = 30^\circ, 150^\circ, 210^\circ\ 330^\circ$

8)

$$y = 6x^3 + \sqrt{x} + 3$$

$$y = 6x^3 + x^{\frac{1}{2}} + 3$$

$$\frac{dy}{dx} = 18x^2 + \frac{1}{2}x^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = 36x - \frac{1}{4}x^{-3/2}$$

9) $5^{3x} = 100$

$\Rightarrow \log_{10}(5^{3x}) = \log_{10} 100$

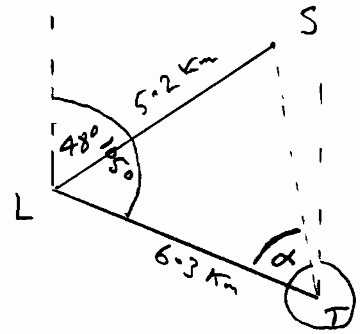
$\Rightarrow 3x\log_{10} 5 = 2$

$\Rightarrow x = \dfrac{2}{3\log_{10} 5}$

$\Rightarrow x = 0.954$

Section B

10) i)



A) $\angle SLT = 105^\circ - 48^\circ = 57^\circ$

Cosine Rule

$$ST^2 = 5.2^2 + 6.3^2 - 2\times 5.2\times 6.3\cos 57^\circ$$

$$ST^2 = 31.0452$$

$ST = 5.57$ Km

B) Bearing of S from T

$= 360 - 75 + \alpha$

Using cosine rule

$$\cos\alpha = \frac{6.3^2 + 5.57^2 - 5.2^2}{2\times 6.3\times 5.57}$$

$$\cos\alpha = 0.6223$$

$$\Rightarrow \alpha = 51.515^\circ \approx 52^\circ$$

10 i B) cont)

Bearing $= 360 - 75 + 52$

$= 337°$

10 ii)

Arc length $= r\theta$

$24 \times \frac{26}{60} = 5.2\,\theta$

$\Rightarrow \theta = 24 \times \frac{26}{60} \div 5.2$

$\Rightarrow \theta = 2$ radians

$\Rightarrow \theta = 2 \times \frac{180°}{\pi} = 114.59°$

$\approx 115°$

Bearing of ship from lighthouse

$= 360 - (115 - 48)$

$= 293°$

11)

$y = x^3 - 3x^2 + 1$

i)

$\frac{dy}{dx} = 3x^2 - 6x$

$\frac{d^2y}{dx^2} = 6x - 6$

At t.p $\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 6x = 0$

$3x(x-2) = 0$

$\Rightarrow x = 0$ or $x = 2$

When $x = 0$, $y = 1$

$\frac{d^2y}{dx^2} = -6 < 0$

$\therefore$ a maximum at $(0,1)$

when $x = 2$, $y = 2^3 - 3 \times 2^2 + 1$

$y = -3$

$\frac{d^2y}{dx^2} = +6 > 0$

$\therefore$ a minimum at $(2,-3)$

11 ii)

When $x = -1$

$\frac{dy}{dx} = 3(-1)^2 - 6(-1)$

$= 3 + 6 = 9$

$\therefore$ tgt at this point has gradient $= 9$

Solve $3x^2 - 6x = 9$

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$\Rightarrow x = -1$ or $x = 3$

At P $x = 3$

$y = 3^3 - 3(3)^2 + 1 = 1$

P is point $(3,1)$

11ii) cont)

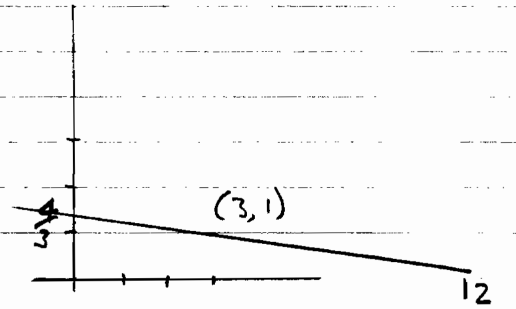
Normal has gradient $= -\frac{1}{9}$

Using $y - y_1 = m(x - x_1)$

$$y - 1 = -\frac{1}{9}(x - 3)$$

$$y - 1 = -\frac{1}{9}x + \frac{1}{3}$$

$$y = -\frac{1}{9}x + \frac{4}{3}$$



When $x = 0$, $y = \frac{4}{3}$

When $y = 0$,

$$0 = -\frac{1}{9}x + \frac{4}{3}$$

$$\frac{1}{9}x = \frac{4}{3}$$

$$x = 12$$

Area of Δ

$= \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} \times 12 \times \frac{4}{3}$

$= 8$ units^2

12)

$$P = a \times 10^{bt}$$

$$\Rightarrow \log_{10} P = \log_{10}(a \times 10^{bt})$$

$$\Rightarrow \log_{10} P = \log_{10} a + \log_{10} 10^{bt}$$

$$\Rightarrow \log_{10} P = \log_{10} a + bt \log_{10} 10$$

$$\Rightarrow \log_{10} P = \log_{10} a + bt$$

This compares with $y = mx + c$

Gradient is b

Intercept $= \log_{10} a$

See Insert

12 (i) $P = a \times 10^{bt}$

$\Rightarrow \log_{10} P = \log_{10}(a \times 10^{bt})$

$\Rightarrow \log_{10} P = \log_{10} a + \log_{10} 10^{bt}$

$\Rightarrow \log_{10} P = \log_{10} a + bt \log_{10} 10$

$\Rightarrow \log_{10} P = \log_{10} a + bt$

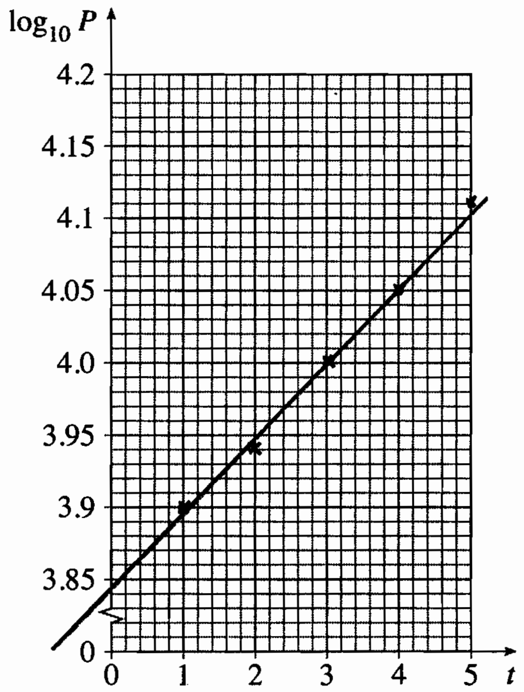
This compares to $y = mx + c$

Gradient $= b$

Vertical Intercept $= \log_{10} a$

(ii)

Year	2001	2002	2003	2004	2005
t	1	2	3	4	5
P	7900	8800	10000	11300	12800
$\log_{10} P$	3.90	3.94	4.00	4.05	4.11



(iii) $b = \dfrac{4.10 - 3.84}{5 - 0} = 0.052$ $\quad \log_{10} a = 3.84 \Rightarrow a = 6918$

$\therefore P = 6918 \times 10^{0.052t}$

(iv) In 2008 $P = 6918 \times 10^{0.052 \times 8}$

$P = 18,029$