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1) i)

$$y = 2x^{-5}$$
$$\frac{dy}{dx} = -10x^{-6}$$

ii)

$$y = \sqrt[3]{x} = x^{\frac{1}{3}}$$
$$\frac{dy}{dx} = \frac{1}{3}x^{-2/3}$$

2)

$$u_n = 12 - \tfrac{1}{2}n$$

i)

$$u_1 = 12 - \tfrac{1}{2}\times 1 = 11\tfrac{1}{2}$$
$$u_2 = 12 - \tfrac{1}{2}\times 2 = 11$$
$$u_3 = 12 - \tfrac{1}{2}\times 3 = 10\tfrac{1}{2}$$

Arithmetic sequence

ii)

$$a = 11.5$$
$$d = -0.5$$
$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$
$$S_{30} = \frac{30}{2}\left(2\times 11.5 + 29(-0.5)\right)$$
$$S_{30} = 127.5$$

3)

$$\frac{dy}{dx} = \frac{18}{x^3} + 2$$

Passes through $(3, 6)$

$$\frac{dy}{dx} = 18x^{-3} + 2$$
$$\Rightarrow y = \frac{18x^{-2}}{-2} + 2x + c$$
$$y = -\frac{9}{x^2} + 2x + c$$

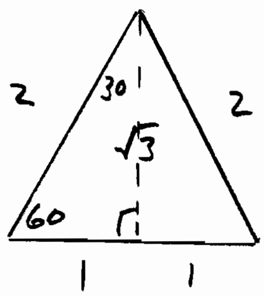
Subst $(3, 6)$

$$6 = -\frac{9}{3^2} + 2\times 3 + c$$
$$6 = -1 + 6 + c$$
$$6 - 6 + 1 = c$$
$$c = 1$$

Curve is

$$y = -\frac{9}{x^2} + 2x + 1$$

4) i)



By Pythagoras height $= \sqrt{2^2 - 1^2}$
$= \sqrt{3}$

$$\cos 30^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$$

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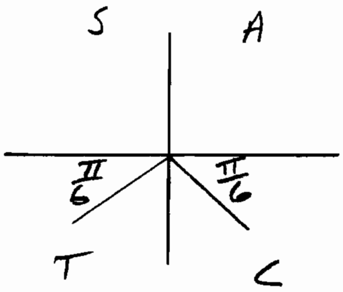
4 ii)

$$2\sin\theta = -1$$

$$\sin\theta = -\tfrac{1}{2}$$

$$\sin^{-1}\tfrac{1}{2} = \frac{\pi}{6}$$



$$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

or $$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

5) See next page

6)

$$S = \frac{a}{1-r}$$

i) For new sequence

$$\text{Sum to } \infty = \frac{2a}{1-r} = 25$$

ii) For third sequence

$$\text{Sum to } \infty = \frac{a}{1-r^2}$$

$$= \frac{a}{(1+r)(1-r)}$$

$$= \frac{S}{1+r}$$

7)

x_n	y_n
0	2.3
1.5	2.9
3	4.0
4.5	4.6
6	4.2
7.5	3.0
9	0.0

$$A \approx \frac{h}{2}\left[y_0 + 2(y_1 + \dots + y_5) + y_6\right]$$

$$= \frac{1.5}{2}\left[2.3 + 2(2.9 + 4.0 + 4.6 + 4.2 + 3.0) + 0.0\right]$$

$$A \approx 29.775 \text{ units}^2$$

$$A \approx 29.8 \text{ units}^2$$

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5)

i)

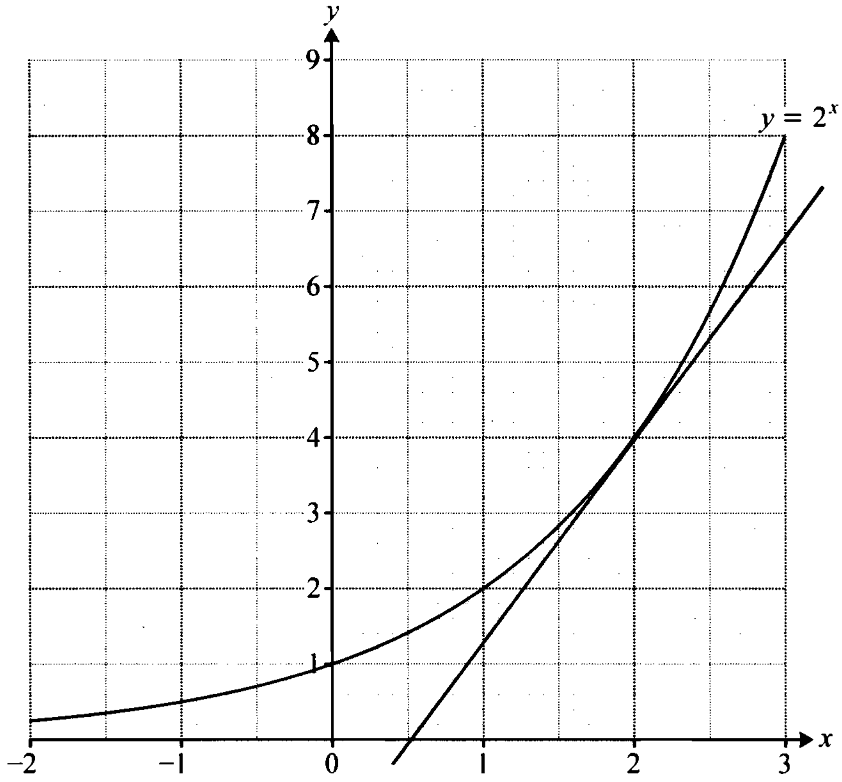

Fig. 5

Gradient of tangent

$= \frac{6.6 - 0}{3 - 0.5}$

$= 2.64$

Estimate of gradient of $y = 2^x$ when $x = 2$ is

2.64

ii) When $x = 1.8$, $y = 2^{1.8} = 3.4822$

when $x = 2.2$ $y = 2^{2.2} = 4.5948$

Gradient of chord between these two points

$= \frac{4.5948 - 3.4822}{2.2 - 1.8} = 2.7815$

Estimate of gradient of curve at $x = 2$ is

2.78

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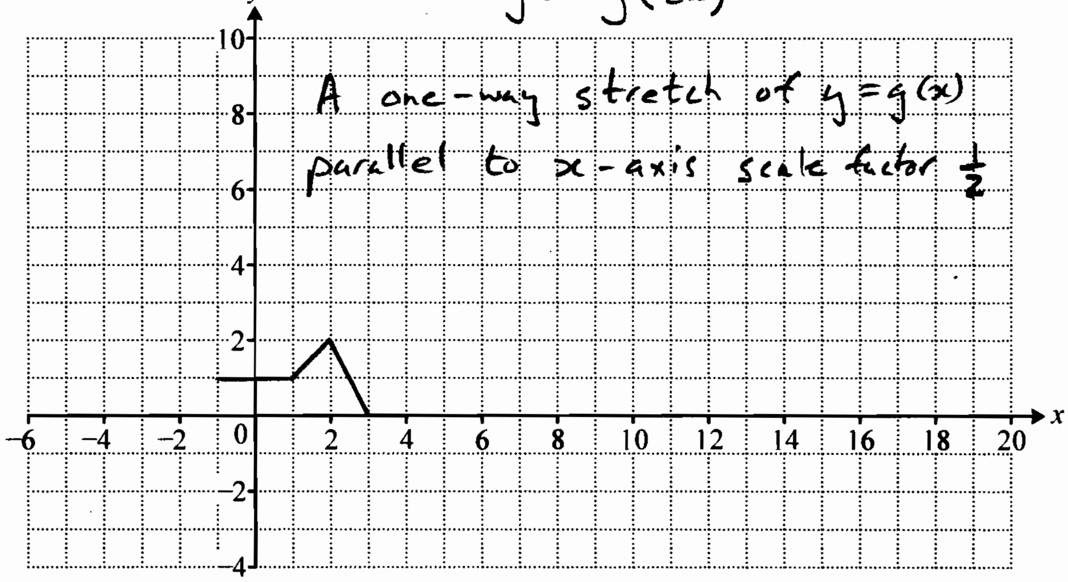
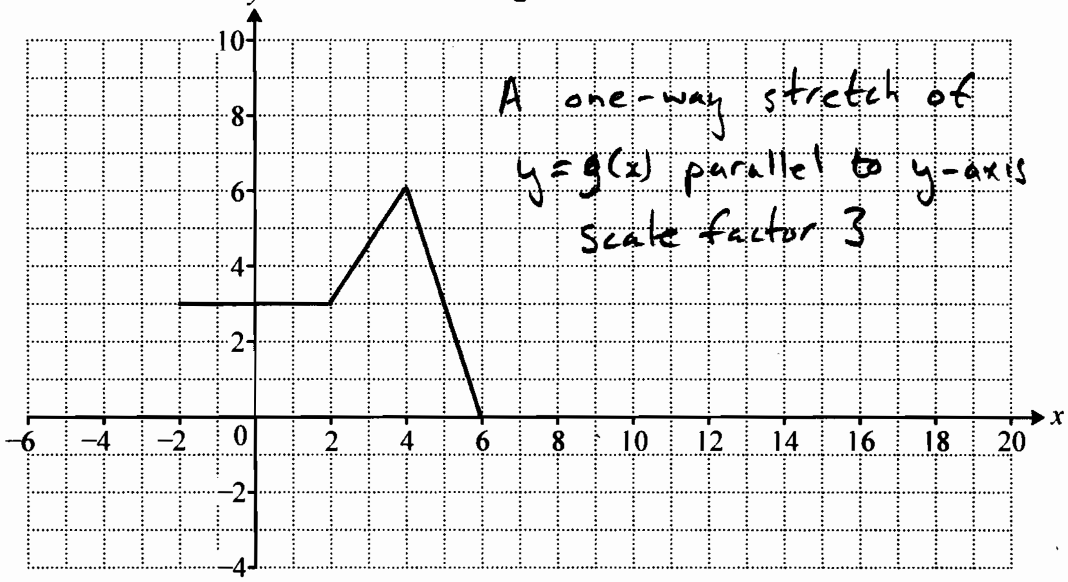
8 i)

$y = g(2x)$



8 ii)

$y = 3g(x)$



Note: The descriptions of the transformations were not asked for.

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Section B

9) i)

$y = x^3 - 3x^2 - 22x + 24$

$\frac{dy}{dx} = 3x^2 - 6x - 22$

At t.p. $\frac{dy}{dx} = 0$

$3x^2 - 6x - 22 = 0$

$x = \frac{6 \pm \sqrt{36 + 4 \times 3 \times 22}}{6}$

$x = \frac{6 \pm \sqrt{300}}{6}$

$x = 3.89, \quad x = -1.89$

9 ii) Solve

$x^3 - 3x^2 - 22x + 24 = 6x + 24$

$x^3 - 3x^2 - 28x = 0$

$x(x^2 - 3x - 28) = 0$

$x(x+4)(x-7) = 0$

$x = 0$
or $x = -4$
or $x = 7$

$x = 7$ at other point of intersection

9 iii)

$A = \int_{-4}^{0} \left((x^3 - 3x^2 - 22x + 24) - (6x + 24) \right) dx$

$= \int_{-4}^{0} (x^3 - 3x^2 - 28x)\, dx$

$= \left[\frac{x^4}{4} - \frac{3x^3}{3} - \frac{28x^2}{2} \right]_{-4}^{0}$

$= \left[\frac{x^4}{4} - x^3 - 14x^2 \right]_{-4}^{0}$

$= 0 - \left(\frac{(-4)^4}{4} - (-4)^3 - 14(-4)^2 \right)$

$= 0 - (64 + 64 - 224)$

$= 96 \text{ units}^2$

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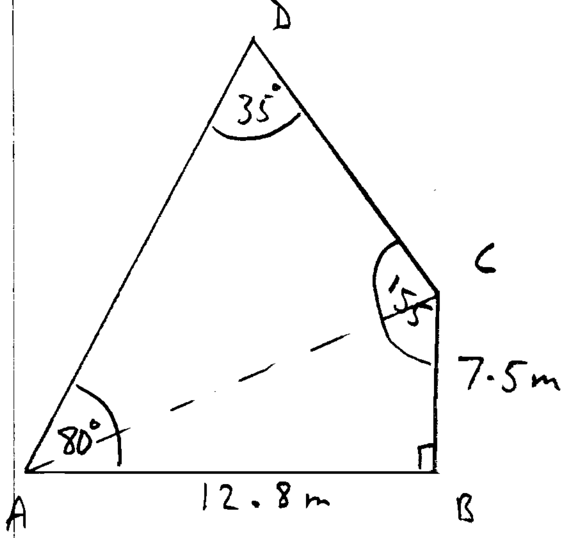
i)



A) $AC^2 = 7.5^2 + 12.8^2$ (Pythagoras)

$= AC = \sqrt{7.5^2 + 12.8^2}$

$AC = 14.8$ m

$\tan(\angle ACB) = \frac{12.8}{7.5}$

$\angle ACB = \tan^{-1}\left(\frac{12.8}{7.5}\right)$

$\angle ACB = 59.6°$

$\angle ACD = 155° - 59.6°$

$\angle ACD = 95.4°$

Sine Rule

$\frac{AC}{\sin 35°} = \frac{AD}{\sin 95.4°}$

$\frac{AC}{\sin 35°} \times \sin 95.4° = AD$

$AD = \frac{14.8}{\sin 35°} \times \sin 95.4°$

$AD = 25.7$ m

B) Area of garden = sum of areas of Δs

Area of $\Delta ABC = \frac{1}{2} \times 12.8 \times 7.5$

$= 48 \text{ m}^2$

$\angle CAD = 180 - (35 + 95.4)$

$= 49.6°$

Using Area of $\Delta = \frac{1}{2}ab\sin C$

Area of ΔACD

$= \frac{1}{2} \times AD \times AC \times \sin(\angle CAD)$

$= \frac{1}{2} \times 25.7 \times 14.8 \times \sin 49.6°$

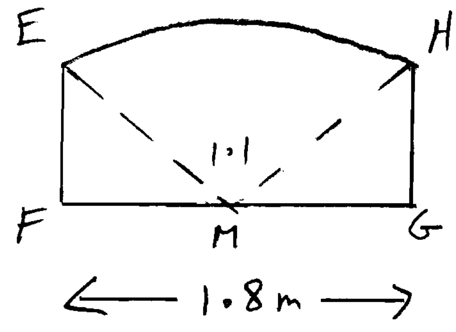
$= 144.8 \text{ m}^2$

Area of garden $= 144.8 + 48$

$= 192.8 \text{ m}^2$

$= 193 \text{ m}^2$ to 3 s.f.

10ii)



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10ii) cont)

$\angle FME = \frac{\pi - 1.1}{2} = 1.0208$

$\tan 1.0208 = \frac{EF}{FM} = \frac{EF}{0.9}$

$\Rightarrow EF = 0.9 \tan 1.0208$

$EF = 1.468$ m

Area of both triangles together

$= 2 \times \frac{1}{2} \times 0.9 \times 1.468$

$= 1.3212 \text{ m}^2$

Area of sector

$= \frac{1}{2} r^2 \theta$

Now $r^2 = EM^2$

$= 0.9^2 + 1.468^2$

so Area of sector

$= \frac{1}{2}(0.9^2 + 1.468^2) \times 1.1$

$= 1.6308 \text{ m}^2$

Area of panel

$= 1.6308 + 1.3212$

$= 2.952 \text{ m}^2$

$= 2.95 \text{ m}^2$ to 3 s.f.

11 i) After 3 mins

$d = 65 \times 0.983^3$

$= 61.741$

$= 61.7$ to 3 s.f.

11 ii) $d = 65 \times 0.983^n$

11 iii) If $d < 3$

$65 \times 0.983^n < 3$

$\Rightarrow \log_{10}(65 \times 0.983^n) < \log_{10} 3$

$\Rightarrow \log_{10} 65 + \log_{10} 0.983^n < \log_{10} 3$

$\Rightarrow n \log_{10} 0.983 < \log_{10} 3 - \log_{10} 65$

$\Rightarrow n > \frac{\log_{10} 3 - \log_{10} 65}{\log_{10} 0.983}$

(inequality reversed since we are dividing by a negative number ie $\log_{10} 0.983 < 0$)

$\therefore n > 179.386$

least integer

$n = 180$

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11iv)

$$d = 65 \times 10^{-kt}$$

After 1 minute

$$d = 65 \times 0.983$$

$$d = 63.895$$

$$\therefore \quad 63.895 = 65 \times 10^{-k \times 1}$$

$$\frac{63.895}{65} = 10^{-k}$$

$$\log_{10}\left(\frac{63.895}{65}\right) = -k$$

$$-0.007446 = -k$$

$$k = 0.007446$$

$$\therefore \quad d = 65 \times 10^{-0.007446t}$$

When $t = 25.3$

$$d = 65 \times 10^{-0.007446 \times 25.3}$$

$$d = 42.1^{\circ}$$

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