MEI CORE 2 MAY 2005

1) $\frac{d}{dx}\ x + \sqrt{x}^3$

$\frac{d}{dx}\ x + x^{3/2} = 1 + \frac{3}{2}x^{\frac{1}{2}}$

$= 1 + \frac{3}{2}\sqrt{x}$

2) n^{th} term $= 6 + 5n$

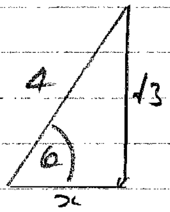
1st term $a = 6 + 5 = 11$
2nd term $a+d = 6 + 10 = 16$
$a = 11,\ d = 5$

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$S_{20} = \frac{20}{2}\left(22 + 19\times 5\right)$

$= 1170$

3) $\sin\theta = \frac{\sqrt{3}}{4}$



$x = \sqrt{16-3} = \sqrt{13}$

$\cos\theta = \pm\frac{\sqrt{13}}{4}$

4) $y = x + \frac{1}{x}$

$y = x + x^{-1}$

$\frac{dy}{dx} = 1 - x^{-2}$

$= 1 - \frac{1}{x^2}$

At t.p $\frac{dy}{dx} = 0$

$\therefore\ 1 - \frac{1}{x^2} = 0$

$x^2 - 1 = 0$

$x^2 = 1 \Rightarrow x = 1$ or $x = -1$

$\therefore$ a t.p at $x = 1$

$\frac{d^2y}{dx^2} = 2x^{-3} = \frac{2}{x^3}$

when $x = 1$ $\frac{d^2y}{dx^2} = \frac{2}{1} > 0$

$\therefore$ t.p is a minimum

5)

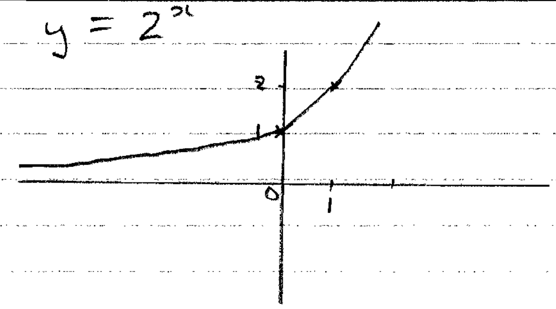
i) $\log_5 5 = 1$

ii) $\log_3\left(\frac{1}{9}\right) = \log_3 1 - \log_3 9$
$= 0 - 2$
$= -2$

iii) $\log_a x + \log_a(x^5)$

$= \log_a x + 5\log_a x = 6\log_a x$

6) $y = 2^x$



$2^x = 50$

$\log_{10} 2^x = \log_{10} 50$

$x\log_{10} 2 = \log_{10} 50$

$x = \frac{\log_{10} 50}{\log_{10} 2}$

$x = 5.64$ to 2dp

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7) $\frac{dy}{dx} = \frac{6}{x^3}$ passes thro (1,4)

$y = \int \frac{6}{x^3}\,dx = \int 6x^{-3}\,dx$

$y = \frac{6x^{-2}}{-2} + c$

$y = -\frac{3}{x^2} + c$

(1,4) on curve

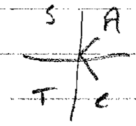
$\therefore\ 4 = -\frac{3}{1^2} + c$

$\Rightarrow c = +7$

Curve $y = -\frac{3}{x^2} + 7$

8) i) $\cos x = 0.4$ $\quad 0 \le x \le 360°$

$x = \cos^{-1} 0.4$



$x = 66.4°$

or $x = 360° - 66.4°$

Ans $x = 66.4°,\ x = 293.6°$

ii) $y = \cos x$ is mapped onto $y = \cos 2x$ by a stretch by a factor of $\frac{1}{2}$ parallel to the x axis.

SECTION B

9) i)

$y = x^3 - 10x^2 + 12x + 72$

$\frac{dy}{dx} = 3x^2 - 20x + 12$

ii) When $x = 2$, $y = 2^3 - 10\times2^2 + 12\times2 + 72$
$= 64$

Point on curve $= (2, 64)$

When $x = 2$ $\frac{dy}{dx} = 3\times2^2 - 20\times2 + 12$

$= -16$

Using $y - y_1 = m(x - x_1)$

Tgt is $y - 64 = -16(x - 2)$

$y - 64 = -16x + 32$

$y = -16x + 96$

9iii)

When $x = -2$, $y = (-2)^3 - 10(-2)^2 + 12(-2) + 72$

$= -8 - 40 - 24 + 72 = 0$

$\therefore$ curve cuts axis when $x = -2$

When $x = 6$ $y = (6)^3 - 10(6^2) + 12(6) + 72$
$= 216 - 360 + 72 + 72 = 0$

$\therefore$ touches x axis at $x = 6$

(When $x = 6$ $\frac{dy}{dx} = 3\times6^2 - 20\times6 + 12$
$= 108 - 120 + 12 = 0$
This confirms curve touches x axis rather than crossing it)

9iv)

$$\text{Area} = \int_{-2}^{6} x^3 - 10x^2 + 12x + 72\,dx$$

$$= \left[\frac{x^4}{4} - \frac{10x^3}{3} + 6x^2 + 72x\right]_{-2}^{6}$$

$$= \left(\frac{6^4}{4} - \frac{10\times6^3}{3} + 6\times6^2 + 72\times6\right) - \left(\frac{(-2)^4}{4} - \frac{10\times(-2)^3}{3} + 6(-2)^2 + 72(-2)\right)$$

9iv) (cont) $= (252) - (-89\frac{1}{3})$

$= 341\frac{1}{3}$ units^2

10) i) Sine Rule

$$\frac{AB}{\sin 42} = \frac{11.4}{\sin 102}$$

$$AB = \frac{11.4}{\sin 102} \times \sin 42 = 7.8\text{cm}$$ to 1dp

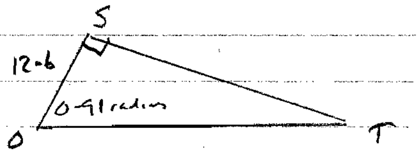
Area of $\triangle ABC$ using $\frac{1}{2}ac\sin B$

$= \frac{1}{2} \times 7.8 \times 11.4 \times \sin 36°$

$= 26.1$ cm^2

Symmetrical $\therefore$ area of logo $= 52.2$ cm^2

ii)



$\angle OST = 90°$ (radius / tgt)

$\angle SOT = 0.91$ radians

$$\tan 0.91 = \frac{ST}{12.6}$$

$\therefore ST = 12.6 \tan 0.91°$

$= 16.2$ cm to 3 sig fig

ii)

Area of $\triangle$ OST $= \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} \times 12.6 \times 16.2$

$= 102.06$ cm^2

Area of quadrilateral OSTR

$= 204.1$ cm^2

Area of minor sector SOR

$= \frac{1}{2}r^2\theta = \frac{1}{2} \times 12.6^2 \times 1.82$

$= 144.5$ cm^2

Area of logo $= 204.1 - 144.5$

$= 59.6$ cm^2

Minor Arc SR $= r\theta$

$= 12.6 \times 1.82$

$= 22.9$ cm

Perimeter of logo

$= 22.9 + 2 \times 16.2$

$= 55.3$ cm

11)

i)

YR	1	2	3	4	5
Heads	1	3	9	27	81

81 flowerheads in YR5

ii) In Year n there are 3^{n-1} flowerheads

iii) This is a GP $a = 1$ $r = 3$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{1(3^n - 1)}{3 - 1}$$

$$= \frac{3^n - 1}{2}$$

iv)

$$\frac{3^n - 1}{2} = 364$$

$3^n - 1 = 728$

$3^n = 729$

11 cont) iv A) $3^6 = 729$

$\therefore n = 6$

Age = 6 years

iv B)

From part (i) flowerheads $= 3^{n-1}$

$= 3^{6-1} = 3^5 = 243$

v)

If age is y

$3^{y-1} > 900$

$\log_{10} 3^{y-1} > \log_{10} 900$

$(y-1)\log_{10} 3 > \log_{10} 900$

$y - 1 > \dfrac{\log_{10} 900}{\log_{10} 3}$

$y > \dfrac{\log_{10} 900}{\log_{10} 3} + 1$

$y > 7.19$

$y = 8$ is smallest integer value

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