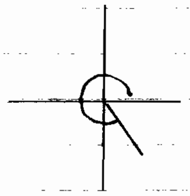
1) i)



$\tan 300^\circ$

$= -\tan 60^\circ$

$= -\sqrt{3}$

ii)

$300^\circ = 300 \times \frac{2\pi}{360}$ radians

$= \frac{5\pi}{3}$ radians

2)

$y = 6x^{3/2}$

$\frac{dy}{dx} = \frac{3}{2} \times 6x^{\frac{1}{2}} = 9x^{\frac{1}{2}}$

$\frac{d^2y}{dx^2} = \frac{1}{2} \times 9x^{-\frac{1}{2}}$

$= \frac{9}{2}x^{-\frac{1}{2}}$

Answers: $\frac{dy}{dx} = 9x^{\frac{1}{2}}$, $\frac{d^2y}{dx^2} = \frac{9}{2}x^{-\frac{1}{2}}$

When $x = 36$, $\frac{d^2y}{dx^2} = \frac{9}{2\sqrt{x}}$

$= \frac{9}{2 \times 6}$

$= \frac{9}{12} = \frac{3}{4}$

3)

A $y = f(x)$

i) B $y = 2f(x)$

ii) C $y = f(x-3)$

4) $t_{n+1} = 2t_n + 5$

i) $t_1 = 3$

$t_2 = 2 \times 3 + 5 = 11$

$t_3 = 2 \times 11 + 5 = 27$

4ii) $\sum_{k=1}^{3} k(k+1)$

$= 1 \times 2 + 2 \times 3 + 3 \times 4$

$= 2 + 6 + 12 = 20$

5) $A = \frac{1}{2}r^2\theta$

$9 = \frac{1}{2} \times 5^2 \times \theta$

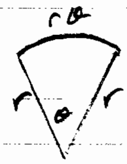
$18 = 25\theta$

$\theta = \frac{18}{25}$ radians

Arc length $= r\theta$

$= 5 \times \frac{18}{25} = \frac{18}{5}$ cm

Perimeter $=$ Arc length $+ 2r$



$= \frac{18}{5} + 2 \times 5$

$= 13.6$ cm

6) i)

$\log_a 1 = 0$

$\log_a a = 1$

ii)

$\log_a x^{10} - 2\log_a\left(\frac{x^3}{4}\right)$

$= 10\log_a x - \log_a\left(\frac{x^3}{4}\right)^2$

$= 10\log_a x - \log_a\left(\frac{x^6}{16}\right)$

$= 10\log_a x - \left(\log_a(x^6) - \log_a 16\right)$

$= 10\log_a x - 6\log x + \log_a 16$

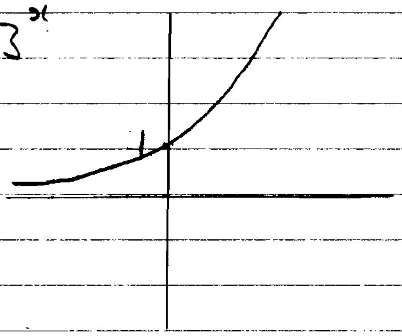
$= 4\log_a x + \log_a(2^4)$

$= 4\log_a x + 4\log_a 2$

$= 4\log_a 2x$

7i)

$y = 3^x$



7ii)

$3^x = 20$

$\ln 3^x = \ln 20$

$x\ln 3 = \ln 20$

$x = \frac{\ln 20}{\ln 3} = 2.73$ to 2 d.p.

8i) $2\cos^2\theta + 7\sin\theta = 5$

$\Rightarrow 2(1-\sin^2\theta) + 7\sin\theta = 5$

$\Rightarrow 2 - 2\sin^2\theta + 7\sin\theta = 5$

$0 = 2\sin^2\theta - 7\sin\theta + 3$

8ii) $(2\sin\theta - 1)(\sin\theta - 3) = 0$

Either $2\sin\theta - 1 = 0$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30^\circ$ or $\theta = 150^\circ$

or $\sin\theta - 3 = 0$

~~$\sin\theta = 3$~~ impossible

Answer $\theta = 30^\circ$ or $\theta = 150^\circ$

9) $y = 2x^3 - 9x^2 + 12x - 2$

i) $\frac{dy}{dx} = 6x^2 - 18x + 12$

When $x = 3$, $\frac{dy}{dx} = 6(3)^2 - 18(3) + 12$

$\frac{dy}{dx} = 12$

When $x = 3$

$y = 2(3)^3 - 9(3)^2 + 12(3) - 2$

9i) cont)

$y = 2\times27 - 9\times9 + 36 - 2$

$y = 54 - 81 + 36 - 2 = 7$

Eqn of tgt thro $(3,7)$ with gradient 12

$y - y_1 = m(x - x_1)$

$y - 7 = 12(x-3)$

$y - 7 = 12x - 36$

$y = 12x - 29$

Verify $(-1,-41)$ on tgt

when $x = -1$

$y = -12 - 29 = -41$

$\therefore (-1,-41)$ is on tgt.

9ii) At t.p. $\frac{dy}{dx} = 0$

$\Rightarrow 6x^2 - 18x + 12 = 0$

$6(x^2 - 3x + 2) = 0$

$6(x-2)(x-1) = 0$

$\Rightarrow x = 2$ or $x = 1$

when $x = 2$, $y = 2(2)^3 - 9(2)^2 + 12(2) - 2$

$y = 2\times8 - 9\times4 + 24 - 2$

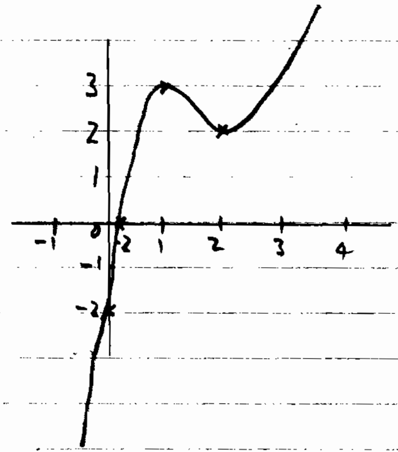
$y = 2$

when $x = 1$, $y = 2(1)^3 - 9(1)^2 + 12(1) - 2$

$y = 2 - 9 + 12 - 2 = 3$

Turning points at $(2,2)$ and $(1,3)$

iii)



10) i)

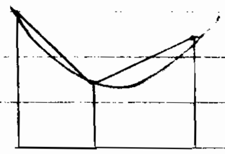
Area $= \frac{h}{2}\left[y_0 + 2(y_1 + \dots + y_{n-1}) + y_n\right]$

$= \frac{10}{2}\left[28 + 2(19+14+11+12+16) + 22\right]$

$= 5\left[194\right] = 970$ units^2

Car travels 970 m

10ii)



Most trapezia include bits above the curve as shown. $\therefore$ an over-estimate.

Underestimate with rectangles

$10 \times \left[19 + 14 + 11 + 11 + 12 + 16\right]$

$= 830$ units^2

10 iii)

$$v = 28 - t + 0.015t^2$$

When $t = 10$

$$v = 28 - 10 + 0.015 \times 100$$

$$v = 28 - 10 + 1.5 = 19.5\,ms^{-1}$$

Measured value $= 19\ ms^{-1}$

$$\frac{0.5}{19} = 0.0263 < 3\%$$

Difference < 3% of measured value.

10 iv)

$$\int_0^{60} (28 - t + 0.015t^2)\,dt$$

$$= \left[28t - \frac{t^2}{2} + 0.015\frac{t^3}{3}\right]_0^{60}$$

$$= \left[28t - \frac{t^2}{2} + 0.005t^3\right]_0^{60}$$

$$= \left(28 \times 60 - \frac{60^2}{2} + 0.005 \times 60^3\right)$$

$$- (0 - 0 + 0)$$

$$= 960\ m$$

11) a) i) AP $a = 3$ $d = 2$

6^{th} pile $= a + 5d$

$= 3 + 5 \times 2 = 13$

ii)

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2 \times 3 + 9 \times 2\right]$$

$$= 5\left[24\right] = 120 \text{ counters}$$

11b)

i) $P_4 = \frac{1}{6} \times \left(\frac{5}{6}\right)^3$

$$P_4 = \frac{125}{1296}$$

ii)

$a = \frac{1}{6}$, $r = \frac{5}{6}$

$$S_\infty = \frac{a}{1-r} = \frac{\frac{1}{6}}{1 - \frac{5}{6}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6}} = 1$$

iii)

$$P_n < 0.001$$

$$\Rightarrow \frac{1}{6}\left(\frac{5}{6}\right)^{n-1} < 0.001$$

$$\Rightarrow \left(\frac{5}{6}\right)^{n-1} < 0.006$$

$$\Rightarrow \log_{10}\left(\frac{5}{6}\right)^{n-1} < \log_{10}(0.006)$$

$$\Rightarrow (n-1)\log_{10}(5/6) < \log_{10}(0.006)$$

$$\Rightarrow n - 1 > \frac{\log_{10}(0.006)}{\log_{10}(5/6)}$$

(Since $\log_{10}(5/6) < 0$)

$$\Rightarrow n > \frac{\log_{10}(0.006)}{\log_{10}(5/6)} + 1$$

$\Rightarrow n > 29.06$ Least value $n = 30$