1) i)

$$\frac{d}{dx}(1+2x)^{\frac{1}{2}}$$

$$= \frac{1}{2}(1+2x)^{-\frac{1}{2}} \times 2$$

$$= \frac{1}{\sqrt{1+2x}}$$

ii)

$$\frac{d}{dx}\ln(f(x)) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}\ln(1-e^{-x}) = \frac{e^{-x}}{1-e^{-x}}$$

$$= \frac{e^{-x}}{(1-e^{-x})} \times \frac{e^{x}}{e^{x}}$$

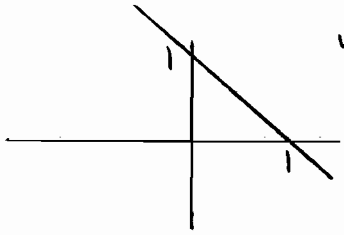
$$= \frac{1}{e^{x}-1}$$

2)

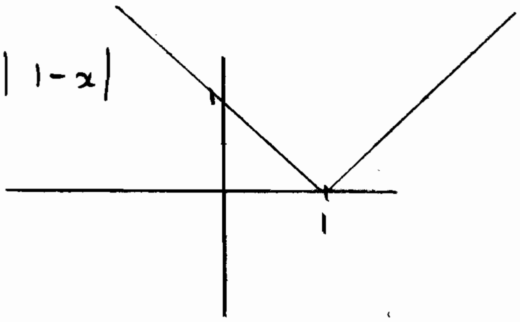
$f(x) = 1-x$, $g(x) = |x|$

$gf(x) = g(1-x) = |1-x|$

$y = 1-x$



$y = |1-x|$



3) i)

$$2y^2 + y = 9x^2 + 1$$

$$(4y+1)\frac{dy}{dx} = 18x$$

$$\frac{dy}{dx} = \frac{18x}{4y+1}$$

At $(1,2)$

$$\frac{dy}{dx} = \frac{18}{8+1} = 2$$

Gradient $= 2$

3 ii)

$$\frac{dy}{dx} = 0 \Rightarrow 18x = 0$$

$$\Rightarrow x = 0$$

$$2y^2 + y = 0 + 1$$

$$2y^2 + y - 1 = 0$$

$$(2y-1)(y+1) = 0$$

$$\Rightarrow y = \tfrac{1}{2} \text{ or } y = -1$$

$\frac{dy}{dx} = 0$ at $(0, \frac{1}{2})$ and $(0,-1)$

4) i)

$$T = 25 + ae^{-kt}$$

Given $T = 100$ when $t = 0$
$T = 80$ when $t = 3$

$$100 = 25 + ae^{0}$$

$$\Rightarrow a = 75$$

$$80 = 25 + 75e^{-3k}$$

$$55 = 75e^{-3k}$$

$$\frac{55}{75} = e^{-3k}$$

4 i) cont

$$\ln\left(\frac{55}{75}\right) = -3k$$

$$k = -\frac{1}{3}\ln\left(\frac{55}{75}\right) = 0.1034$$

Answer

$a = 75$, $k = 0.1034$

4 ii)

A) When $t = 5$

$$T = 25 + 75e^{-0.1034\times 5}$$

$$T = 69.7 \ ^\circ C$$

B) As $t \to \infty$ $T \to 25^\circ C$

5)

Disprove n^2+3n+1 is prime

n	n^2+3n+1	
1	$1+3+1$	$=5$
2	$4+6+1$	$=11$
3	$9+9+1$	$=19$
4	$16+12+1$	$=29$
5	$25+15+1$	$=41$
6	$36+18+1$	$=55$

$55 = 5\times 11$ not prime

$\therefore$ formula does not always produce primes

6)

$$y = f(x) = \frac{1}{2}\tan^{-1}x$$

i)

$$-\frac{\pi}{4} < f(x) < \frac{\pi}{4}$$

ii) $y = \frac{1}{2}\tan^{-1}x$

Swap variables

$$x = \frac{1}{2}\tan^{-1}y$$

$$2x = \tan^{-1}y$$

$$\tan 2x = y$$

Inverse fn $f^{-1}(x) = \tan 2x$

iii)

$$\frac{d}{dx}\tan 2x = 2\sec^2 2x$$

$$= \frac{2}{\cos^2 2x}$$

At origin gradient of $f^{-1}(x)$

$$= \frac{2}{1^2} = 2$$

$\therefore$ gradient of $f(x) = \frac{1}{2}$

7)

$$y = \frac{x^2}{1+2x^3}$$

i)

Asymptote when $1+2x^3=0$

$$2x^3 = -1$$

$$x^3 = -\frac{1}{2}$$

$$x = \sqrt[3]{-\frac{1}{2}}$$

$x = -0.794$ to 3 s.f.

7ii) $\frac{dy}{dx} = \frac{(1+2x^3)2x - x^2(6x^2)}{(1+2x^3)^2}$

$\frac{dy}{dx} = \frac{2x + 4x^4 - 6x^4}{(1+2x^3)^2}$

$\frac{dy}{dx} = \frac{2x - 2x^4}{(1+2x^3)^2}$

At t.p. $\frac{dy}{dx} = 0$

$\Rightarrow 2x - 2x^4 = 0$

$\Rightarrow 2x(1-x^3) = 0$

$\Rightarrow x = 0$ or $x = 1$

when $x = 0$, $y = \frac{0}{1+0} = 0$

when $x = 1$ $y = \frac{1}{1+2} = \frac{1}{3}$

Turning points

$(0,0)$ and $(1, \frac{1}{3})$

7iii) Area $= \int_0^1 y\,dx$

$= \int_0^1 \frac{x^2}{1+2x^3}\,dx$

$= \frac{1}{6}\int_0^1 \frac{6x^2}{1+2x^3}\,dx$

$= \frac{1}{6}\Big[\ln(1+2x^3)\Big]_0^1$

$= \frac{1}{6}\Big[\ln(3) - \ln(1)\Big]$

$= \frac{1}{6}\ln 3$

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8) $y = x\cos 2x$

i) Crosses x axis when

$x\cos 2x = 0$

$\Rightarrow x = 0$ or $\cos 2x = 0$

$\cos(2 \times \frac{\pi}{4}) = 0$

P is point $(\frac{\pi}{4}, 0)$

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ii) Let $f(x) = x\cos 2x$

then $f(-x) = -x\cos(-2x)$

$= -x\cos 2x$

$= -f(x)$

$\therefore f(x)$ is an odd function

Graph has rotational symmetry of order 2 about the origin

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iii) $y = x\cos 2x$

$\frac{dy}{dx} = x(-2\sin 2x) + \cos 2x \times 1$

$= -2x\sin 2x + \cos 2x$

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iv) Turning points when $\frac{dy}{dx} = 0$

8 iv) (cont)

$$\Rightarrow \cos 2x - 2x\sin 2x = 0$$

$$\cos 2x = 2x\sin 2x$$

$$1 = \frac{2x\sin 2x}{\cos 2x}$$

$$1 = 2x\tan 2x$$

$$\frac{1}{2} = x\tan 2x$$

8 v)

When $x = 0$

$$\frac{dy}{dx} = \cos 0 - 2\times 0\times\sin 0$$
$$= 1$$

Gradient at origin $= 1$

$$\frac{dy}{dx} = \cos 2x - 2x\sin 2x$$

$$\frac{d^2y}{dx^2} = -2\sin 2x - \left[2x\times 2\cos 2x + \sin 2x\times 2\right]$$

$$= -2\sin 2x - 4x\cos 2x - 2\sin 2x$$

when $x = 0$

$$\frac{d^2y}{dx^2} = -2\sin 0 - 4\times 0\cos 0 - 2\sin 0$$
$$= 0 - 0 - 0 = 0$$

8 vi)

$$\int_0^{\frac{\pi}{4}} x\cos 2x\,dx$$

Let $u = x \Rightarrow \frac{du}{dx} = 1$

Let $\frac{dv}{dx} = \cos 2x \Rightarrow v = \frac{1}{2}\sin 2x$

$$\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$$

$$\int_0^{\frac{\pi}{4}} x\cos 2x\,dx$$

$$= \left[\frac{1}{2}x\sin 2x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{1}{2}\sin 2x\,dx$$

$$= \left[\frac{1}{2}x\sin 2x\right]_0^{\frac{\pi}{4}} - \left[-\frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{4}}$$

$$= \left[\frac{1}{2}\cdot\frac{\pi}{4}\sin\frac{\pi}{2} - 0\right] - \left[\left(-\frac{1}{4}\cos\frac{\pi}{2}\right) - \left(-\frac{1}{4}\cos 0\right)\right]$$

$$= \frac{\pi}{8} - \left[0 + \frac{1}{4}\right]$$

$$= \frac{\pi}{8} - \frac{1}{4}$$

This is equal to the area under the curve and above the x-axis between 0 and P