1)

$$\frac{d}{dx}\sqrt[3]{1+6x^2}$$

$$= \frac{d}{dx}\left(1+6x^2\right)^{\frac{1}{3}}$$

$$= \frac{1}{3}\left(1+6x^2\right)^{-\frac{2}{3}} \times 12x$$

$$= 4x\left(1+6x^2\right)^{-2/3}$$

2)

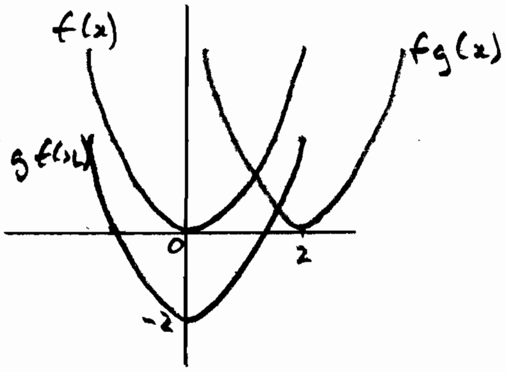
$$f(x) = x^2, \quad g(x) = x-2$$

i)

$$fg(x) = f(x-2) = (x-2)^2$$

$$gf(x) = g(x^2) = x^2 - 2$$

ii)



3)

$$P = Ae^{bn}$$

$P = 10000$ when $n = 1$
$P = 16000$ when $n = 2$

$$10000 = Ae^{b} \quad 1$$

$$16000 = Ae^{2b} \quad 2$$

$2 \div 1$

$$1\cdot 6 = \frac{Ae^{2b}}{Ae^{b}} = e^{b}$$

Subst for e^b in 1

$$10000 = A \times 1.6$$

$$\frac{10000}{1.6} = A$$

$$\Rightarrow A = 6250$$

Since $e^b = 1.6$

$$b = \ln 1.6$$

$$\Rightarrow b = 0.470 \text{ to 3 s.f.}$$

3ii)

$$P = Ae^{bn}$$

$$P = 6250e^{0\cdot 470n}$$

When $n = 20$

$$P = 6250e^{0\cdot 470 \times 20}$$

$$P = 75,552,379.56$$

$$P = £75.6\text{m to 3 s.f.}$$

4)

$$P = \frac{k}{V}$$

When $V = 100\,m^3$, $P = 5$ atmos

and $\frac{dV}{dt} = 10\,m^3s^{-1}$

i) Substituting gives

$$5 = \frac{k}{100}$$

$$\Rightarrow k = 500$$

4ii)

$$P = \frac{500}{V}$$

$$\frac{dP}{dV} = -\frac{500}{V^2}$$

4iii) Find $\frac{dP}{dt}$ when $V = 100$

$$\frac{dP}{dt} = \frac{dP}{dV} \cdot \frac{dV}{dt}$$

$$\frac{dP}{dt} = -\frac{500}{V^2} \times 10$$

$$= \frac{-500}{100^2} \times 10$$

$= -0.5$ atmospheres/second

5)

i) $2^2 - 1 = 3$

$2^3 - 1 = 7$

$2^5 - 1 = 31$

$2^7 - 1 = 127$

All prime, ∴ true for all prime numbers less than 11

ii) $23 \times 89 = 2047$

$= 2^{11} - 1$

∴ $2^p - 1$ is not prime when $p = 11$

It is therefore not true for all prime numbers p

6) $e^{2y} = x^2 + y$

i) Differentiate with respect to x

$$2e^{2y}\frac{dy}{dx} = 2x + \frac{dy}{dx}$$

$$2e^{2y}\frac{dy}{dx} - \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}\left(2e^{2y} - 1\right) = 2x$$

$$\frac{dy}{dx} = \frac{2x}{2e^{2y} - 1}$$

ii) Infinite gradient when

$$2e^{2y} - 1 = 0$$

$$\Rightarrow 2e^{2y} = 1$$

$$e^{2y} = \tfrac{1}{2}$$

$$2y = \ln\left(\tfrac{1}{2}\right)$$

$$y = \tfrac{1}{2}\ln\left(\tfrac{1}{2}\right)$$

$$y = -0.34657$$

From original eqn $x^2 = e^{2y} - y$

$$x = \pm\sqrt{e^{2y} - y}$$

$$x = \pm\sqrt{e^{2\times -0.34657} + 0.34657}$$

$$x = \pm 0.92009$$

To 3 s.f. P is point

$$(0.920, -0.347)$$

7) $y = 2x\ln(1+x)$

i) $\frac{dy}{dx} = 2x \times \frac{1}{(1+x)} + 2\ln(1+x)$

$\frac{dy}{dx} = \frac{2x}{1+x} + 2\ln(1+x)$

When $x = 0$

$\frac{dy}{dx} = \frac{0}{1} + 2\ln 1 = 0$

Also when $x = 0$ $y = 2\times 0 \ln 1 = 0$

$\therefore$ origin is a st. pt. on curve

ii) $\frac{d^2y}{dx^2} = \frac{2(1+x) - 2x}{(1+x)^2} + \frac{2}{1+x}$

When $x = 0$

$\frac{d^2y}{dx^2} = \frac{2-0}{1^2} + \frac{2}{1} = 4$

Since $\frac{d^2y}{dx^2} > 0$

st. pt at $(0,0)$ is a min.

iii) $\int \frac{x^2}{1+x}\,dx$

Let $u = 1+x$

$\frac{du}{dx} = 1$

$du = dx$

Also $x = u - 1$

$\int \frac{x^2}{1+x}\,dx = \int \frac{(u-1)^2}{u}\,du$

$= \int \frac{u^2 - 2u + 1}{u}\,du$

$= \int \left(u - 2 + \frac{1}{u}\right) du$

Find $\int_0^1 \frac{x^2}{1+x}\,dx$

when $x = 1$, $u = 2$
when $x = 0$, $u = 1$

$= \int_1^2 \left(u - 2 + \frac{1}{u}\right) du$

$= \left[\frac{u^2}{2} - 2u + \ln u\right]_1^2$

$= \left(\frac{4}{2} - 4 + \ln 2\right) - \left(\frac{1}{2} - 2 + \ln 1\right)$

$= (-2 + \ln 2) - \left(-\frac{3}{2} + 0\right)$

$= -2 + \ln 2 + \frac{3}{2}$

$= \ln 2 - \frac{1}{2}$

iv) $\int_0^1 2x\ln(1+x)\,dx$

Let $u = \ln(1+x)$ $\Rightarrow \frac{du}{dx} = \frac{1}{1+x}$

Let $\frac{dv}{dx} = 2x$ $\Rightarrow v = x^2$

7iv) cont

$$\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$$

$$\int_0^1 2x\ln(1+x)\,dx$$

$$= \left[x^2\ln(1+x)\right]_0^1 - \int_0^1 \frac{x^2}{1+x}\,dx$$

$$= \left[1^2\ln 2 - 0\right] - \left[\ln 2 - \tfrac{1}{2}\right]$$

$$= \ln 2 - \ln 2 + \tfrac{1}{2}$$

$$= \tfrac{1}{2}$$

8)

$$f(x) = 1 + \sin 2x \qquad -\tfrac{\pi}{4} \le x \le \tfrac{\pi}{4}$$

i) Translation by $\begin{pmatrix}0\\1\end{pmatrix}$ and

a stretch by a scale factor of $\frac{1}{2}$ parallel to the x axis. These could be in either order

ii)

$$\text{Area} = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (1 + \sin 2x)\,dx$$

$$= \left[x - \tfrac{1}{2}\cos 2x\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= \left(\tfrac{\pi}{4} - \tfrac{1}{2}\cos\tfrac{\pi}{2}\right) - \left(-\tfrac{\pi}{4} - \tfrac{1}{2}\cos\left(-\tfrac{\pi}{2}\right)\right)$$

$$= \left(\tfrac{\pi}{4} - 0\right) - \left(-\tfrac{\pi}{4} - 0\right)$$

$$= \tfrac{\pi}{2}$$

iii) $f'(x) = 2\cos 2x$

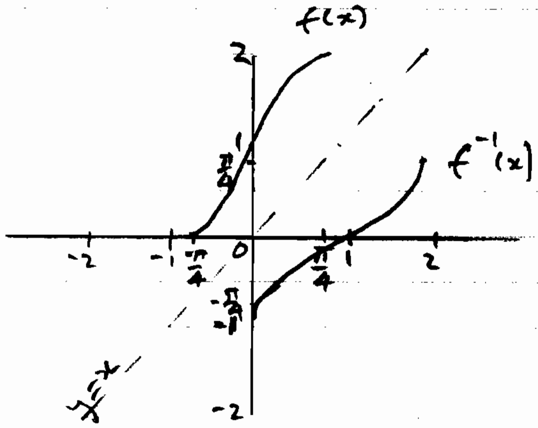
when $x = 0$, $f'(x) = 2\cos 0 = 2$

Gradient at $(0,1) = 2$

Gradient of $f^{-1}(x)$ at $(1,0) = \frac{1}{2}$

iv) Domain of $f^{-1}(x)$ given by

$$0 \le x \le 2$$



v) Let $y = 1 + \sin 2x$

Swap variables $x = 1 + \sin 2y$

Make y the subject

$$x - 1 = \sin 2y$$

$$\sin^{-1}(x-1) = 2y$$

$$y = \tfrac{1}{2}\sin^{-1}(x-1)$$

$$f^{-1}(x) = \tfrac{1}{2}\sin^{-1}(x-1)$$