1)

$$|3x-2| = x$$

Either $3x - 2 = x$
$3x - x = 2$
$2x = 2$
$x = 1$

or $-3x + 2 = x$
$-3x - x = -2$
$-4x = -2$
$x = \frac{1}{2}$

2)

$$\int_0^{\frac{\pi}{6}} x \sin 2x \, dx$$

Let $u = x$ $\Rightarrow \frac{du}{dx} = 1$

Let $\frac{dv}{dx} = \sin 2x$ $\Rightarrow v = -\frac{1}{2}\cos 2x$

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

$$\int_0^{\frac{\pi}{6}} x \sin 2x \, dx$$

$$= \left[-\frac{1}{2} x \cos 2x\right]_0^{\frac{\pi}{6}} + \int_0^{\pi/6} \frac{1}{2}\cos 2x \, dx$$

$$= \left[-\frac{1}{2} x \cos 2x\right]_0^{\frac{\pi}{6}} + \left[\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{6}}$$

$$= \left(-\frac{\pi}{12}\cos\frac{\pi}{3} - -0\right) + \left(\frac{1}{4}\sin\frac{\pi}{3} - 0\right)$$

$$= -\frac{\pi}{24} + \frac{\sqrt{3}}{8}$$

$$= -\frac{\pi}{24} + \frac{3\sqrt{3}}{24}$$

$$= \frac{3\sqrt{3} - \pi}{24}$$

3)

i) $y = \sin^{-1}(x-1)$ $\quad 0 \leq x \leq 2$

$\sin y = x - 1$

$x = 1 + \sin y$

$\frac{dx}{dy} = \cos y$

3 ii) when $x = 1.5$

$y = \sin^{-1}(1.5 - 1)$

$y = \sin^{-1}(0.5) = \frac{\pi}{6}$

At point $\left(1.5, \frac{\pi}{6}\right)$

$\frac{dx}{dy} = \cos y = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\therefore \frac{dy}{dx} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

4)

$$V = \frac{1}{3}\pi h^2(3-h)$$

i) $V = \pi h^2 - \frac{1}{3}\pi h^3$

$\frac{dV}{dh} = 2\pi h - \pi h^2$

$= \pi h(2-h)$

ii)

$\frac{dV}{dt} = 0.02 \text{ m}^3 \text{ min}^{-1}$

4ii) cont)

$$\frac{dh}{dt} = \frac{dh}{dV}\frac{dV}{dt}$$

$$= \frac{dV}{dt} \Big/ \frac{dV}{dh}$$

$$\frac{dh}{dt} = \frac{0.02}{\pi h(2-h)}$$

when $h = 0.4$

$$\frac{dh}{dt} = \frac{0.02}{\pi \times 0.4(1.6)}$$

$$\frac{dh}{dt} = 0.00995 \text{ m per min}$$

to 3 s.f.

5)

i) $(2t)^2 + (t^2-1)^2$

$= 4t^2 + t^4 - 2t^2 + 1$

$= t^4 + 2t^2 + 1 = (t^2+1)^2$

$\therefore$ $2t$, t^2-1 and t^2+1 form a Pythagorean triple for an integer $t > 1$

5ii)

$20^2 + 21^2 = 841$

$\sqrt{841} = 29$

Third integer $= 29$

20, 21, 29

Since $(t^2+1) - (t^2-1) = 2$

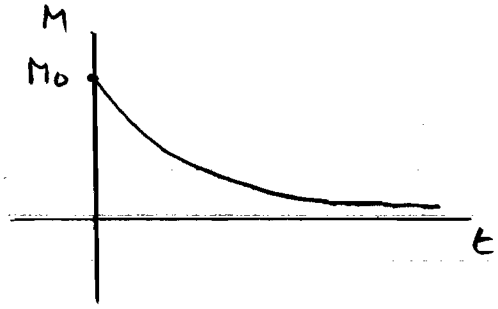
triples of the type in part (i) must have two integers a distance of 2 apart.

20, 21, 29

do not have two integers a distance 2 apart and so represent a Pythagorean triple which cannot be written in that form.

6)

i) $M = M_0 e^{-kt}$



ii)

$M = M_0 e^{-0.000121 \times 5730}$

$M = M_0 \times 0.4999$

$\therefore$ $M \approx \frac{1}{2} M_0$

iii) For half-life T

$M = \frac{1}{2} M_0$

$\Rightarrow$ $e^{-kT} = \frac{1}{2}$

$\ln(e^{-kT}) = \ln(\frac{1}{2})$

$-kT = -\ln 2$

$T = \frac{-\ln 2}{-k}$

$T = \frac{\ln 2}{k}$

6 iv) $T = \dfrac{\ln 2}{2.88 \times 10^{-5}}$

$= 24068$ years

7) i) $y = \dfrac{x^2+3}{x-1}$

Asymptote $x = 1$

ii) $\dfrac{dy}{dx} = \dfrac{(x-1)\times 2x - (x^2+3)\times 1}{(x-1)^2}$

$\dfrac{dy}{dx} = \dfrac{2x^2-2x-x^2-3}{(x-1)^2}$

$\dfrac{dy}{dx} = \dfrac{x^2-2x-3}{(x-1)^2}$

$\dfrac{dy}{dx} = \dfrac{(x-3)(x+1)}{(x-1)^2}$

At t.p. $\dfrac{dy}{dx} = 0$

$\Rightarrow x = 3$ or $x = -1$

From graph P is point where $x=3$

$y = \dfrac{3^2+3}{3-1} = \dfrac{12}{2} = 6$

P is point $(3, 6)$

iii) Area $= \displaystyle\int_2^3 y\,dx$

$= \displaystyle\int_2^3 \frac{x^2+3}{x-1}\,dx$

Let $u = x-1$ then $x = u+1$

$\dfrac{du}{dx} = 1$

$du = dx$

When $x = 3$, $u = 2$
when $x = 2$, $u = 1$

$\displaystyle\int_2^3 \frac{x^2+3}{x-1}\,dx = \int_1^2 \left(\frac{(u+1)^2+3}{u}\right) du$

$= \displaystyle\int_1^2 \left(\frac{u^2+2u+1+3}{u}\right) du$

$= \displaystyle\int_1^2 \left(u + 2 + \frac{4}{u}\right) du$

$= \left[\dfrac{u^2}{2} + 2u + 4\ln u\right]_1^2$

$= \left(\dfrac{2^2}{2} + 2(2) + 4\ln 2\right) - \left(\dfrac{1}{2} + 2 + 0\right)$

$= 2 + 4 + 4\ln 2 - \dfrac{5}{2}$

$= \dfrac{7}{2} + 4\ln 2$

iv) $e^y = \dfrac{x^2+3}{x-1}$

$e^y \dfrac{dy}{dx} = \dfrac{(x-3)(x+1)}{(x-1)^2}$

$\dfrac{dy}{dx} = \dfrac{(x-3)(x+1)}{e^y (x-1)^2}$

7 iv cont)

$$\frac{dy}{dx} = \frac{(x-3)(x+1)}{\left(\frac{x^2+3}{x-1}\right)(x-1)^2}$$

$$\frac{dy}{dx} = \frac{(x-3)(x+1)}{(x^2+3)(x-1)}$$

when $x = 2$

$$\frac{dy}{dx} = \frac{(2-3)(2+1)}{(4+3)(2-1)}$$

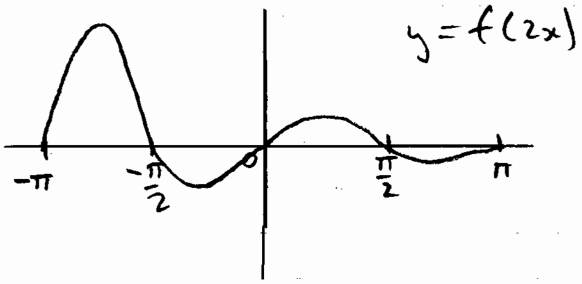
$$= \frac{-1 \times 3}{7 \times 1} = -\frac{3}{7}$$

8)

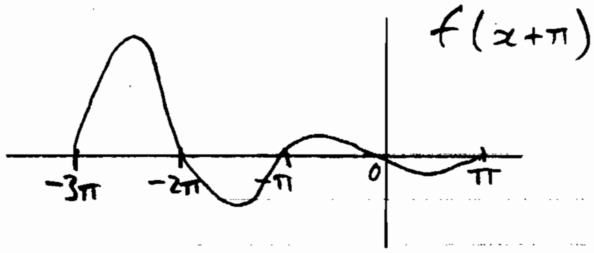
$$y = f(x) = e^{-\frac{x}{5}} \sin x$$

i)

A)



B)



ii)

$$\frac{dy}{dx} = e^{-\frac{x}{5}} \cos x - \frac{1}{5} e^{-\frac{x}{5}} \sin x$$

$$= e^{-\frac{x}{5}}\left(\cos x - \frac{1}{5}\sin x\right)$$

At t.p $\frac{dy}{dx} = 0$

$$\Rightarrow \cos x - \frac{1}{5}\sin x = 0$$

$$\cos x = \frac{1}{5}\sin x$$

$$5\cos x = \sin x$$

$$5 = \frac{\sin x}{\cos x} = \tan x$$

$\therefore \tan x = 5$

$$x = 1.3734$$

$$y = e^{-\frac{1.3734}{5}} \sin(1.3734)$$

$$= 0.7451$$

P is point $(1.373, 0.745)$

iii)

$$f(x+\pi) = e^{-\frac{(x+\pi)}{5}} \sin(x+\pi)$$

$$= e^{-\frac{\pi}{5}} e^{-\frac{x}{5}} \times (-\sin x)$$

$$= -e^{-\frac{\pi}{5}} f(x)$$

Let $u = x - \pi$ then $x = u + \pi$

$$\frac{du}{dx} = 1$$

$$du = dx$$

when $x = 2\pi$, $u = \pi$
when $x = \pi$ $u = 0$

$$\int_{\pi}^{2\pi} f(x)\,dx = \int_0^{\pi} f(u+\pi)\,du$$

$$= -e^{-\frac{\pi}{5}} \int_0^{\pi} f(u)\,du$$

The area enlosed by a loop of the curve and the x-axis is the area of the previous one multiplied by $e^{-\frac{\pi}{5}}$. The minus signs indicate that loops alternate above and below the axis.