1) i) $P(2,1)$

ii) For $y = |x|$

when $y = \frac{3}{2}$, $x = \pm \frac{3}{2}$

For $y = |x-2| + 1$

when $y = \frac{3}{2}$

$$\frac{3}{2} = |x-2| + 1$$

$$\frac{1}{2} = |x-2|$$

$$x - 2 = \pm \frac{1}{2}$$

Either $x = +\frac{1}{2} + 2 = \frac{5}{2}$

or $x = -\frac{1}{2} + 2 = \frac{3}{2}$

$\therefore$ graphs intersect at

$$\left(\frac{3}{2}, \frac{3}{2}\right)$$

so y-coord of Q is $1\frac{1}{2}$

2)

$$\int_1^2 x^2 \ln x \, dx$$

Let $u = \ln x$ $\Rightarrow \frac{du}{dx} = \frac{1}{x}$

Let $\frac{dv}{dx} = x^2$ $\Rightarrow v = \frac{x^3}{3}$

Using $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

$$\int_1^2 x^2 \ln x \, dx = \left[\frac{x^3}{3} \ln x\right]_1^2 - \int_1^2 \frac{x^3}{3} \cdot \frac{1}{x} \, dx$$

$$= \left[\frac{x^3}{3} \ln x\right]_1^2 - \int_1^2 \frac{x^2}{3} \, dx$$

$$= \left[\frac{x^3}{3} \ln x\right]_1^2 - \left[\frac{x^3}{9}\right]_1^2$$

$$= \left[\frac{8}{3} \ln 2 - \frac{1}{3} \ln 1\right] - \left[\frac{8}{9} - \frac{1}{9}\right]$$

$$= \frac{8}{3} \ln 2 - \frac{7}{9}$$

3) i)

$$V = A e^{-kt}$$

$V = 10000$ when $t = 0$

$$10000 = A e^0 \Rightarrow A = 10000$$

$V = 6000$ when $t = 3$

$$6000 = 10000 e^{-3k}$$

$$\frac{6000}{10000} = e^{-3k}$$

$$\ln(0.6) = -3k$$

$$k = -\frac{1}{3} \ln 0.6$$

$k = 0.1702752$ to 7 s.f.

3 ii)

$$2000 = 10000 e^{-0.1702752t}$$

$$0.2 = e^{-0.1702752t}$$

$$\ln(0.2) = -0.1702752t$$

$t = -\frac{\ln(0.2)}{0.1702752} = 9.45$ years to 3 s.f.

4) Perfect squares with 1 or 2 digits

0, 1, 4, 9, 16, 25, 36, 49, 64, 81

None of these end in 2, 3, 7 or 8

Any integer squared does not end in 2, 3, 7 or 8

5) i)

$$y = \frac{x^2}{2x+1}$$

$$\frac{dy}{dx} = \frac{(2x+1)2x - x^2(2)}{(2x+1)^2}$$

$$\frac{dy}{dx} = \frac{4x^2 + 2x - 2x^2}{(2x+1)^2}$$

$$\frac{dy}{dx} = \frac{2x^2+2x}{(2x+1)^2}$$

$$\frac{dy}{dx} = \frac{2x(x+1)}{(2x+1)^2}$$

5ii) At st. pt. $\frac{dy}{dx} = 0$

$\Rightarrow \quad 2x(x+1) = 0$

$\Rightarrow \quad x = 0$ or $x = -1$

when $x = 0$, $y = \frac{0}{1} = 0$

when $x = -1$, $y = \frac{(-1)^2}{2(-1)+1}$

$y = \frac{1}{-1} = -1$

Stationary points at $(0, 0)$ and $(-1, -1)$

6) i)

$AQ = 3 - y$

$AP = 6 - AQ$

$\therefore \quad AP = 6 - (3 - y)$

$AP = 3 + y$

By Pythagoras $OP^2 + OA^2 = AP^2$

$\Rightarrow \quad x^2 + 3^2 = (y+3)^2$

6ii)

$(y+3)^2 = x^2 + 9$

$2(y+3)\frac{dy}{dx} = 2x$

$$\frac{dy}{dx} = \frac{2x}{2(y+3)} = \frac{x}{y+3}$$

6iii) When $x = 4$ and $y = 2$, $\frac{dx}{dt} = 2$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

In above case

$$\frac{dy}{dt} = \frac{x}{y+3} \times 2$$

$$= \frac{4}{2+3} \times 2$$

$$= \frac{8}{5}$$

7) i)

$f(x) = x\sqrt{1+x}$

$f(-1) = -1\sqrt{1-1} = -1 \times 0 = 0$

$\therefore P(-1, 0)$

Domain $x \geq -1$

7ii)

$y = x\sqrt{1+x}$

$\frac{dy}{dx} = x\frac{1}{2}(1+x)^{-\frac{1}{2}} + (1+x)^{\frac{1}{2}} \times 1$

$= \frac{x}{2\sqrt{1+x}} + \sqrt{1+x}$

$= \frac{x + 2(1+x)}{2\sqrt{1+x}}$

$\frac{dy}{dx} = \frac{2+3x}{2\sqrt{1+x}}$

7iii)

At t.p. $\frac{dy}{dx} = 0 \Rightarrow 2+3x=0$

$x = -\frac{2}{3}$

When $x = -\frac{2}{3}$ $y = -\frac{2}{3}\sqrt{\frac{1}{3}}$

Turning point is $\left(-\frac{2}{3}, -\frac{2}{3\sqrt{3}}\right)$

Range $f(x) \geq -\frac{2}{3\sqrt{3}}$

7iv)

$\int_{-1}^{0} x\sqrt{1+x}\,dx$

Let $u = 1+x$ then $\frac{du}{dx} = 1$

$du = dx$ and $x = u-1$

when $x = -1$, $u = 0$
$x = 0$, $u = 1$

$\int_{-1}^{0} x\sqrt{1+x}\,dx = \int_{0}^{1} (u-1)u^{\frac{1}{2}}\,du$

$= \int_{0}^{1} \left(u^{3/2} - u^{1/2}\right) du$

$= \left[\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right]_0^1$

$= \left[\frac{2u^{5/2}}{5} - \frac{2u^{3/2}}{3}\right]_0^1$

$= \left(\frac{2}{5} - \frac{2}{3}\right) - (0 - 0)$

$= \frac{6}{15} - \frac{10}{15} = -\frac{4}{15}$

Area $= \frac{4}{15}$ units^2

Minus sign indicates that area is below x-axis.

8i) $f(x) = (e^x - 1)^2$ for $x \geq 0$

$f'(x) = 2(e^x - 1)e^x$

$f'(0) = 2(e^0 - 1)e^0 = 0$

$f'(\ln 2) = 2(e^{\ln 2} - 1)e^{\ln 2}$

$= 2(2-1)2 = 4$

gradient at $(0,0) = 0$

gradient at $(\ln 2, 1) = 4$

8ii)

$g(x) = \ln(1 + \sqrt{x})$ for $x \geq 0$

Let $y = (e^x - 1)^2$

Swap variables

$x = (e^y - 1)^2$

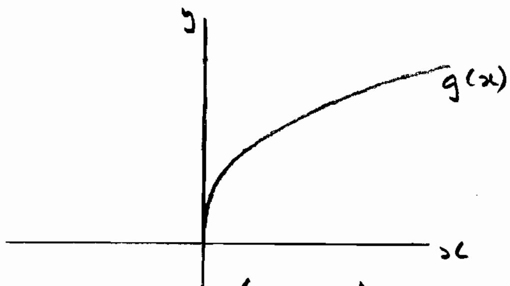
Make y the subject

$\sqrt{x} = e^y - 1$

$\sqrt{x} + 1 = e^y$

$\ln(\sqrt{x} + 1) = y$

$\therefore f^{-1}(x) = \ln(1 + \sqrt{x}) = g(x)$



Gradient at $(1, \ln 2) = \frac{1}{4}$

(since gradient of $f(x)$ at $(\ln 2, 1) = 4$)

8iii) $\int (e^x - 1)^2 dx$

$= \int (e^{2x} - 2e^x + 1)\, dx$

$= \frac{1}{2}e^{2x} - 2e^x + x + c$

$\int_0^{\ln 2} (e^x - 1)^2$

$= \left[\frac{1}{2}e^{2x} - 2e^x + x\right]_0^{\ln 2}$

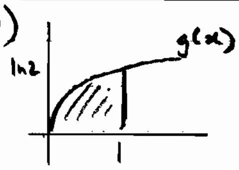
$= \left(\frac{1}{2}e^{2\ln 2} - 2e^{\ln 2} + \ln 2\right) - \left(\frac{1}{2}e^0 - 2e^0 + 0\right)$

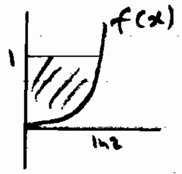
$= \left(\frac{1}{2}e^{\ln 4} - 4 + \ln 2\right) - \left(\frac{1}{2} - 2\right)$

$= (2 - 4 + \ln 2) + \frac{3}{2}$

$= -\frac{1}{2} + \ln 2$

8iv)





Shaded areas are equal

For $y = f(x)$ the shaded area

$= \text{rectangle}(1 \times \ln 2) - \int_0^{\ln 2} f(x)\, dx$

$= \ln 2 - \left(-\frac{1}{2} + \ln 2\right)$

$= \frac{1}{2}$