MEI CORE 2 MAY 2012

1.) $y = \sqrt{x} + \frac{3}{x}$

$y = x^{\frac{1}{2}} + 3x^{-1}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 3x^{-2}$

or $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} - \frac{3}{x^2}$

2) $u_1 = 5, \quad u_{n+1} = u_n + 3$

$u_2 = 5 + 3 = 8$

$u_3 = 8 + 3 = 11$

AP $a = 5, \quad d = 3$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{50} = \frac{50}{2}(10 + 49 \times 3)$

$S_{50} = 3{,}925$

3) Cosine Rule

i) $AD^2 = 9.8^2 + 6.4^2 - 2 \times 9.8 \times 6.4 \cos 53.4°$

$AD^2 = 62.20955$

$AD = 7.9$ cm to 2 s.f.

ii) $\angle ACB = 180 - 53.4 = 126.6°$

Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 9.8 \times 7.3 \sin 126.6°$

$= 28.7 \text{ cm}^2$

4) $P(6,3)$ on $y = f(x)$

i) $y = 3f(x) \quad (6,3) \mapsto (6,9)$

ii) $y = f(4x) \quad (6,3) \longmapsto (\frac{3}{2}, 3)$

5)

$A = \frac{1}{2}r^2\theta \quad \Rightarrow r^2 = \frac{2A}{\theta}$

$r = \sqrt{\frac{2A}{\theta}}$

$r = \sqrt{\frac{2 \times 45}{1.6}} = 7.5$ cm

$r = 7.5$ cm

Perimeter $=$ arc $+ 2 \times r$

$= r\theta + 2r$

$= 7.5 \times 1.6 + 2 \times 7.5$

$= 27$ cm

6) $\log y$ against $\log x$ a straight line

$\Rightarrow \quad y = ax^b$

$\log y = \log(ax^b)$

$\log y = \log a + \log x^b$

$\log y = \log a + b\log x$

Gradient of line $= b = \frac{17-5}{5-1} = 3$

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6 cont so $\log y = \log a + 3\log x$

$(1,5)$ on line so

$$5 = \log a + 3\times 1$$

$$\Rightarrow \log a = 2 \Rightarrow a = 10^2$$

$$a = 100$$

$$\therefore \quad y = ax^b = 100x^3$$

7) $$\frac{dy}{dx} = 6x^{\frac{1}{2}} - 5$$

$$\Rightarrow y = \frac{6x^{3/2}}{3/2} - 5x + c$$

$$y = 4x^{3/2} - 5x + c$$

Thro $(4, 20)$

$$20 = 4\times 4^{3/2} - 5(4) + c$$

$$20 = 32 - 20 + c$$

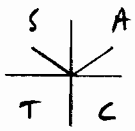
$$20 = 12 + c$$

$$\Rightarrow c = 8$$

$$y = 4x^{3/2} - 5x + 8$$

8) $$\sin 2\theta = 0.7$$

$$2\theta = \sin^{-1} 0.7$$



$$2\theta = 0.7754$$
$$2\theta = \pi - 0.7754$$
$$2\theta = 2\pi + 0.7754$$
$$2\theta = 3\pi - 0.7754$$

$\therefore$
$\theta = 0.388$ radians
$\theta = 1.18$ radians
$\theta = 3.53$ radians
$\theta = 4.32$ radians

9) i) $$A \approx \frac{h}{2}\left[y_0 + 2(y_1+y_2+y_3+y_4+y_5)+y_6\right]$$

$$A \approx \frac{0.2}{2}\left[0 + 2(0.5+0.7+0.75+0.7+0.5)+0\right]$$

$$A \approx 0.63 \text{ m}^2$$

$$V = A\times \text{length} = 0.63\times 50$$

$$V = 31.5 \text{ m}^3$$

ii) $$y = 3.8x^4 - 6.8x^3 + 7.7x^2 - 4.2x$$

A) $$y = 3.8\times 0.2^4 - 6.8\times 0.2^3 + 7.7\times 0.2^2 - 4.2\times 0.2$$

$$y = -0.58032$$

Model depth 0.58032 m

Difference $0.6 - 0.58032$ m

Model is 0.01968 m less deep than actual ditch

B) $$\int_0^{0.9}\left(3.8x^4 - 6.8x^3 + 7.7x^2 - 4.2x\right)dx$$

$$= \left[\frac{3.8}{5}x^5 - \frac{6.8}{4}x^4 + \frac{7.7}{3}x^3 - \frac{4.2x^2}{2}\right]_0^{0.9}$$

$$= \frac{3.8\times 0.9^5}{5} - \frac{6.8\times 0.9^4}{4} + \frac{7.7\times 0.9^3}{3} - \frac{4.2\times 0.9^2}{2}$$

$$- 0$$

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9 ii B) cont

$= -0.4965$

$\therefore$ Area $= 0.4965\ m^2$

$V = 0.4965 \times 50$

$V = 24.8\ m^3$ to 3 s.f.

10) $y = x^3 - 5x$

i) $\frac{dy}{dx} = 3x^2 - 5$

At t.p. $\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 5 = 0$

$3x^2 = 5$

$x^2 = 5/3$

$x = \sqrt{5/3} = \pm 1.29 = \pm 1.3$ to 1 dp.

when $x = 1.29$

$y = 1.29^3 - 5 \times 1.29 = -4.3$

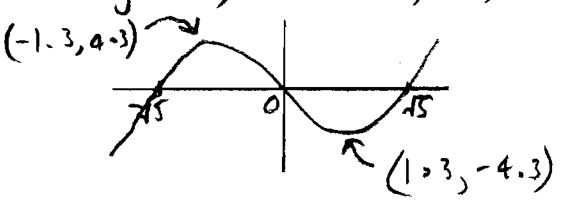
when $x = -1.29$, $y = 4.3$

Turning points to 1 d.p

$(1.3, -4.3)$, $(-1.3, 4.3)$

ii) $y = x^3 - 5x = x(x^2 - 5)$

When $y = 0$, $x = 0, \sqrt{5}, -\sqrt{5}$



iii) when $x = 1$, $\frac{dy}{dx} = 3(1)^2 - 5 = -2$

Point $(1, -4)$

$y - y_1 = m(x - x_1)$

$y - -4 = -2(x - 1)$

$y + 4 = -2x + 2$

$y = -2x - 2$ is tgt

Meets curve when

$-2x - 2 = x^3 - 5x$

$0 = x^3 - 5x + 2x + 2$

$0 = x^3 - 3x + 2$

$$\begin{array}{r|l} & x^2 + x - 2 \\ \hline x - 1 & x^3 \quad\quad - 3x + 2 \\ & x^3 - x^2 \\ \hline & x^2 - 3x \\ & x^2 - x \\ \hline & -2x + 2 \\ & -2x + 2 \\ \hline \end{array}$$

$0 = (x-1)(x^2 + x - 2)$

$0 = (x-1)(x-1)(x+2)$

Meets curve again when

$x = 2$

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11)

i) ① $ar = 6$ 2nd term

② $\dfrac{a}{1-r} = 25$ S_∞

If $a = 10$, from ①

$10r = 6 \Rightarrow r = 0.6$

Sub in ② $\dfrac{10}{1-0.6} = \dfrac{10}{0.4} = 25$ ✓

$\therefore$ $a = 10$, $r = 0.4$

is a solution

Alternatively $r = \dfrac{6}{a}$

Sub in ②

$$\frac{a}{1 - 6/a} = 25$$

$$\frac{a^2}{a-6} = 25$$

$$a^2 = 25(a-6)$$

$$a^2 - 25a + 150 = 0$$

$$(a-10)(a-15) = 0$$

$\Rightarrow$ $a = 10$ or $a = 15$

When $a = 15$, $r = \dfrac{6}{15} = 0.4$

$a = 15$, $r = 0.4$

is the other solution

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ii) n^{th} term $= ar^{n-1}$

$$10 \times 0.6^{n-1} : 15 \times 0.4^{n-1}$$

$$= 2 \times 0.6^{n-1} : 3 \times 0.4^{n-1}$$

$$= 2 \times (3 \times 0.2)^{n-1} : 3 \times (2 \times 0.2)^{n-1}$$

$$= 2 \times 3^{n-1} \times 0.2^{n-1} : 3 \times 2^{n-1} \times 0.2^{n-1}$$

$$= 2 \times 3^{n-1} : 3 \times 2^{n-1}$$

$$= 3^{n-2} : 2^{n-2}$$

or $2^{n-2} : 3^{n-2}$