OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4752/01**

MATHEMATICS (MEI)

Concepts for Advanced Mathematics (C2)

TUESDAY 16 JANUARY 2007

Morning
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- There is an **insert** for use in Question **13**.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages, **2** blank pages and an insert.

Section A (36 marks)

1 Differentiate $6x^{\frac{5}{2}} + 4$. [2]

2 A geometric progression has 6 as its first term. Its sum to infinity is 5.

Calculate its common ratio. [3]

3 Given that $\cos\theta = \frac{1}{3}$ and θ is acute, find the exact value of $\tan\theta$. [3]

4 Sequences A, B and C are shown below. They each continue in the pattern established by the given terms.

A: 1, 2, 4, 8, 16, 32, …

B: 20, –10, 5, –2.5, 1.25, –0.625, …

C: 20, 5, 1, 20, 5, 1, …

(i) Which of these sequences is periodic? [1]

(ii) Which of these sequences is convergent? [1]

(iii) Find, in terms of n, the nth term of sequence A. [1]

5 A is the point $(2, 1)$ on the curve $y = \dfrac{4}{x^2}$.

B is the point on the same curve with x-coordinate 2.1.

(i) Calculate the gradient of the chord AB of the curve. Give your answer correct to 2 decimal places. [2]

(ii) Give the x-coordinate of a point C on the curve for which the gradient of chord AC is a better approximation to the gradient of the curve at A. [1]

(iii) Use calculus to find the gradient of the curve at A. [2]

6 Sketch the curve $y = \sin x$ for $0° \leqslant x \leqslant 360°$.

Solve the equation $\sin x = -0.68$ for $0° \leqslant x \leqslant 360°$. [4]

7 The gradient of a curve is given by $\frac{dy}{dx} = x^2 - 6x$. Find the set of values of x for which y is an increasing function of x. [3]

8 The 7th term of an arithmetic progression is 6. The sum of the first 10 terms of the progression is 30.

Find the 5th term of the progression. [5]

9 A curve has gradient given by $\frac{dy}{dx} = 6x^2 + 8x$. The curve passes through the point $(1, 5)$. Find the equation of the curve. [4]

10 **(i)** Express $\log_a x^4 + \log_a\left(\frac{1}{x}\right)$ as a multiple of $\log_a x$. [2]

(ii) Given that $\log_{10} b + \log_{10} c = 3$, find b in terms of c. [2]

[Section B starts on the next page.]

Section B (36 marks)

11 Fig. 11.1 shows a village green which is bordered by 3 straight roads AB, BC and CA. The road AC runs due North and the measurements shown are in metres.

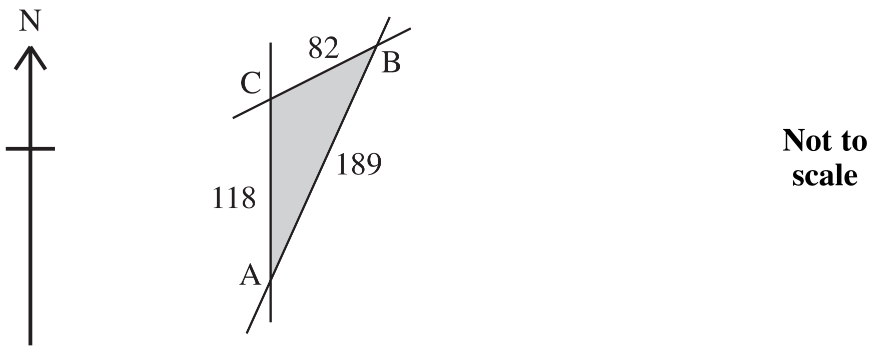


Fig. 11.1

(i) Calculate the bearing of B from C, giving your answer to the nearest 0.1°. [4]

(ii) Calculate the area of the village green. [2]

The road AB is replaced by a new road, as shown in Fig. 11.2. The village green is extended up to the new road.

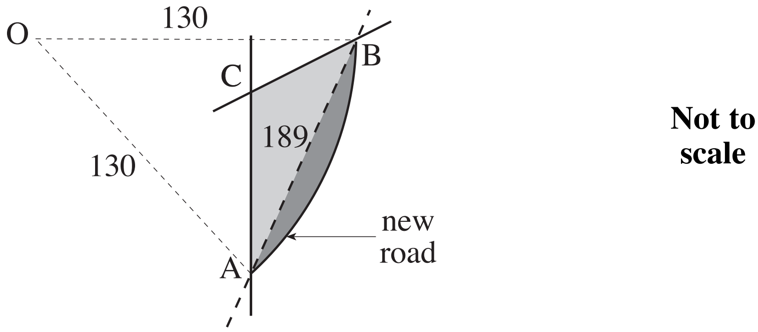


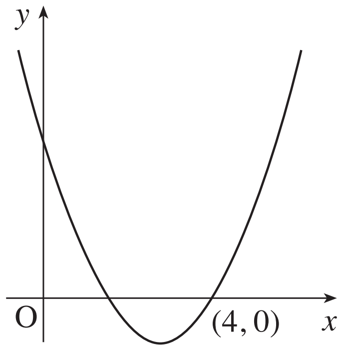
Fig. 11.2

The new road is an arc of a circle with centre O and radius 130 m.

(iii) (*A*) Show that angle AOB is 1.63 radians, correct to 3 significant figures. [2]

(*B*) Show that the area of land added to the village green is 5300 m^2 correct to 2 significant figures. [4]

12 Fig. 12 is a sketch of the curve $y = 2x^2 - 11x + 12$.



Not to scale

Fig. 12

(i) Show that the curve intersects the x-axis at $(4, 0)$ and find the coordinates of the other point of intersection of the curve and the x-axis. [3]

(ii) Find the equation of the normal to the curve at the point $(4, 0)$.

Show also that the area of the triangle bounded by this normal and the axes is 1.6 units^2. [6]

(iii) Find the area of the region bounded by the curve and the x-axis. [3]

13 Answer part (ii) of this question on the insert provided.

The table gives a firm's monthly profits for the first few months after the start of its business, rounded to the nearest £100.

Number of months after start-up (x)	1	2	3	4	5	6
Profit for this month (£y)	500	800	1200	1900	3000	4800

The firm's profits, £y, for the xth month after start-up are modelled by

$$y = k \times 10^{ax}$$

where a and k are constants.

(i) Show that, according to this model, a graph of $\log_{10} y$ against x gives a straight line of gradient a and intercept $\log_{10} k$. [2]

(ii) **On the insert**, complete the table and plot $\log_{10} y$ against x, drawing by eye a line of best fit. [3]

(iii) Use your graph to find an equation for y in terms of x for this model. [3]

(iv) For which month after start-up does this model predict profits of about £75 000? [3]

(v) State one way in which this model is unrealistic. [1]

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MATHEMATICS (MEI)

Concepts for Advanced Mathematics (C2)

INSERT

TUESDAY 16 JANUARY 2007

Morning
Time: 1 hour 30 minutes

Candidate Name	

Centre Number						Candidate Number				

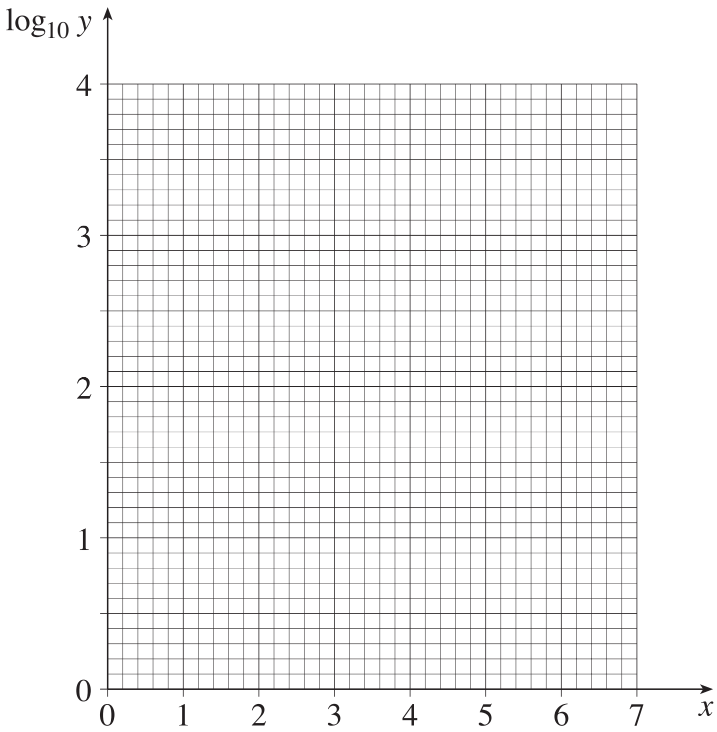
INSTRUCTIONS TO CANDIDATES

- This insert should be used in Question **13(ii)**.
- Write your name, centre number and candidate number in the spaces provided above and **attach the page to your answer booklet**.

This insert consists of 2 printed pages.

13 (ii)

Number of months after start-up (x)	1	2	3	4	5	6
Profit for this month (£y)	500	800	1200	1900	3000	4800
$\log_{10} y$	2.70					



Mark Scheme 4752
January 2007

Section A

1	$\frac{5}{2}\times 6x^{\frac{3}{2}}$	1+1	- 1 if extra term	2
2	-0.2	3	M1 for $5=\frac{6}{1-r}$ and M1 dep for correct constructive step	3
3	√8 or 2√2 not ±√8	3	M1 for use of $\sin^2\theta + (1/3)^2 = 1$ and M1for sinθ = √8/3 (ignore ±) Diag.: hypot = 3, one side =1 M1 3rd side √8 M1	3
4	(i) C (ii) B (iii) 2^{n-1}	1 1 1		3
5	(i) −0.93, -0.930, -0.9297… (ii) answer strictly between 1.91 and 2 or 2 and 2.1 (iii) $y' = -8/x^3$, gradient = −1	2 B1 M1A1	M1 for grad = (1 – their y_B)/(2 − 2.1) if M0, SC1 for 0.93 don't allow 1.9 recurring	5
6	At least one cycle from (0, 0) amplitude 1 and period 360[°] indicated 222.8 to 223 and 317 to 317.2 [°]	G1 G1dep 2	1 each, ignore extras	4
7	$x < 0$ and $x > 6$	3	B2 for one of these or for 0 and 6 identified or M1 for $x^2 - 6x > 0$ seen (M1 if y found correctly and sketch drawn)	3
8	$a + 6d = 6$ correct $30=\frac{10}{2}(2a+9d)$ correct o.e. elimination using their equations $a = -6$ and $d = 2$ 5th term = 2	M1 M1 M1f.t. A1 A1	Two equations in a and d	5
9	(y =) $2x^3 + 4x^2 - 1$ accept $2x^3 + 4x^2 + c$ <u>and</u> c = - 1	4	M2 for (y =) $2x^3 + 4x^2 + c$ (M1 if one error) and M1 for subst of (1, 5) dep on their y =, +c, integration attempt.	4
10	(i) $3 \log_a x$ ii) $b=\frac{1000}{c}$	2 2	M1 for $4\log_a x$ or $-\log_a x$; or $\log x^3$ M1 for 1000 or 10^3 seen	4

Section B

11	**i**	Correct attempt at cos rule correct full method for C C = 141.1… bearing = [0]38.8 cao	M1 M1 A1 A1	any vertex, any letter or B4	4
	ii	½ × 118 × 82 × sin their C or supp. 3030 to 3050 [m^2]	M1 A1	or correct use of angle A or angle B	2
	iiiA	sin (θ/2) = (½ × 189)/130 1.6276 → 1.63	M1 A1	or cosθ = (130^2+130^2-189^2)/(2x130x130) In all methods, the more accurate number to be seen.	2
	iiiB	$0.5 \times 130^2 \times \sin 1.63$ $0.5 \times 130^2 \times 1.63$ their sector – their triangle AOB 5315 to 5340	M1 M1 M1 A1	condone their θ (8435) condone their θ in radians (13770) dep on sector > triangle	4
12	**i**	(2*x* − 3)(*x* − 4) *x* = 4 or 1.5	M1 A1A1	or (11 ±√(121 – 96))/4 if M0, then B1 for showing *y* = 0 when *x* = 4 and B2 for x = 1.5	3
	ii	*y*′ = 4*x* − 11 = 5 when *x* = 4 c.a.o. grad of normal = −1/their *y*′ *y*[− 0]=<u>their</u> −0.2 (*x* − 4) y-intercept for <u>their</u> normal area = ½ × 4 × 0.8 c.a.o.	M1 A1 M1f.t. M1 B1f.t. A1	condone one error or 0 = their (-0.2)x4 + c dep on normal attempt s.o.i. normal must be linear or integrating <u>their</u> f(x) from 0 to 4 M1	6
	iii	$\frac{2}{3}x^3 - \frac{11}{2}x^2 + 12x$ attempt difference between value at 4 and value at 1.5 [−]$5\frac{5}{24}$ o.e. or [−]5.2(083..)	M1 M1 A1	condone one error, ignore + c ft their (i), dep on integration attempt. c.a.o.	3
13	**i**	$\log_{10} y = \log_{10} k + \log_{10} 10^{ax}$ $\log_{10} y = ax + \log_{10} k$ compared to y = mx+c	M1 M1		2
	ii	2.9(0), 3.08, 3.28, 3.48, 3.68 plots [tol 1 mm] ruled line of best fit drawn	T1 P1f.t L1f.t.	condone one error	3
	iii	intercept = 2.5 approx gradient = 0.2 approx y = their 300x $10^{x(\text{their } 0.2)}$ or y = $10^{(\text{their } 2.5 + \text{their } 0.2x)}$	M1 M1 M1f.t.	or y – 2.7 = m(x – 1)	3
	iv	subst 75000 in any x/y eqn subst in a correct form of the relationship 11,12 or 13	M1 M1 A1	 B3 with evidence of valid working	3
	v	"Profits change" or any reason for this.	R1	too big, too soon	1

4752 - Concepts for Advanced Mathematics (C2)

General Comments

The paper was well received with a full range of achievement. There were many excellent scripts as usual and there were fewer scripts scoring under ten marks. There was some evidence that a few candidates were short of time as the last question was incomplete or missing on some scripts. This was possibly due to inefficient working or inefficient use of the calculator in questions 5(i), 7, 8, 11 and 12. Three pieces of advice, which would help to enhance candidates' scores are as follows.

- When a question asks for an exact answer, keep off the calculator; in question 3, 2.828 usually led to a score of zero.
- When using the cosine rule, the examiner wants to see $189^2 = 118^2 + 82^2 - 2\times118\times82\cos C$, or equivalent, to check the method is right, but then all that is needed is the correct answer; there is no need to show all the numbers in the intermediate steps.
- When a question gives the answer, e.g. "show that the angle AOB is 1.63 radians to 3 s.f." it is clear that the examiner will not award the final mark for 1.63, it will be earned for 1.628, the number which rounds to 1.63.

Comments on Individual Questions

Section A

1 This was very well done; just a few did not know the basic rule for differentiation; just a few integrated.

2 Again, this was very well done. Very few candidates misquoted the formula as $S = a/(r - 1)$. Weaker candidates made errors in the algebra.

3 Too many put 1/3 in their calculators to find θ, then took their calculator answer for $\tan\theta$ as 2.828…; this received no credit. The majority put 1/3 onto a right angled triangle, used the theorem of Pythagoras (usually correctly) and wrote down that $\tan\theta = \sqrt{8}$. Those who used $(1/3)^2 + \sin^2\theta = 1$ were less successful as $\sqrt{(1 - 1/9)}$ sometimes defeated them.

4 All three parts were very well done by the majority of candidates. Some candidates did not understand the words periodic or convergent; some could not cope with the expression ar^{n-1}.

5 (i) The main problem was in finding the y value at x = 2.1. It is 0.9070 and at least 0.907 is needed to achieve the correct answer to two decimal places. Many took it to be 0.91 or even 0.9, thus losing the accuracy mark. The method for this part was usually good, a few lost the minus sign and a few used run/rise for the gradient.

(ii) Not all candidates understood the situation here; the response 2 did not score, nor did 1.9.

(iii) This was very well done, the one frequent error being $dy/dx = -8x^{-1}$

6 Almost all candidates produced a sine curve, and the majority convinced the examiner that they knew the period and amplitude for the second mark. The majority found one, usually both, of the correct angles satisfying the equation.

7 This was not well done. The majority of candidates had not met the phrase "increasing function" and many of those that had met it failed to use the simple condition y' > 0. A small minority coped perfectly to obtain x < 0 and x > 6. A few of these presented the answer as 6 < x < 0; this received full marks, but the examiner's disapproval must be noted. Many tried to solve $x^2 – 6x = 0$, but lost the zero root. Many tried to work with y or y'', but usually without any progress. Many worked out a series of y values and things like x ≥ 7 appeared, not elegant and not correct.

8 Some had no idea how to tackle this question. Some treated it as a geometric progression and quoted appropriate formulae. Some quoted the correct A.P. formulae, but could not substitute the given data properly. Some quoted the expression $\frac{n}{2}(a + l)$, which is not wrong, but the third letter muddied the waters somewhat and they extricated themselves with great difficulty, or more usually, did not. Many did everything correctly and scored full marks, some very neatly, some less so. There were attempts of varying ingenuity that did not involve the standard formulae. Some guessed that the common difference was 2 and produced a list of numbers which could be checked for a sum of 30. A delightful method was seen: 5 pairs add up to 30, 6 each, 4^{th} and 7^{th} is a pair, hence 4^{th} is 0, 5^{th} and 6^{th} must be 2 and 4.

9 A few calculated y' at x = 1 and said the curve was y = 14x + c. Similarly a few got involved with $y = (6x^2 + 8x)x + c$. The majority knew what had to be done and duly scored full marks. A common slip was $6x^3/3 = 3x^3$.

10 (i) There was some good work done here, but it did not always lead to full marks. There was a mark for 4logx or –logx or $\log x^3$, but for both marks the multiple of logx was needed.

(ii) logbc = 3 did not score any marks. The candidates had to deal with the 3 to earn a mark and they found that remarkably difficult. $bc = 10^3$ scored 1 mark and $b = 10^3/c$ scored the other.. A common attempt, which only scored 1, was logb = 3 – logc, hence $b = 10^{(3 - \log c)}$, correct, but not nice. Some suggested b = 10 and c = 100. This is not wrong, but it is not good enough. A sizeable minority suggested $bc = e^3$

Section B

11 (i) Most candidates recognised the need for the cosine rule and most applied it correctly to find C, B or A. Many did not appreciate that there are three forms, depending on which angle one needs. A few, calculating C, lost the minus sign to obtain 38.8, thus reading rather a lot into "not to scale" on the diagram. Some did excellent work but threw away a mark by giving the bearing as 038.84.

(ii) The formula (absinC)/2 was well known and many earned both marks for this part; some by using the appropriate numbers having found A or B in part (i)

(iii) Part A was very well done by various methods and most arrived correctly at 1.63. Unfortunately many lost a mark for failing to show 1.628.
In part B, most good candidates scored full marks; they knew how to find the areas of the sector and the triangle and correctly did both and subtracted. Just a few could not organise their areas and calculated sector minus triangle ABC.

12 (i) Most candidates scored full marks for this part, some taking two lines, some taking two pages. They solved the quadratic by formula and then laboriously substituted both roots to confirm y = 0.

(ii) Many scored full marks, doing the six or seven operations perfectly correctly. Some created difficulties by taking the gradient as – 1/ (4x – 11) and substituting this into a line formula. Some recovered, some did not. A few did not convert 5 to – 1/5; it was left as 5 or converted to 1/5. The area of the triangle was well done using (base × height)/2 or integration from 0 to 4.

(iii) The method for this part was well known and most candidates scored 2 marks, 3 if they coped with the arithmetic – many did. A complication here was the interpretation of "bounded by the curve and the x-axis". Some deemed the area between x = 0 and x = 1.5 to be "between the curve and the x-axis" and so included it in their calculations.

13 (i) All candidates knew that logarithms had to be taken, but a very common error was $\log(k10^{ax}) = \log k \times \log 10^{ax}$. Many did convert correctly to ax + logk and this, with any mention of a and c, earned full marks.

(ii) This part was very well done. For the table mark we required e.g. 3.28 or 3.3. Drawing the line of best fit freehand cost a mark.

(iii) Candidates had to find the gradient of the line and the logy intercept (or an equivalent calculation using a point on the line) and put these into Y = mx + c, where Y = y or logy. To score the third mark they had to produce $y = 316 \times 10^{0.2x}$ approximately.

(iv) There was a method mark for substituting 75 000 into any x/y equation and a second method mark if the x/y equation was of the correct form i.e. logy = 0.2x + 2.5 or $y = 316 \times 10^{0.2x}$ Covering all previous small inaccuracies, 11,12 or 13 scored the third mark. Full marks were awarded if the correct answer came from a convincing attempt at extrapolating 48 000 to 75 000 with appropriate increments.

(v) Anything suggesting that profits are not impervious to any one of hundreds of outside influences scored. A few hit on the serious point that indefinite exponential growth is never realistic.