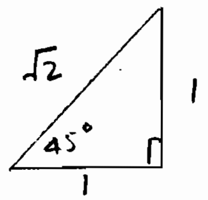
MEI CORE 2 MAY 2009

1)



By Pythagoras hypotenuse $= \sqrt{2}$

$\therefore \cos 45 = \dfrac{1}{\sqrt{2}}$

2)

$$\int_1^2 (12x^5 + 5)\,dx$$

$$= \left[2x^6 + 5x\right]_1^2$$

$$= (2 \times 64 + 5 \times 2) - (2 + 5)$$

$$= 138 - 7 = 131$$

3 i)

$$\sum_{k=3}^{8} (k^2 - 1)$$

$$= (3^2-1) + (4^2-1) + (5^2-1) + (6^2-1) + (7^2-1) + (8^2-1)$$

$$= 8 + 15 + 24 + 35 + 48 + 63$$

$$= 193$$

3 ii) k^{th} term is divergent because successive terms get ever further apart.

4 i)

$r = 18.0$ cm $\quad S = 43.2$ cm

Arc length $S = r\theta$

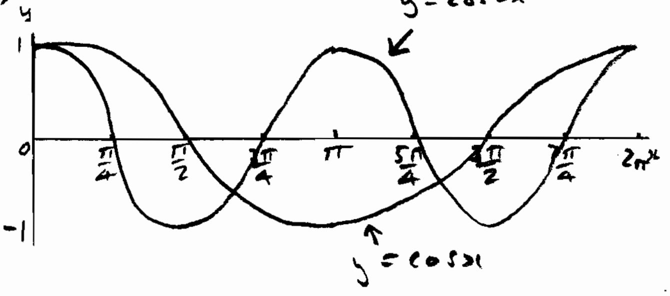
$$\theta = \frac{S}{r} = \frac{43.2}{18.0} = 2.4 \text{ radians}$$

4 ii)

$$2.4 \times \frac{180}{\pi} = 137.50987°$$

$$= 138° \text{ to n'est degree}$$

5 i)



5 ii)

$y = \cos x$ is mapped to $y = 3\cos x$ by a stretch scale factor 3 parallel to the y-axis.

6)

$$y = x^3 - 6x^2 - 15x$$

$$\frac{dy}{dx} = 3x^2 - 12x - 15$$

At t.p. $\dfrac{dy}{dx} = 0$

$$\Rightarrow 3x^2 - 12x - 15 = 0$$

$$3(x^2 - 4x - 5) = 0$$

$$3(x - 5)(x + 1) = 0$$

$$\Rightarrow x = 5 \text{ or } x = -1$$

Sketch

Increasing function when $x < -1$ and also when $x > 5$

7) $4\cos^2\theta = 4 - \sin\theta$

$\Rightarrow 4(1-\sin^2\theta) = 4 - \sin\theta$

$\Rightarrow 4 - 4\sin^2\theta = 4 - \sin\theta$

$\Rightarrow 0 = 4\sin^2\theta - \sin\theta$

$\Rightarrow \sin\theta(4\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = 0$ or $\sin\theta = \frac{1}{4}$

When $\sin\theta = 0$ $\theta = 0, 180°$

When $\sin\theta = \frac{1}{4}$ $\theta = 14.5°, 165.5°$

Solution $\theta = 0, 14.5°, 165.5°, 180°$

8)

$\frac{dy}{dx} = 3\sqrt{x} - 5$

$\frac{dy}{dx} = 3x^{\frac{1}{2}} - 5$

$\Rightarrow y = \frac{3x^{3/2}}{\frac{3}{2}} - 5x + c$

$y = 2x^{3/2} - 5x + c$

Passes through $(4,6)$ so

$6 = 2 \times 4^{3/2} - 5(4) + c$

$6 = 2 \times 8 - 20 + c$

$10 = c$

Curve is given by:

$y = 2x^{3/2} - 5x + 10$

9) i) $10 - 3\log_a a$

$= 10 - 3 = 7$

9ii)

$\frac{\log_{10} a^5 + \log_{10}\sqrt{a}}{\log_{10} a}$

$= \frac{5\log_{10} a + \log_{10} a^{\frac{1}{2}}}{\log_{10} a}$

$= \frac{5\log_{10} a + \frac{1}{2}\log_{10} a}{\log_{10} a}$

$= \frac{\frac{11}{2}\log_{10} a}{\log_{10} a} = \frac{11}{2}$

10 i) See insert

ii) $h = a\log_{10} t + b$

From graph $b = h$ intercept $= -9$

$a = \text{gradient} = \frac{26 - -9}{1.6 - 0} = \frac{35}{1.6}$

$a = 21.875$

Eqn $h = 21.9\log_{10} t - 9$

iii) When $t = 100$ $h = 21.9\log_{10} 100 - 9$

$h = 34.8$ m

iv)

$29 = 21.9\log_{10} t - 9$

$38 = 21.9\log_{10} t$

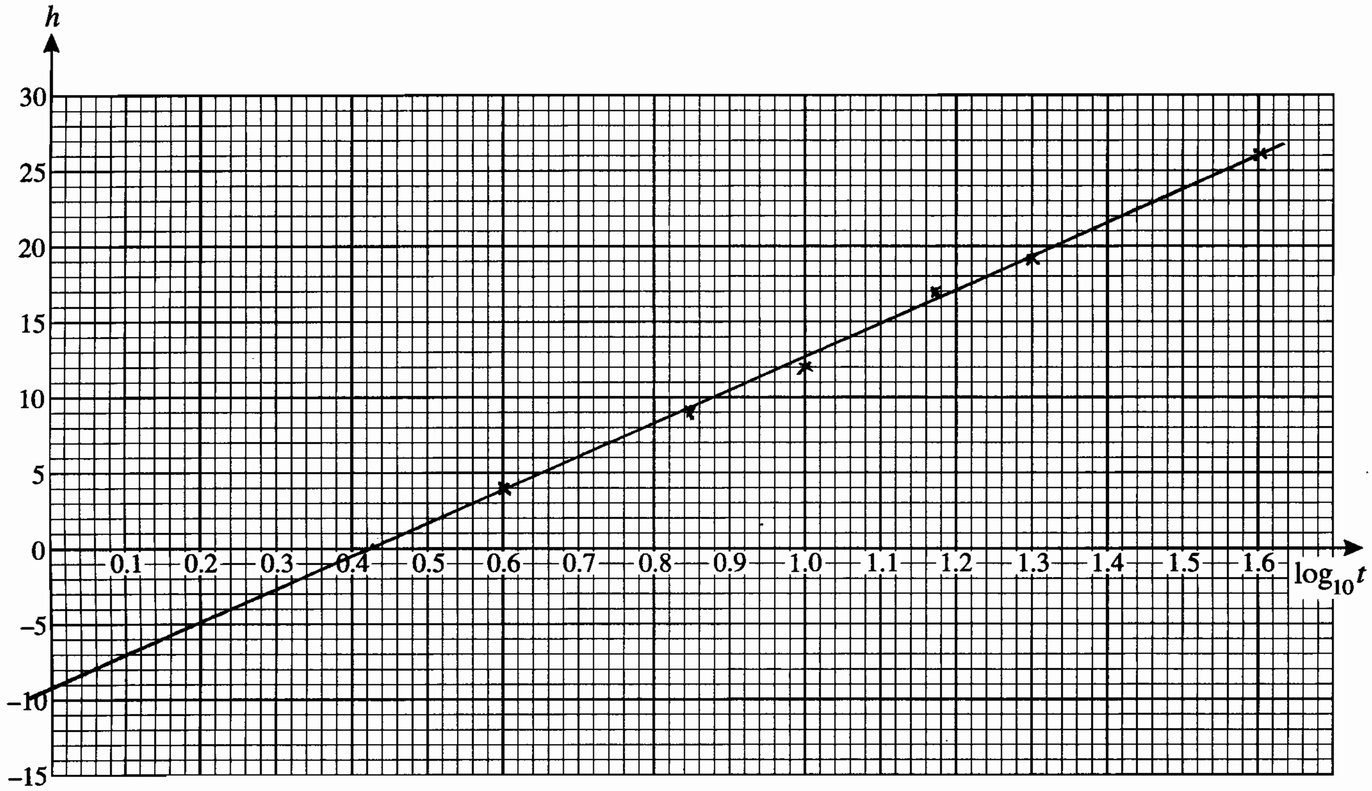
$\frac{38}{21.9} = \log_{10} t$

$t = 10^{\left(\frac{38}{21.9}\right)} = 54.3$ years

③

10 (i)

Age (t years)	4	7	10	15	20	40
$\log_{10} t$	0.602	0.845	1	1.176	1.301	1.602
Height (h m)	4	9	12	17	19	26



10v) Model is unsuitable for very young trees which will be predicted to have negative heights.
eg when $t = 2$ years model predicts
$h = 21.9 \log_{10} 2 - 9 = -2.4\text{m}$

11 i)
A) $10 + 20 + 30 + 40 + 50 + 60$
$= £210$

B) AP $a = 10$, $d = 10$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}\left(20 + 10(n-1)\right)$$

$$S_n = \frac{n}{2}(10n + 10)$$

$$S_n = 5n(n+1)$$

$$10{,}350 = 5n^2 + 5n$$

$$2070 = n^2 + n$$

$$n^2 + n - 2070 = 0$$

$$n = \frac{-1 \pm \sqrt{1^2 + 4 \times 2070}}{2}$$

$$n = \frac{-1 \pm 91}{2}$$

$$n = -46 \text{ or } n = 45$$

In this context

$$n = 45$$

11 ii)
A) $5 + 10 + 20 + 40 = £75$

4 questions correct

B) GP $a = 5$ $r = 2$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_9 = \frac{5(2^9 - 1)}{2 - 1} = £2555$$

c) $$S_n = 5(2^n - 1)$$

$$2{,}621{,}435 = 5(2^n - 1)$$

$$524{,}287 = 2^n - 1$$

$$2^n = 524{,}288$$

$$n \log_{10} 2 = \log_{10} 524{,}288$$

$$n = \frac{\log_{10} 524{,}288}{\log_{10} 2}$$

$$n = 19$$

12)
i) $$y = x^2 - 7$$

When $x = 3.1$, $y = 3.1^2 - 7 = 2.61$

when $x = 3$ $y = 3^2 - 7 = 2$

Points are $(3, 2)$ and $(3.1, 2.61)$

$$\text{Gradient} = \frac{2.61 - 2}{3.1 - 3} = 6.1$$

12ii) $f(x) = x^2 - 7$

$$\frac{f(3+h) - f(3)}{h}$$

$$= \frac{(3+h)^2 - 7 - (3^2 - 7)}{h}$$

$$= \frac{9 + 6h + h^2 - 7 - 2}{h}$$

$$= \frac{6h + h^2}{h}$$

$$= \frac{h(6+h)}{h}$$

$$= 6 + h$$

12iii) Letting $h \to 0$ gives the gradient of the tangent and therefore the function at $x = 3$

$\therefore$ gradient $= 6 + 0 = 6$

12iv) When $x = 3$, $y = 2$
gradient $= 6$

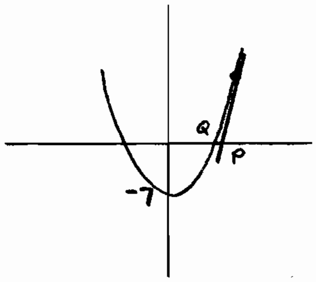
$$y - y_1 = m(x - x_1)$$

$$y - 2 = 6(x - 3)$$

$$y - 2 = 6x - 18$$

$$y = 6x - 16$$

12v)



$y = 6x - 16$

At P $y = 0$ $\therefore$ $0 = 6x - 16$

$$6x = 16$$

$$x = \frac{16}{6}$$

At Q $y = 0$ $\therefore$ $x^2 - 7 = 0$

$$x^2 = 7$$

$$x = +\sqrt{7}$$

$$PQ = \frac{16}{6} - \sqrt{7} = 0.0209$$

$PQ = 0.021$ to 3 d.p.