OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4752

Concepts for Advanced Mathematics (C2)

QUESTION PAPER

Candidates answer on the Printed Answer Book

OCR Supplied Materials:
- Printed Answer Book 4752
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Thursday 27 May 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- **The questions are on the inserted Question Paper.**
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your Candidate Number, Centre Number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 You are given that

$$u_1 = 1,$$
$$u_{n+1} = \frac{u_n}{1 + u_n}.$$

Find the values of u_2, u_3 and u_4. Give your answers as fractions. **[2]**

2 **(i)** Evaluate $\sum_{r=2}^{5} \frac{1}{r-1}$. **[2]**

(ii) Express the series $2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6 + 6 \times 7$ in the form $\sum_{r=2}^{a} \mathrm{f}(r)$ where $\mathrm{f}(r)$ and a are to be determined. **[2]**

3 **(i)** Differentiate $x^3 - 6x^2 - 15x + 50$. **[2]**

(ii) Hence find the x-coordinates of the stationary points on the curve $y = x^3 - 6x^2 - 15x + 50$. **[3]**

4 In this question, $\mathrm{f}(x) = x^2 - 5x$. Fig. 4 shows a sketch of the graph of $y = \mathrm{f}(x)$.

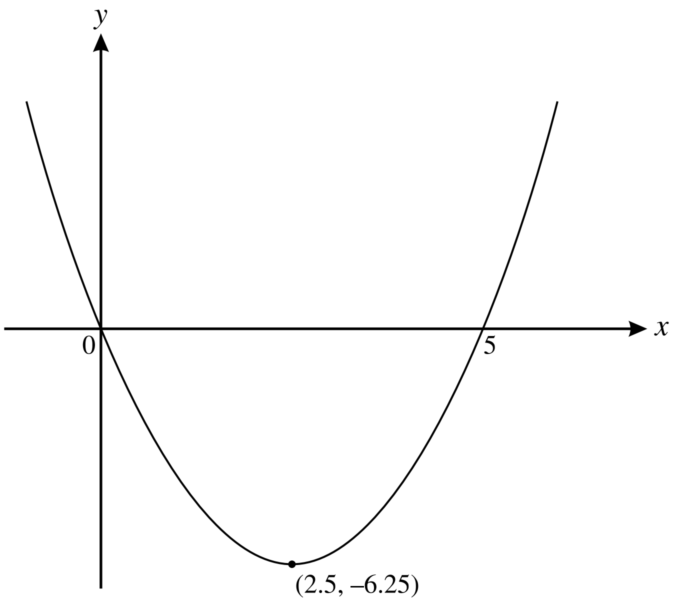

Fig. 4

On separate diagrams, sketch the curves $y = \mathrm{f}(2x)$ and $y = 3\mathrm{f}(x)$, labelling the coordinates of their intersections with the axes and their turning points. **[4]**

5 Find $\int_2^5 \left(1 - \frac{6}{x^3}\right) dx$. **[4]**

6 The gradient of a curve is $6x^2 + 12x^{\frac{1}{2}}$. The curve passes through the point (4, 10). Find the equation of the curve. **[5]**

7 Express $\log_a x^3 + \log_a \sqrt{x}$ in the form $k \log_a x$. **[2]**

8 Showing your method clearly, solve the equation $4\sin^2\theta = 3 + \cos^2\theta$, for values of θ between $0°$ and $360°$. **[5]**

9 The points (2, 6) and (3, 18) lie on the curve $y = ax^n$.

Use logarithms to find the values of a and n, giving your answers correct to 2 decimal places. **[5]**

Section B (36 marks)

10 **(i)** Find the equation of the tangent to the curve $y = x^4$ at the point where $x = 2$. Give your answer in the form $y = mx + c$. **[4]**

(ii) Calculate the gradient of the chord joining the points on the curve $y = x^4$ where $x = 2$ and $x = 2.1$. **[2]**

(iii) (A) Expand $(2 + h)^4$. **[3]**

(B) Simplify $\frac{(2+h)^4 - 2^4}{h}$. **[2]**

(C) Show how your result in part **(iii)** (B) can be used to find the gradient of $y = x^4$ at the point where $x = 2$. **[2]**

11 (a)

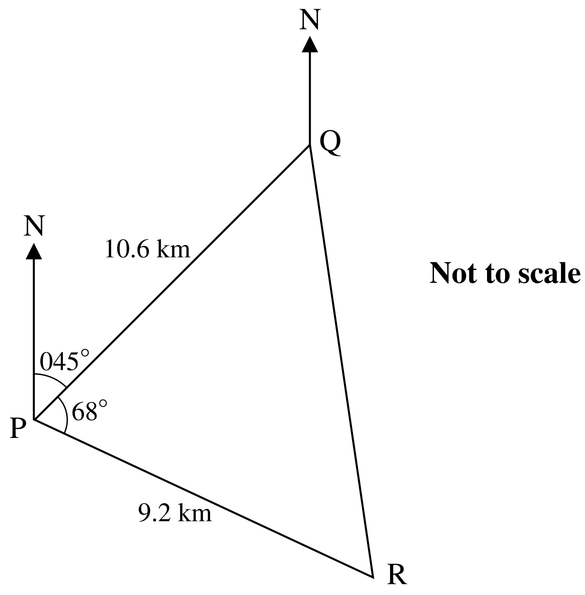

Fig. 11.1

A boat travels from P to Q and then to R. As shown in Fig. 11.1, Q is 10.6 km from P on a bearing of 045°. R is 9.2 km from P on a bearing of 113°, so that angle QPR is 68°.

Calculate the distance and bearing of R from Q. **[5]**

(b) Fig. 11.2 shows the cross-section, EBC, of the rudder of a boat.

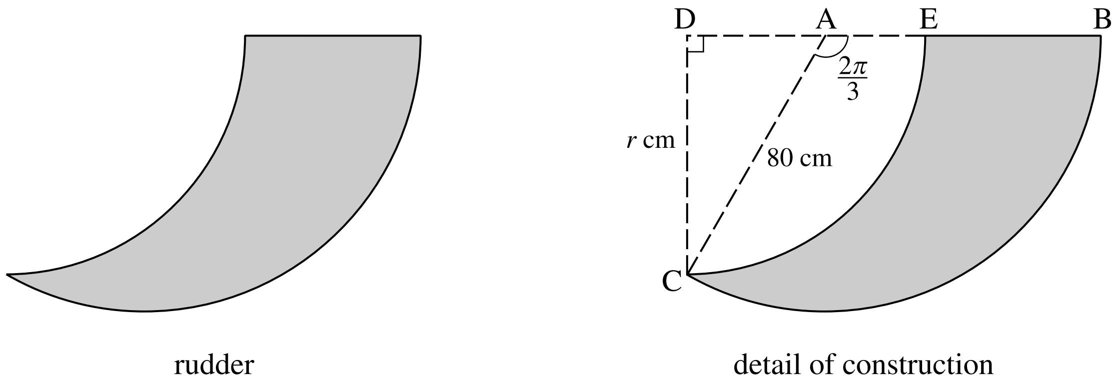

Fig. 11.2

BC is an arc of a circle with centre A and radius 80 cm. Angle CAB = $\frac{2\pi}{3}$ radians.

EC is an arc of a circle with centre D and radius r cm. Angle CDE is a right angle.

(i) Calculate the area of sector ABC. **[2]**

(ii) Show that $r = 40\sqrt{3}$ and calculate the area of triangle CDA. **[3]**

(iii) Hence calculate the area of cross-section of the rudder. **[3]**

12

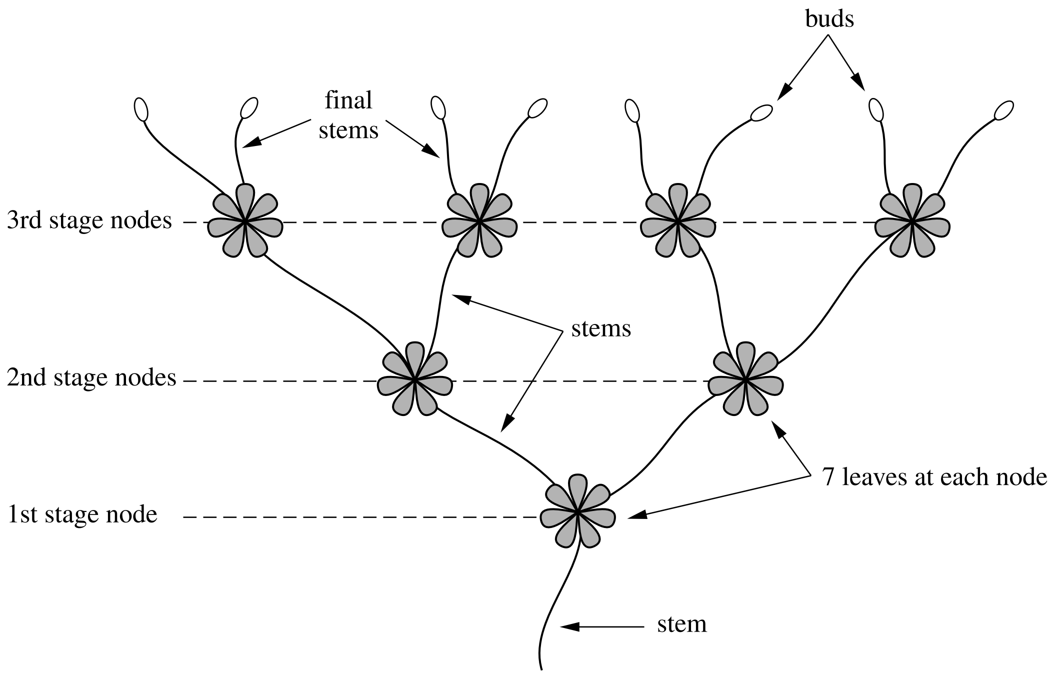

Fig. 12

A branching plant has stems, nodes, leaves and buds.

- There are 7 leaves at each node.
- From each node, 2 new stems grow.
- At the end of each final stem, there is a bud.

Fig. 12 shows one such plant with 3 stages of nodes. It has 15 stems, 7 nodes, 49 leaves and 8 buds.

(i) One of these plants has 10 stages of nodes.

(*A*) How many buds does it have? **[2]**

(*B*) How many stems does it have? **[2]**

(ii) (*A*) Show that the number of leaves on one of these plants with n stages of nodes is

$$7(2^n - 1).$$ **[2]**

(*B*) One of these plants has n stages of nodes and more than 200 000 leaves. Show that n satisfies the inequality $n > \dfrac{\log_{10} 200\,007 - \log_{10} 7}{\log_{10} 2}$. Hence find the least possible value of n. **[4]**

GCE

Mathematics (MEI)

Advanced GCE **4752**

Concepts for Advanced Mathematics (C2)

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

SECTION A

1	$[1], \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$	**2**	**B1** for $[1], \frac{1}{2}, \frac{1}{3}$
2 (i)	$2\frac{1}{12}$ or $\frac{25}{12}$ or 2.08(3…)	**2**	**M1** for $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$
2 (ii)	$\sum_{r=2}^{6} r(r+1)$ o.e.	**2**	**M1** for [f(r) =] $r(r + 1)$ o.e. **M1** for [a =] 6
3 (i)	$3x^2 - 12x - 15$	**2**	**M1** if one term incorrect or an extra term is included.
3 (ii)	Their $\frac{dy}{dx} = 0$ s.o.i. $x = 5$ $x = -1$	**M1** **B1** **B1**	
4	crossing x-axis at 0 and 2.5 min at (1.25, −6.25) crossing x-axis at 0 and 5 min at (2.5, −18.75)	**1** **1** **1** **1**	
5	$x - \frac{6x^{-2}}{-2}$ o.e. their $[5 + \frac{3}{25}] - [2 + \frac{3}{4}]$ = 2.37 o.e. c.a.o.	**2** **M1** **A1**	M1 for 1 term correct Dependent on at least **M1** already earned i.s.w.
6	attempt to integrate $6x^2 + 12x^{\frac{1}{2}}$ [y =] $2x^3 + 8x^{1.5}$ + c Substitution of (4, 10) [y =] $2x^3 + 8a^{1.5}$ −182 or c = −182	**M1** **A2** **M1** **A1**	accept un-simplified; **A1** for 2 terms correct dependent on attempted integral with + c term
7	3.5 $\log_a x$ or k = 3.5	**2**	**B1** for 3 $\log_a x$ or ½ $\log_a x$ or $\log_a x^{3½}$ seen

8	Subst. of $1-\cos^2\theta$ or $1-\sin^2\theta$ $5\cos^2\theta = 1$ or $5\sin^2\theta = 4$ $\cos\theta = \pm\sqrt{\text{their } \frac{1}{5}}$ or $\sin\theta = \pm\sqrt{\text{their } \frac{4}{5}}$ o.e. 63.4, 116.6, 243.4, 296.6	**M1** **A1** **M1** **B2**	 Accept to nearest degree or better; **B1** for 2 correct (ignore any extra values in range).
9	log 18 = log a + n log 3 <u>and</u> log 6 = log a + n log 2 log 18 − log 6 = n (log 3 − log 2) n = 2.71 to 2 d.p. c.a.o. log 6 = log a + 2.70951…log 2 o.e. a = 0.92 to 2 d.p. c.a.o.	 **M1*** **DM1** **A1** **M1** **A1**	or $18 = a \times 3^n$ <u>and</u> $6 = a \times 2^n$ $3 = \left(\frac{3}{2}\right)^n$ $n = \frac{\log 3}{\log 1.5} = 2.71$ c.a.o. $6 = a \times 2^{2.70951}$ o.e. = 0.92 c.a.o.

Section A Total: 36

SECTION B

10 (i)	$\frac{dy}{dx} = 4x^3$ when $x = 2$, $\frac{dy}{dx} = 32$ s.o.i. when $x = 2$, $y = 16$ s.o.i. $y = 32x - 48$ c.a.o.	**M1** **A1** **B1** **A1**	 i.s.w.
10 (ii)	34.481	**2**	**M1** for $\frac{2.1^4 - 2^4}{0.1}$
10 (iii) (*A*)	$16 + 32h + 24h^2 + 8h^3 + h^4$ c.a.o.	**3**	**B2** for 4 terms correct **B1** for 3 terms correct
10 (iii) (*B*)	$32 + 24h + 8h^2 + h^3$ or ft	**2**	**B1** if one error
10 (iii) (*C*)	as $h \to 0$, result $\to$ their 32 from (iii) (B) gradient of tangent is limit of gradient of chord	**1** **1**	

11	(a)	$10.6^2 + 9.2^2 - 2\times10.6\times9.2\times\cos68°$ o.e. QR = 11.1(3…) $\frac{\sin 68}{\text{their QR}} = \frac{\sin Q}{9.2}$ or $\frac{\sin R}{10.6}$ o.e. Q = 50.01..° or R = 61.98..° bearing = 174.9 to 175°	**M1** **A1** **M1** **A1** **B1**	 Or correct use of Cosine Rule 2 s.f. or better
11	(b) (i)	(*A*) ½ × 80^2 × $\frac{2\pi}{3}$ $= \frac{6400\pi}{3}$	**M1** **A1**	 6702.(…) to 2 s.f. or more
11	(b) (ii)	DC = $80\sin(\frac{\pi}{3}) = 80\frac{\sqrt{3}}{2}$ Area = ½×their DA×$40\sqrt{3}$ or ½×$40\sqrt{3}$×80×sin(their DCA) o.e. area of triangle = $800\sqrt{3}$ or 1385.64… to 3s.f. or more	**B1** **M1** **A1**	both steps required s.o.i.
11	(b) (iii)	area of ¼ circle = ½ × $\frac{\pi}{2}$ × $(40\sqrt{3})^2$ o.e. "6702" + "1385.6" – "3769.9" = 4300 to 4320	**M1** **M1** **A1**	[=3769.9…] i.e. their(b) (i) + their (b) (ii) – their ¼ circle o.e. $933\frac{1}{3}\pi + 800\sqrt{3}$

12	**(i) (*A*)**	1024	**2**	**M1** for number of buds = 2^{10} s.o.i.
12	**(i) (*B*)**	2047	**2**	**M1** for 1+2+4+…2^{10} or for $2^{11} - 1$ or (their 1024) + 512 + 256 +…+ 1
12	**(ii) (*A*)**	no. of nodes = 1 + 2 +..+ 2^{n-1} s.o.i. $\frac{7\times(2^n-1)}{2-1}$	**1** **1**	no. of leaves = 7 + 14 +…+ $7\times2^{n-1}$
12	**(ii) (*B*)**	$7(2^n - 1) > 200\,000$ $2^n > \frac{200\,000}{7} + 1$ or $\frac{200\,007}{7}$ $n \log 2 > \log(\frac{200\,007}{7})$ and completion to given ans [n =] 15 c.a.o.	**M1** **M1** **M1** **B1**	or $\log 7 + \log 2^n > \log 200\,007$

Section B Total: 36